



CS 574: Randomized Algorithms Practice Midterm 1

20 questions Suggested time: 60 minutes Copy 1

Choose **exactly one answer** for each question. Each correct answer earns 1 point; incorrect, blank, or multiple answers earn 0 points. Record your answers on the separate answer sheet. Try the questions before consulting the solutions.

Question 1 You want to estimate the expected running time of a randomized algorithm on one particular input I . Which experiment matches this expectation?

- ☐ A Keep only the fastest run on I .
☐ B Run once and use that running time as the exact expectation.
☐ C Use a different input each time and fix all random choices.
☒ D Keep I fixed and run the algorithm repeatedly with fresh random choices.
☐ E Keep only the slowest run on I .

Explanation: For a fixed input the expectation is over the algorithm's random choices. Repeated runs with fresh randomness sample that running-time distribution.

mock-q001 Easy; estimated 1 min. Source: Lecture 1.1.3.

Question 2 An alias table for n labels takes $\Theta(n)$ time to build and $\Theta(1)$ time per sample, under the lecture's random-real and floor model. If it is built once and then used for m samples, what is the tight asymptotic total time, as a function of n and m ?

- ☐ A $\Theta(n^2 + m)$ ☐ B $\Theta(\log n + m)$ ☐ C $\Theta(n + m^2)$ ☒ D $\Theta(n + m)$
☐ E $\Theta(nm)$

Explanation: The one-time construction cost is added to the cost of the m queries: $\Theta(n) + m\Theta(1) = \Theta(n + m)$.

mock-q003 Easy; estimated 1 min. Source: Lecture 2.2.

Question 3 A sampler chooses one of the three rows uniformly. In its chosen row it returns the primary label with probability q , and otherwise the alias.

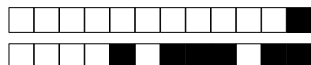
Row	Primary	q	Alias
1	red	$1/2$	blue
2	blue	x	green
3	green	1	—

What value of x makes the overall probability of returning green equal to $1/2$?

- ☐ A $1/4$ ☐ B 0 ☒ C $1/2$ ☐ D $3/4$ ☐ E 1

Explanation: Green comes from row 3 with probability $1/3$ and from row 2 with probability $(1 - x)/3$. Solving $1/3 + (1 - x)/3 = 1/2$ gives $x = 1/2$.

mock-q004 Short application; estimated 3 min. Source: Lecture 2.2; Homework 1B, inverse table query.



Question 4 A fair six-sided die is rolled. Given that the result is even, what is the probability that it is at least 4?

- ☐ A $1/3$ ☒ B $2/3$ ☐ C $1/2$ ☐ D $1/6$ ☐ E $5/6$

Explanation: Conditional on an even result, the equally likely possibilities are 2, 4 and 6. Two of these three possibilities are at least 4.

mock-q005 Easy; estimated 1 min. Source: Lecture 3.1.1–3.1.2.

Question 5 Let X equal -1 or $+1$, each with probability $1/2$. A student writes $\Pr(X \geq 1) \leq \mathbb{E}[X]/1 = 0$ by Markov's inequality. Why is this application invalid?

- ☐ A The threshold must be negative.
☒ B X is not nonnegative.
☐ C The expectation of X is undefined.
☐ D Markov can only be applied to independent sums.
☐ E Markov requires an infinite sample space.

Explanation: The usual first-moment Markov bound requires a nonnegative random variable. Here X can be negative, and the actual event probability is $1/2$.

mock-q007 Easy; estimated 1 min. Source: Lecture 3.4.1.

Question 6 A clause with two literals on two distinct Boolean variables has weight 4, and a clause with three literals on three distinct Boolean variables has weight 8. Each clause is an OR of its literals, and the clauses may share variables. Assign all variables independent fair truth values. What is the expected total weight of the satisfied clauses?

- ☐ A 12 ☒ B 10 ☐ C 11 ☐ D 8 ☐ E 6

Explanation: The clauses are satisfied with probabilities $3/4$ and $7/8$. Weighted linearity gives $4(3/4) + 8(7/8) = 3 + 7 = 10$. Clause independence is not needed.

mock-q009 Short application; estimated 3 min. Source: Lecture 3.3; Homework 2B, weighted version.

Question 7 Four distinct keys arrive in a uniformly random order. Count an arrival whenever its key is larger than every key that arrived earlier; the first arrival always counts. What is the expected total count?

- ☐ A 1 ☐ B $3/2$ ☐ C 4 ☒ D $25/12$ ☐ E $5/2$

Explanation: The i th arrival is the largest among the first i with probability $1/i$. Summing the four indicators gives $1 + 1/2 + 1/3 + 1/4 = 25/12$.

mock-q010 Short application; estimated 3 min. Source: Lecture 3.1.2, indicators and permutation symmetry.



Question 8 A QuickSort partition is applied to 11 distinct keys. Its pivot has sorted rank 7 among those keys. After removing the pivot, what are the sizes of the smaller-key and larger-key parts, respectively?

- ☐ A 4 and 6.
- ☐ B 7 and 3.
- ☐ C 7 and 4.
- ☒ D 6 and 4.
- ☐ E 6 and 5.

Explanation: Ranks 1 through 6 are smaller; ranks 8 through 11 are larger. The pivot belongs to neither recursive part.

mock-q011 Easy; estimated 1 min. Source: Lecture 4.1.

Question 9 Consider this algorithm for finding the minimum of nine distinct keys. Choose a pivot uniformly and compare it with every other key. If no key is smaller, return the pivot. Otherwise discard the pivot and all larger keys and repeat on the smaller keys. A single remaining key is returned immediately. What is the probability that after the first partition the algorithm either stops or recurses on at most five keys?

- ☐ A $1/2$
- ☒ B $2/3$
- ☐ C $5/6$
- ☐ D $1/3$
- ☐ E $5/9$

Explanation: A pivot of rank r leaves $r - 1$ smaller keys, unless $r = 1$ and the algorithm stops. The desired event is $r \leq 6$, covering six of the nine equally likely ranks. Its probability is $6/9 = 2/3$.

mock-q014 Short application; estimated 3 min. Source: Supplied algorithm; Lecture 4.2 for instructor reference.

Question 10 A random variable has mean μ and variance 4. What is the largest lower bound on $\Pr(|X - \mu| < 6)$ obtained by applying Chebyshev's inequality to the complementary event?

- ☐ A $2/3$
- ☐ B 1
- ☐ C $1/9$
- ☒ D $8/9$
- ☐ E 0

Explanation: Chebyshev gives $\Pr(|X - \mu| \geq 6) \leq 4/36 = 1/9$. Taking the complement gives at least $1 - 1/9 = 8/9$.

mock-q015 Easy; estimated 1 min. Source: Lecture 5.1.2.



Question 11 Exactly half the people in a population support a proposal. Select one person uniformly and record their fixed support indicator $X \in \{0, 1\}$. A program copies this same observation m times and reports the average of the copies. What is the variance of the reported average, for any positive integer m ?

- ☐ A $1/8$
☐ B 0
☐ C 1
☐ D $1/2$
☒ E $1/4$

Explanation: The average of m copies of X is still X . Its variance is $(1/2)(1/2) = 1/4$; copying an observation does not provide independent samples.

mock-q016 Easy; estimated 1 min. Source: Lecture 5.1.1–5.2.

Question 12 Take 64 independent measurements, each with mean μ and variance 9, and let $\bar{X}$ be their average. What is the smallest upper bound on $\Pr(|\bar{X} - \mu| \geq 3/4)$ obtained by Chebyshev's inequality using the variance of the average?

- ☐ A 1
☐ B $3/4$
☒ C $1/4$
☐ D $9/64$
☐ E $1/16$

Explanation: Independence gives $\text{Var}(\bar{X}) = 9/64$. Dividing by $(3/4)^2 = 9/16$ gives $(9/64)/(9/16) = 1/4$.

mock-q018 Short application; estimated 3 min. Source: Lecture 5.1.1–5.2.

Question 13 Exactly one quarter of an ordered population lies strictly below a threshold L . Draw nine values independently and uniformly with replacement. Let B count the sampled values strictly below L ; the sample median is the fifth smallest sampled value, counting repetitions. Using Markov's inequality on B and its expectation, what is the smallest resulting upper bound on the probability that the sample median is strictly below L ?

- ☐ A $5/9$
☐ B $1/5$
☐ C $4/9$
☒ D $9/20$
☐ E $1/4$

Explanation: The median is below L exactly when $B \geq 5$. Since $\mathbb{E}[B] = 9/4$, Markov gives $\Pr(B \geq 5) \leq (9/4)/5 = 9/20$. This is a bound, not the exact probability.

mock-q019 Short application; estimated 3 min. Source: Lecture 5.3.1; Markov from Lecture 3.4.1; Homework 4C.



Question 14 A polynomial identity test evaluates a real polynomial h at one randomly chosen point and obtains 0. Which statement is justified by this observation alone?

- ☐ A Every later evaluation will also be zero.
- ☐ B h must be a nonzero polynomial.
- ☐ C h must be the zero polynomial.
- ☐ D h has total degree zero.
- ☒ The observation is compatible with either a zero or a nonzero polynomial.

Explanation: The zero polynomial vanishes everywhere, while a nonzero polynomial can vanish at some points. A single zero evaluation does not distinguish these possibilities.

mock-q021 Easy; estimated 1 min. Source: Lecture 6.1.3.

Question 15 All matrix arithmetic is modulo 2. Suppose

$$AB - C = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}.$$

Choose $r = (r_1, r_2)^T$ with independent fair binary coordinates. What is the probability that the matrix-product check $A(Br) = Cr$ passes?

- ☐ A 3/4 ☐ B 0 ☐ C 1/4 ☐ D 1 ☒ 1/2

Explanation: The difference times r is $(r_1, r_1)^T$, which is zero exactly when $r_1 = 0$. The two row conditions are identical, so their probabilities cannot be multiplied as if they were independent.

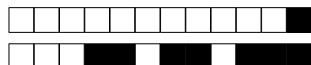
mock-q023 Short application; estimated 3 min. Source: Lecture 6.1.2.

Question 16 A collection has seven equally likely coupon types, with independent draws. You currently hold three distinct types. What is the probability that the next coupon is a type you already hold?

- ☒ 3/7 ☐ B 1/3 ☐ C 1/7 ☐ D 4/7 ☐ E 3/4

Explanation: Three of the seven equally likely types are already in the collection. Their total probability is 3/7.

mock-q024 Easy; estimated 1 min. Source: Lecture 7.4.



Question 17 Place three balls independently and uniformly into four bins. What is the probability that exactly two balls share one bin and the third ball is in a different bin?

- ☐ A $3/16$
☒ B $9/16$
☐ C $3/8$
☐ D $1/16$
☐ E $3/4$

Explanation: Choose the pair in three ways. For a specified pair the two balls share a bin with probability $1/4$, and the third avoids that bin with probability $3/4$. The three cases are disjoint, giving $3(1/4)(3/4) = 9/16$.

mock-q026 Short application; estimated 3 min. Source: Lecture 7.2–7.3.

Question 18 Starting with no coupons, draw independently and uniformly from four coupon types. What is the expected number of draws until you have two distinct types, including the draw that supplies the second type?

- ☐ A $4/3$
☐ B $8/3$
☐ C 4
☐ D 2
☒ E $7/3$

Explanation: The first draw always supplies the first type. Each subsequent draw supplies a new type with probability $3/4$, so the additional expected wait is $4/3$. The total is $1 + 4/3 = 7/3$.

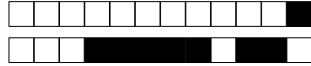
mock-q027 Short application; estimated 3 min. Source: Lecture 7.4.

Question 19 Let X, Y, Z be pairwise independent fair bits. Which statement is *not* guaranteed by these assumptions?

- ☐ A $\text{Var}(X + Y + Z) = 3/4$.
☒ B All eight possible triples are equally likely.
☐ C $\Pr(X = 0, Y = 1) = 1/4$.
☐ D $\mathbb{E}[X + Y + Z] = 3/2$.
☐ E Each bit has expectation $1/2$.

Explanation: Pairwise independence determines all pair distributions and makes variances add, but it need not give a uniform joint distribution on all eight triples. The mean follows from linearity alone.

mock-q028 Easy; estimated 1 min. Source: Lecture 8.1.1; variance from Lecture 5.1.1.



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Question 20 Choose (X, Y, Z) uniformly from the four triples

$(0, 0, 1), (0, 1, 0), (1, 0, 0), (1, 1, 1).$

Given that $X = 1$ and $Y = 1$, what is the conditional probability that $Z = 1$?

☐ 1 ☐ $1/2$ ☐ 0 ☐ $3/4$ ☐ $1/4$

Explanation: Only the triple $(1, 1, 1)$ is consistent with both observations, so the conditional probability is 1. Each pair of bits is independent, but observing two bits together can completely determine the third.

mock-q030 Short application; estimated 3 min. Source: Lecture 8.1.1–8.1.2, explicit example only; conditional probability from Lecture 3.1.2.



Practice Midterm 1 — Answer Sheet

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Fill in one square per question completely with a dark mark.

- 1: ☐ A ☐ B ☐ C ☒ ☐ E
- 2: ☐ A ☐ B ☐ C ☒ ☐ E
- 3: ☐ A ☐ B ☒ ☐ D ☐ E
- 4: ☐ A ☒ ☐ C ☐ D ☐ E
- 5: ☐ A ☒ ☐ C ☐ D ☐ E
- 6: ☐ A ☒ ☐ C ☐ D ☐ E
- 7: ☐ A ☐ B ☐ C ☒ ☐ E
- 8: ☐ A ☐ B ☐ C ☒ ☐ E
- 9: ☐ A ☒ ☐ C ☐ D ☐ E
- 10: ☐ A ☐ B ☐ C ☒ ☐ E

- 11: ☐ A ☐ B ☐ C ☐ D ☒
- 12: ☐ A ☐ B ☒ ☐ D ☐ E
- 13: ☐ A ☐ B ☐ C ☒ ☐ E
- 14: ☐ A ☐ B ☐ C ☐ D ☒
- 15: ☐ A ☐ B ☐ C ☐ D ☒
- 16: ☒ ☐ B ☐ C ☐ D ☐ E
- 17: ☐ A ☒ ☐ C ☐ D ☐ E
- 18: ☐ A ☐ B ☐ C ☐ D ☒
- 19: ☐ A ☒ ☐ C ☐ D ☐ E
- 20: ☒ ☐ B ☐ C ☐ D ☐ E