

CS 574: Randomized Algorithms Practice Midterm 1

20 questions Suggested time: 60 minutes Copy 1

Choose **exactly one answer** for each question. Each correct answer earns 1 point; incorrect, blank, or multiple answers earn 0 points. Record your answers on the separate answer sheet. Try the questions before consulting the solutions.

Question 1 You want to estimate the expected running time of a randomized algorithm on one particular input I . Which experiment matches this expectation?

- ☐ A Keep only the fastest run on I .
- ☐ B Run once and use that running time as the exact expectation.
- ☐ C Use a different input each time and fix all random choices.
- ☐ D Keep I fixed and run the algorithm repeatedly with fresh random choices.
- ☐ E Keep only the slowest run on I .

Question 2 An alias table for n labels takes $\Theta(n)$ time to build and $\Theta(1)$ time per sample, under the lecture's random-real and floor model. If it is built once and then used for m samples, what is the tight asymptotic total time, as a function of n and m ?

- ☐ A $\Theta(n^2 + m)$
- ☐ B $\Theta(\log n + m)$
- ☐ C $\Theta(n + m^2)$
- ☐ D $\Theta(n + m)$
- ☐ E $\Theta(nm)$

Question 3 A sampler chooses one of the three rows uniformly. In its chosen row it returns the primary label with probability q , and otherwise the alias.

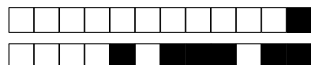
Row	Primary	q	Alias
1	red	1/2	blue
2	blue	x	green
3	green	1	—

What value of x makes the overall probability of returning green equal to $1/2$?

- ☐ A $1/4$
- ☐ B 0
- ☐ C $1/2$
- ☐ D $3/4$
- ☐ E 1

Question 4 A fair six-sided die is rolled. Given that the result is even, what is the probability that it is at least 4?

- ☐ A $1/3$
- ☐ B $2/3$
- ☐ C $1/2$
- ☐ D $1/6$
- ☐ E $5/6$



Question 5 Let X equal -1 or $+1$, each with probability $1/2$. A student writes $\Pr(X \geq 1) \leq \mathbb{E}[X]/1 = 0$ by Markov's inequality. Why is this application invalid?

- ☐ A The threshold must be negative.
- ☐ B X is not nonnegative.
- ☐ C The expectation of X is undefined.
- ☐ D Markov can only be applied to independent sums.
- ☐ E Markov requires an infinite sample space.

Question 6 A clause with two literals on two distinct Boolean variables has weight 4, and a clause with three literals on three distinct Boolean variables has weight 8. Each clause is an OR of its literals, and the clauses may share variables. Assign all variables independent fair truth values. What is the expected total weight of the satisfied clauses?

- ☐ A 12 ☐ B 10 ☐ C 11 ☐ D 8 ☐ E 6

Question 7 Four distinct keys arrive in a uniformly random order. Count an arrival whenever its key is larger than every key that arrived earlier; the first arrival always counts. What is the expected total count?

- ☐ A 1 ☐ B $3/2$ ☐ C 4 ☐ D $25/12$ ☐ E $5/2$

Question 8 A QuickSort partition is applied to 11 distinct keys. Its pivot has sorted rank 7 among those keys. After removing the pivot, what are the sizes of the smaller-key and larger-key parts, respectively?

- ☐ A 4 and 6.
- ☐ B 7 and 3.
- ☐ C 7 and 4.
- ☐ D 6 and 4.
- ☐ E 6 and 5.

Question 9 Consider this algorithm for finding the minimum of nine distinct keys. Choose a pivot uniformly and compare it with every other key. If no key is smaller, return the pivot. Otherwise discard the pivot and all larger keys and repeat on the smaller keys. A single remaining key is returned immediately. What is the probability that after the first partition the algorithm either stops or recurses on at most five keys?

- ☐ A $1/2$ ☐ B $2/3$ ☐ C $5/6$ ☐ D $1/3$ ☐ E $5/9$



Question 10 A random variable has mean μ and variance 4. What is the largest lower bound on $\Pr(|X - \mu| < 6)$ obtained by applying Chebyshev's inequality to the complementary event?

- ☐ A $2/3$ ☐ B 1 ☐ C $1/9$ ☐ D $8/9$ ☐ E 0

Question 11 Exactly half the people in a population support a proposal. Select one person uniformly and record their fixed support indicator $X \in \{0, 1\}$. A program copies this same observation m times and reports the average of the copies. What is the variance of the reported average, for any positive integer m ?

- ☐ A $1/8$ ☐ B 0 ☐ C 1 ☐ D $1/2$ ☐ E $1/4$

Question 12 Take 64 independent measurements, each with mean μ and variance 9, and let $\bar{X}$ be their average. What is the smallest upper bound on $\Pr(|\bar{X} - \mu| \geq 3/4)$ obtained by Chebyshev's inequality using the variance of the average?

- ☐ A 1 ☐ B $3/4$ ☐ C $1/4$ ☐ D $9/64$ ☐ E $1/16$

Question 13 Exactly one quarter of an ordered population lies strictly below a threshold L . Draw nine values independently and uniformly with replacement. Let B count the sampled values strictly below L ; the sample median is the fifth smallest sampled value, counting repetitions. Using Markov's inequality on B and its expectation, what is the smallest resulting upper bound on the probability that the sample median is strictly below L ?

- ☐ A $5/9$ ☐ B $1/5$ ☐ C $4/9$ ☐ D $9/20$ ☐ E $1/4$

Question 14 A polynomial identity test evaluates a real polynomial h at one randomly chosen point and obtains 0. Which statement is justified by this observation alone?

- ☐ A Every later evaluation will also be zero.
☐ B h must be a nonzero polynomial.
☐ C h must be the zero polynomial.
☐ D h has total degree zero.
☐ E The observation is compatible with either a zero or a nonzero polynomial.



Question 15 All matrix arithmetic is modulo 2. Suppose

$$AB - C = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}.$$

Choose $r = (r_1, r_2)^T$ with independent fair binary coordinates. What is the probability that the matrix-product check $A(Br) = Cr$ passes?

- ☐ A 3/4 ☐ B 0 ☐ C 1/4 ☐ D 1 ☐ E 1/2

Question 16 A collection has seven equally likely coupon types, with independent draws. You currently hold three distinct types. What is the probability that the next coupon is a type you already hold?

- ☐ A 3/7 ☐ B 1/3 ☐ C 1/7 ☐ D 4/7 ☐ E 3/4

Question 17 Place three balls independently and uniformly into four bins. What is the probability that exactly two balls share one bin and the third ball is in a different bin?

- ☐ A 3/16 ☐ B 9/16 ☐ C 3/8 ☐ D 1/16 ☐ E 3/4

Question 18 Starting with no coupons, draw independently and uniformly from four coupon types. What is the expected number of draws until you have two distinct types, including the draw that supplies the second type?

- ☐ A 4/3 ☐ B 8/3 ☐ C 4 ☐ D 2 ☐ E 7/3

Question 19 Let X, Y, Z be pairwise independent fair bits. Which statement is *not* guaranteed by these assumptions?

- ☐ A $\text{Var}(X + Y + Z) = 3/4$.
☐ B All eight possible triples are equally likely.
☐ C $\Pr(X = 0, Y = 1) = 1/4$.
☐ D $\mathbb{E}[X + Y + Z] = 3/2$.
☐ E Each bit has expectation $1/2$.

Question 20 Choose (X, Y, Z) uniformly from the four triples

$$(0, 0, 1), \quad (0, 1, 0), \quad (1, 0, 0), \quad (1, 1, 1).$$

Given that $X = 1$ and $Y = 1$, what is the conditional probability that $Z = 1$?

- ☐ A 1 ☐ B 1/2 ☐ C 0 ☐ D 3/4 ☐ E 1/4



Practice Midterm 1 — Answer Sheet

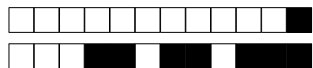
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Fill in one square per question completely with a dark mark.

- 1: ☐ A ☐ B ☐ C ☐ D ☐ E
- 2: ☐ A ☐ B ☐ C ☐ D ☐ E
- 3: ☐ A ☐ B ☐ C ☐ D ☐ E
- 4: ☐ A ☐ B ☐ C ☐ D ☐ E
- 5: ☐ A ☐ B ☐ C ☐ D ☐ E
- 6: ☐ A ☐ B ☐ C ☐ D ☐ E
- 7: ☐ A ☐ B ☐ C ☐ D ☐ E
- 8: ☐ A ☐ B ☐ C ☐ D ☐ E
- 9: ☐ A ☐ B ☐ C ☐ D ☐ E
- 10: ☐ A ☐ B ☐ C ☐ D ☐ E

- 11: ☐ A ☐ B ☐ C ☐ D ☐ E
- 12: ☐ A ☐ B ☐ C ☐ D ☐ E
- 13: ☐ A ☐ B ☐ C ☐ D ☐ E
- 14: ☐ A ☐ B ☐ C ☐ D ☐ E
- 15: ☐ A ☐ B ☐ C ☐ D ☐ E
- 16: ☐ A ☐ B ☐ C ☐ D ☐ E
- 17: ☐ A ☐ B ☐ C ☐ D ☐ E
- 18: ☐ A ☐ B ☐ C ☐ D ☐ E
- 19: ☐ A ☐ B ☐ C ☐ D ☐ E
- 20: ☐ A ☐ B ☐ C ☐ D ☐ E



+1/6/55+