CS 573: Algorithms, Fall 2014
Homework 0, due Tuesday, September 2, 11:00:00, 2014

Name:	
Net ID:	

Neatly print your name (first name first, with no comma), your network ID, and a short alias into the boxes above. **Do not sign your name. Do not write your Social Security number.** Staple this sheet of paper to the top of your homework.

This homework tests your familiarity with the prerequisite material from CS 173, CS 225, and CS 373—many of these problems have appeared on homeworks or exams in those classes—primarily to help you identify gaps in your knowledge. **You are responsible for filling those gaps on your own.** Chapters 1–6 of CLR should be sufficient review, but you may want to consult other texts as well.

Hard copy of homeworks should be submitted in Box 49 on the lower level in Siebel. Also, submit electronically a copy to moodle.

Before you do anything else, read the Homework Instructions and FAQ on the CS 573 course web page (`http://courses.engr.illinois.edu/cs573/fa2014/faq/`), and then check the box below. This web page gives instructions on how to write and submit homeworks—staple your solutions together in order, write your name and netID on every page, don't turn in source code, analyze everything, use good English and good logic, and so forth.

☐ I have read the CS 573 Homework Instructions and FAQ.

Remember to do the quiz on moodle!

Version: 1.06

> "Be that as it may, it is to night school that I owe what education I possess; I am the first to own that it doesn't amount to much, though there is something rather grandiose about the gaps in it." – The tin drum, Gunter Grass

Required Problems

1. The numbers dance. (60 PTS.)

(A) (30 PTS.) The input is a multiset X of n positive integer numbers. Consider the following famous algorithm:

PlayItBen(X) :
  while X contains more than two elements **do**
    two distinct elements x_1, x_2 are chosen arbitrarily from X,
      such that $x_1 \leq x_2$
    if $x_1 = x_2$ or $x_1 + 1 = x_2$ **then**
      $X \leftarrow (X \setminus \{x_1, x_2\}) \cup \{x_1 + x_2\}$
    else
      $X \leftarrow (X \setminus \{x_1, x_2\}) \cup \{x_1 + 1, x_2 - 1\}$

Prove (maybe using induction, but you do not have to) that **PlayItBen** always terminates.

(B) (30 PTS.) (Harder.) Let $N = \sum_{x \in X} x$, and let $n = |X|$. Provide an upper bound, as tight as possible, using n and N on the running time of **PlayItBen**.

2. Random walk. (40 PTS.)

A *random walk* is a walk on a graph G, generated by starting from a vertex $v_0 = v \in V(G)$, and in the i-th stage, for $i > 0$, randomly selecting one of the neighbors of v_{i-1} and setting v_i to be this vertex. A walk $v_0, v_1, \ldots, v_m$ is of length m.

(A) (20 PTS.) For a vertex $u \in V(G)$, let $P_u(m, v)$ be the probability that a random walk of length m, starting from u, visits v (i.e., $v_i = v$ for some i).
Prove that a graph G with n vertices is connected, if and only if, for any two vertices $u, v \in V(G)$, we have $P_u(n-1, v) > 0$.

(B) (20 PTS.) Prove that a graph G with n vertices is connected if and only if for any pair of vertices $u, v \in V(G)$, we have $\lim_{m \to \infty} P_u(m, v) = 1$.