Pixels and Image Filtering

Computer Vision

Derek Hoiem, University of Illinois

Graphic: http://www.notcot.org/post/4068/

Today's Class: Pixels and Linear Filters

- Review of lighting
  - Reflection and absorption

- What is image filtering and how do we do it?

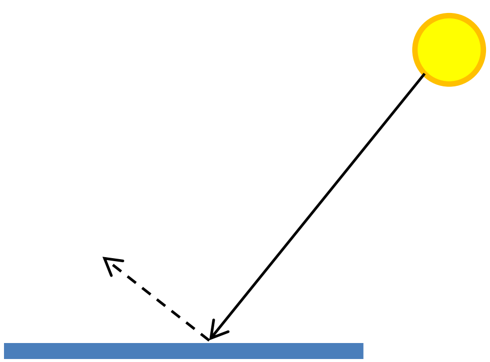
- Color models

Reflection models

- Albedo: fraction of light that is reflected
  - Determines color (amount reflected at each wavelength)

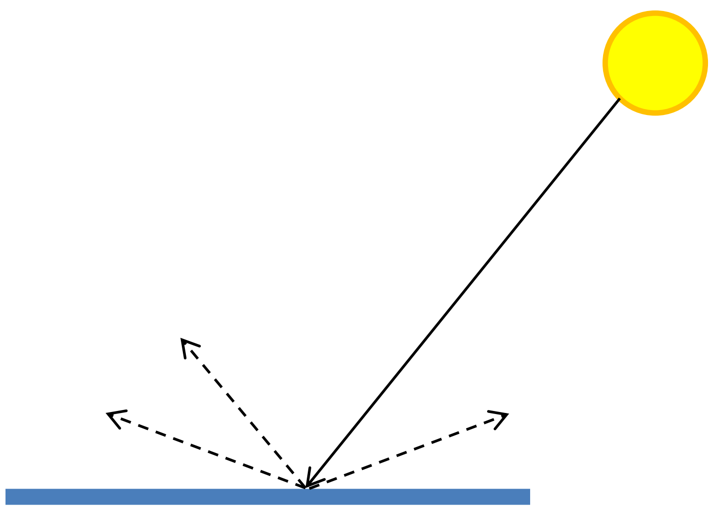
Reflection models

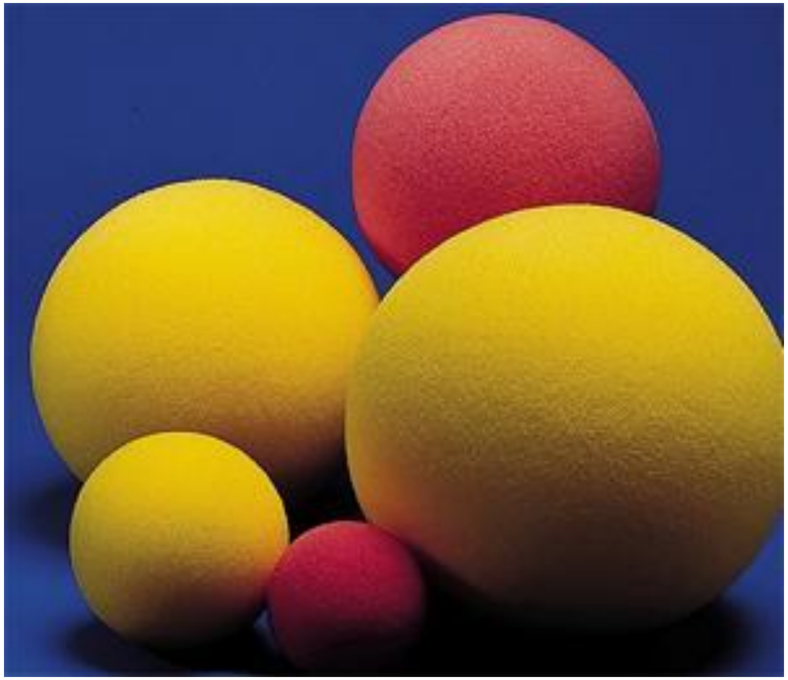
- Specular reflection: mirror-like
  - Light reflects at incident angle
  - Reflection color = incoming light color



Reflection models

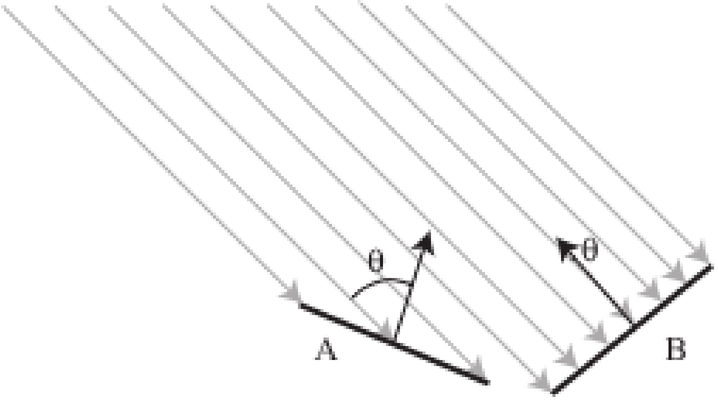
- Diffuse reflection
  - Light scatters in all directions (proportional to cosine between light source and surface normal)
  - Observed intensity is independent of viewing direction
  - Reflection color depends on light color and albedo





Surface orientation and light intensity

- Amount of light that hits surface from distant point source depends on angle between surface normal and source

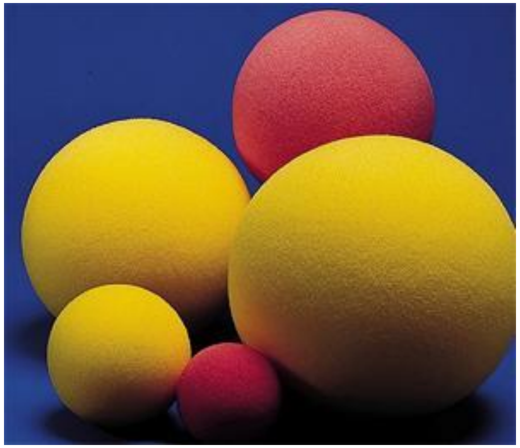


$$I(x) = \rho(x)(\boldsymbol{S} \cdot \boldsymbol{N}(x))$$

proportional to cosine of relative angle

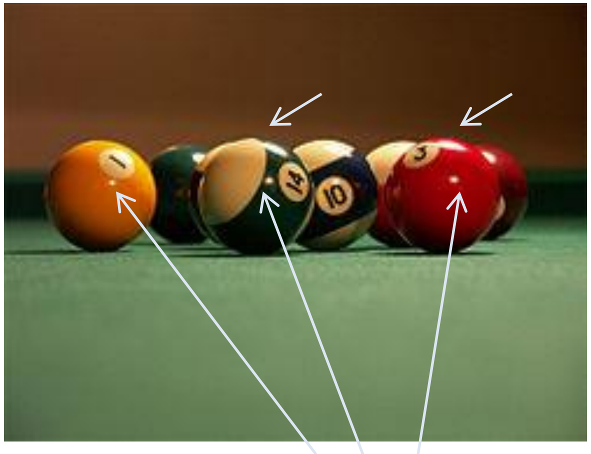
Reflection models

Lambertian: reflection all diffuse



Mirrored: reflection all specular

Glossy: reflection mostly diffuse, some specular



Specularities

Questions

- How many light sources are in the scene?
- How could I estimate the color of the camera's flash?

The plight of the poor pixel

- A pixel's brightness is determined by
  - Light source (strength, direction, color)
  - Surface orientation
  - Surface material and albedo
  - Reflected light and shadows from surrounding surfaces
  - Gain on the sensor
- A pixel's brightness tells us nothing by itself

Basis for interpreting intensity images

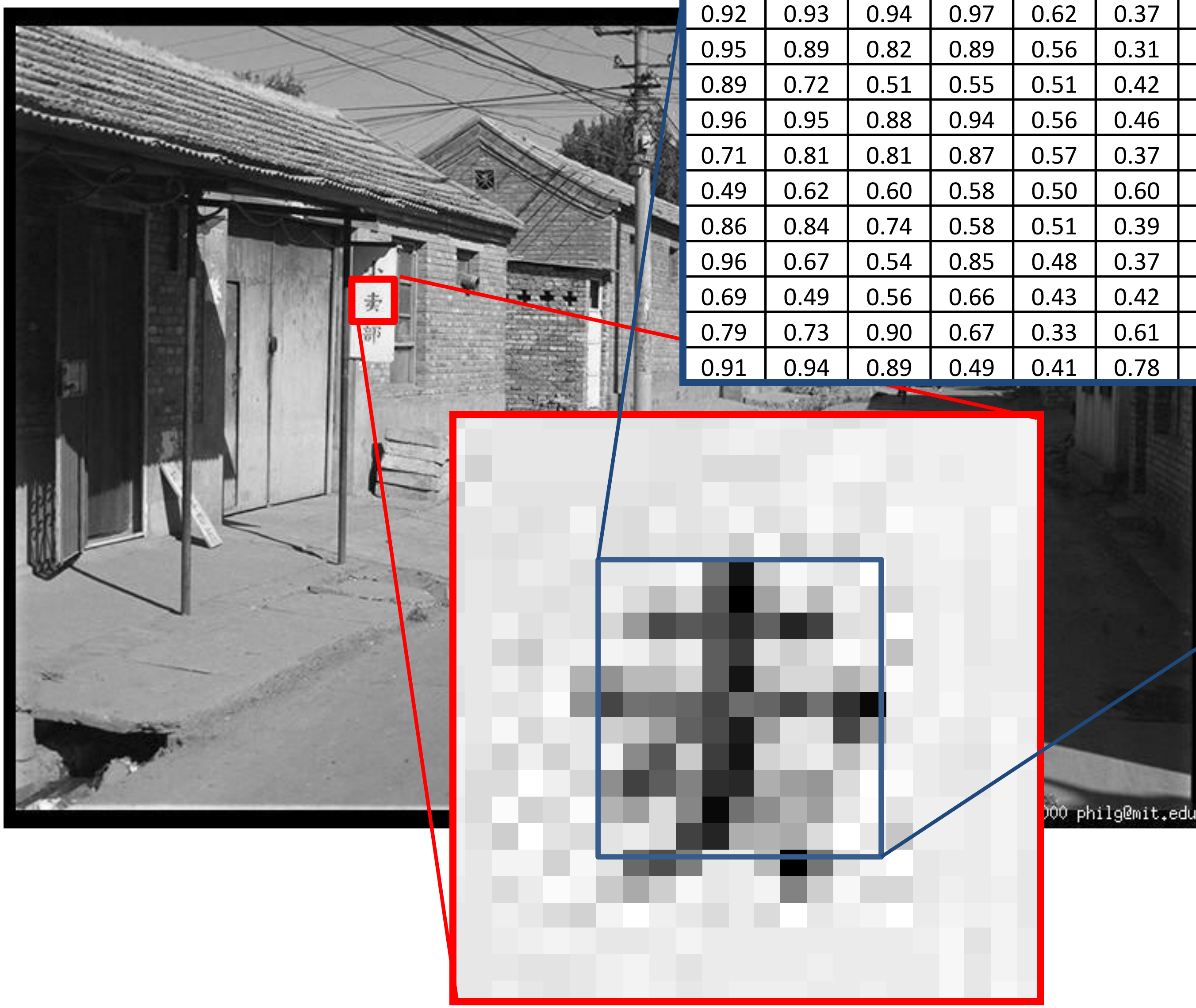
- Key idea: for nearby scene points, most factors do not change much
- The information is mainly contained in *local differences* of brightness

Darkness = Large Difference in Neighboring Pixels

Next three classes: three views of filtering

- Image filters in spatial domain
  - Filter is a mathematical operation on values of each patch
  - Smoothing, sharpening, measuring texture
- Image filters in the frequency domain
  - Filtering is a way to modify the frequencies of images
  - Denoising, sampling, image compression
- Templates and Image Pyramids
  - Filtering is a way to match a template to the image
  - Detection, coarse-to-fine registration

The raster image (pixel matrix)



0.92	0.93	0.94	0.97	0.62	0.37	0.85	0.97	0.93	0.92	0.99
0.95	0.89	0.82	0.89	0.56	0.31	0.75	0.92	0.81	0.95	0.91
0.89	0.72	0.51	0.55	0.51	0.42	0.57	0.41	0.49	0.91	0.92
0.96	0.95	0.88	0.94	0.56	0.46	0.91	0.87	0.90	0.97	0.95
0.71	0.81	0.81	0.87	0.57	0.37	0.80	0.88	0.89	0.79	0.85
0.49	0.62	0.60	0.58	0.50	0.60	0.58	0.50	0.61	0.45	0.33
0.86	0.84	0.74	0.58	0.51	0.39	0.73	0.92	0.91	0.49	0.74
0.96	0.67	0.54	0.85	0.48	0.37	0.88	0.90	0.94	0.82	0.93
0.69	0.49	0.56	0.66	0.43	0.42	0.77	0.73	0.71	0.90	0.99
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97
0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93

Image filtering

- Image filtering: for each pixel, compute function of local neighborhood and output a new value
  - Same function applied at each position
  - Output and input image are typically the same size

Image filtering

- Linear filtering: function is a weighted sum/difference of pixel values

- Really important!
  – Enhance images
    - Denoise, smooth, increase contrast, etc.
  – Extract information from images
    - Texture, edges, distinctive points, etc.
  – Detect patterns
    - Template matching

Example: box filter

$$g[\cdot\,,\cdot\,]$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Slide credit: David Lowe (UBC)

Image filtering

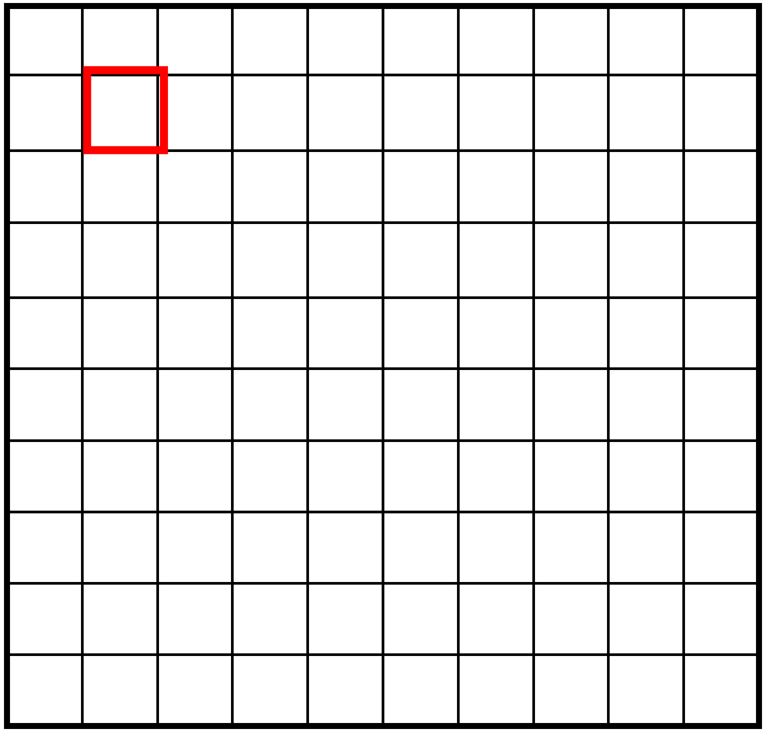
$g[\cdot\,,\cdot\,]\ \frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$



$$h[m,n] = \sum_{k,l} g[k,l]\ f[m+k, n+l]$$

Credit: S. Seitz

Image filtering

$g[\cdot,\cdot]\ \frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10							

$$h[m,n]=\sum_{k,l} g[k,l]\ f[m+k,n+l]$$

Credit: S. Seitz

Image filtering

$g[\cdot,\cdot]\ \frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20						

$$h[m,n]=\sum_{k,l} g[k,l]\ f[m+k,n+l]$$

Credit: S. Seitz

Image filtering

$g[\cdot,\cdot]\ \frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30					

$$h[m,n] = \sum_{k,l} g[k,l]\ f[m+k, n+l]$$

Credit: S. Seitz

Image filtering

$$g[\cdot,\cdot] \; \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30	30				

$$h[m,n] = \sum_{k,l} g[k,l]\; f[m+k, n+l]$$

Credit: S. Seitz

Image filtering

$$g[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30	30				
				?					

$$h[m,n]=\sum_{k,l} g[k,l]\ f[m+k,n+l]$$

Credit: S. Seitz

Image filtering

$g[\cdot,\cdot]\ \frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30	30				
						?			
				50					

$$h[m,n]=\sum_{k,l} g[k,l]\ f[m+k,n+l]$$

Credit: S. Seitz

Image filtering

$g[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

Credit: S. Seitz

Box Filter

What does it do?

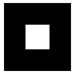
- Replaces each pixel with an average of its neighborhood
- Achieve smoothing effect (remove sharp features)

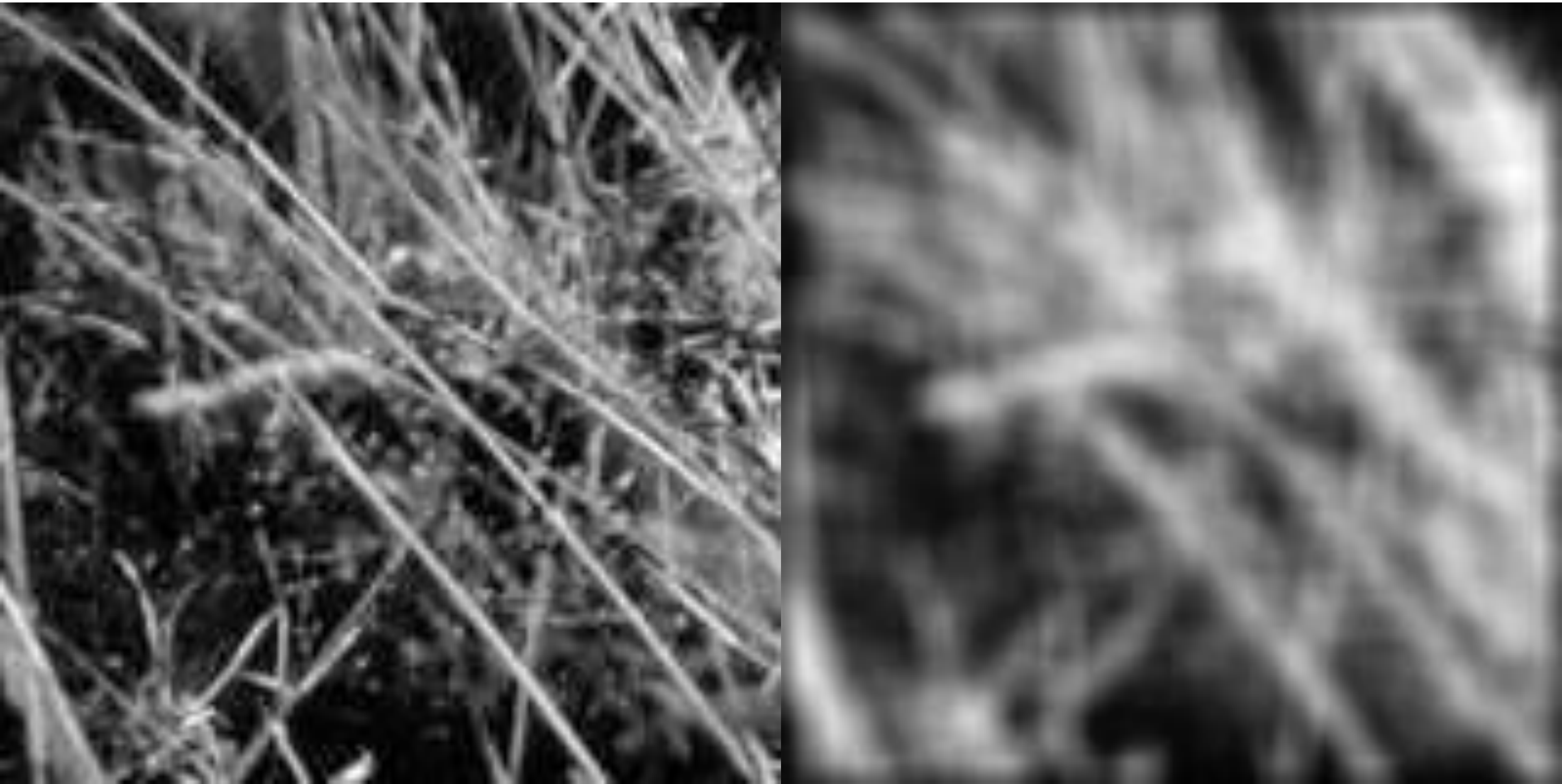
$$g[\cdot\,,\cdot\,]$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Slide credit: David Lowe (UBC)

Smoothing with box filter





Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

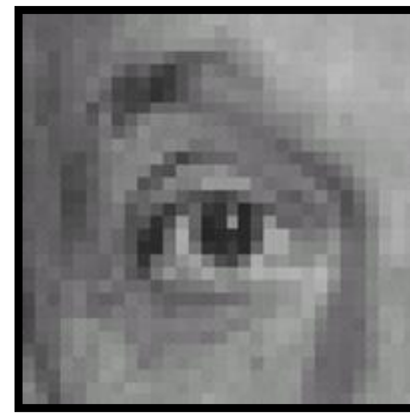
?

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	1	0
0	0	0



Filtered
(no change)

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	0	1
0	0	0

?

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	0	1
0	0	0

Shifted left
By 1 pixel

Source: D. Lowe

Practice with linear filters

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

?

(Note that filter sums to 1)

Source: D. Lowe

Practice with linear filters

Original

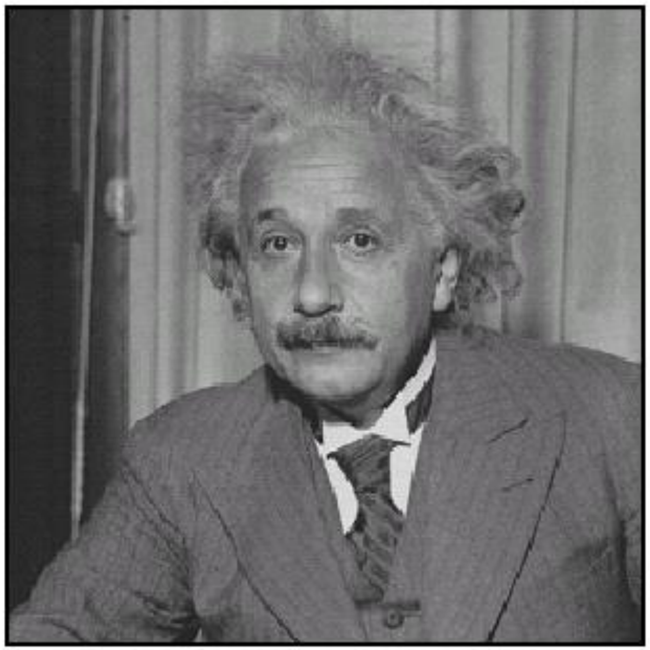
$$\begin{bmatrix}0&0&0\\0&2&0\\0&0&0\end{bmatrix} - \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

Sharpening filter

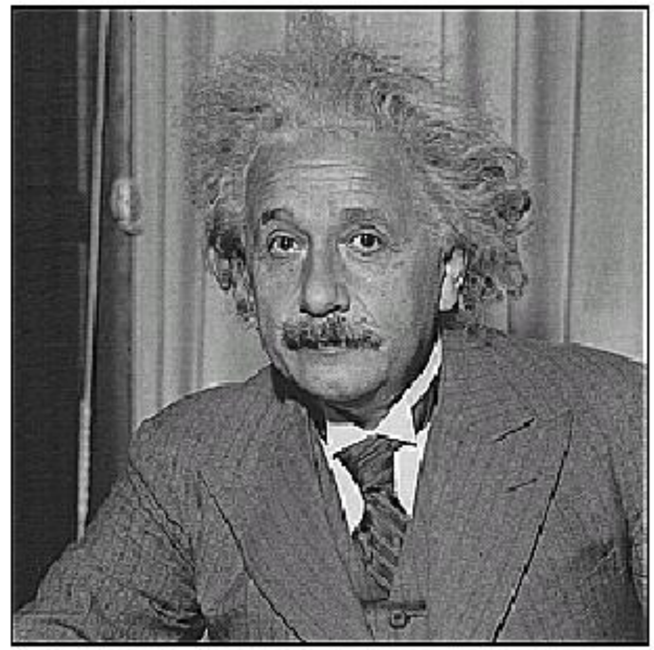
- Accentuates differences with local average

Source: D. Lowe

Sharpening



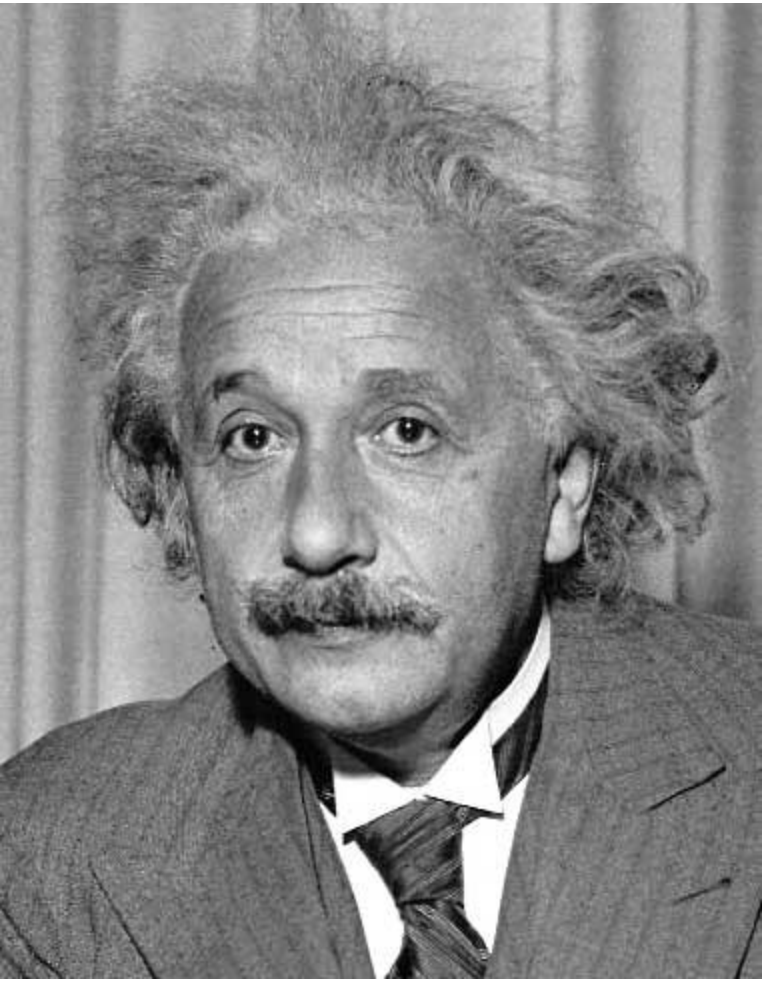
before



after

Source: D. Lowe

Other filters

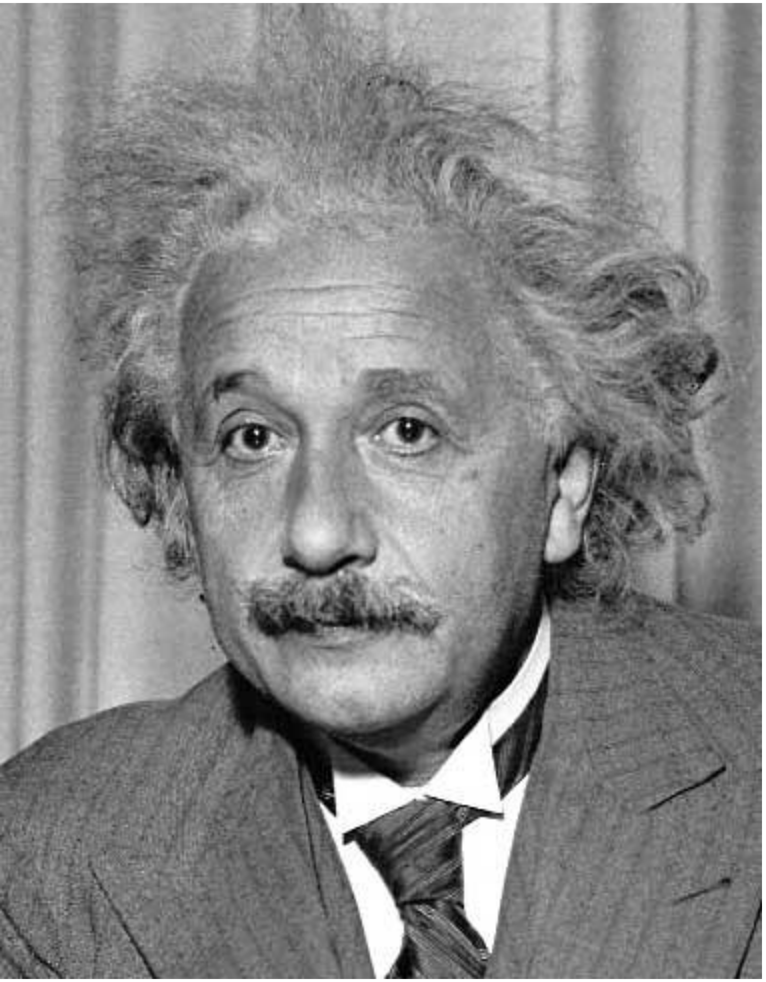


1	0	-1
2	0	-2
1	0	-1

Sobel

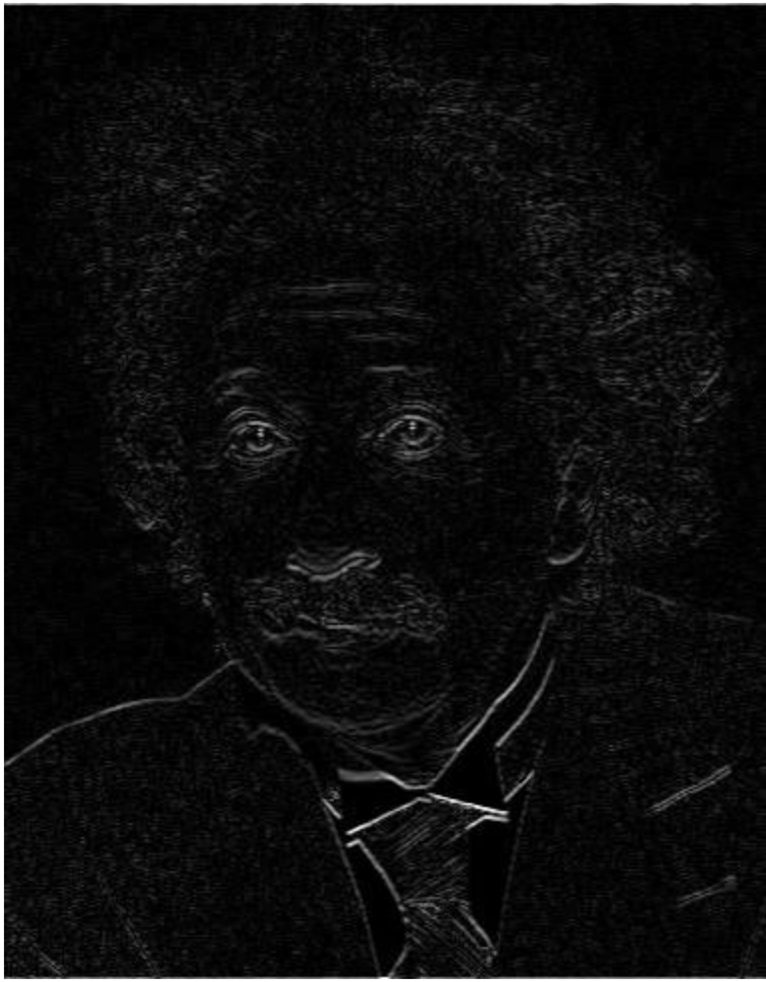
Vertical Edge
(absolute value)

Other filters



1	2	1
0	0	0
-1	-2	-1

Sobel



Horizontal Edge
(absolute value)

Basic gradient filters

Horizontal Gradient

0	0	0
-1	0	1
0	0	0

or

-1	0	1

Vertical Gradient

0	-1	0
0	0	0
0	1	0

or

-1
0
1

Examples

Write as filtering operations, plus some elementwise operations: +, -, .*,>

1. Sum of four adjacent neighbors plus 1

$$out(m,n) = 1 + \sum_{k,l \in \{-1,1\}} in(m+k, n+l)$$

2. Sum of squared values of 3x3 windows around each pixel: $out(m,n) = \sum_{k,l \in \{-1,0,1\}} in(m+k, n+l)^2$

3. Center pixel value is larger than the average of the pixel values to the left and right:

$$out(m,n) = 1 \quad \text{if} \quad in(m,n) > (in(m,n-1) + in(m,n+1))/2$$

$$out(m,n) = 0 \quad \text{if} \quad in(m,n) \leq (in(m,n-1) + in(m,n+1))/2$$

Filtering vs. Convolution

- 2d filtering
  - `h=filter2(g,f);` or `h=imfilter(f,g);`

g=filter f=image

$$h[m,n] = \sum_{k,l} g[k,l]\; f[m+k, n+l]$$

- 2d convolution
  - `h=conv2(g,f);`

$$h[m,n] = \sum_{k,l} g[k,l]\; f[m-k, n-l]$$

Key properties of linear filters

Linearity:

$\texttt{filter}(f_1 + f_2) = \texttt{filter}(f_1) + \texttt{filter}(f_2)$

Shift invariance: same behavior regardless of pixel location

$\texttt{filter}(\texttt{shift}(f)) = \texttt{shift}(\texttt{filter}(f))$

Any linear, shift-invariant operator can be represented as a convolution

Source: S. Lazebnik

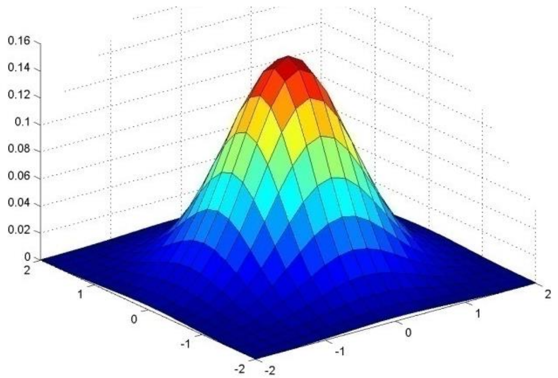
More properties

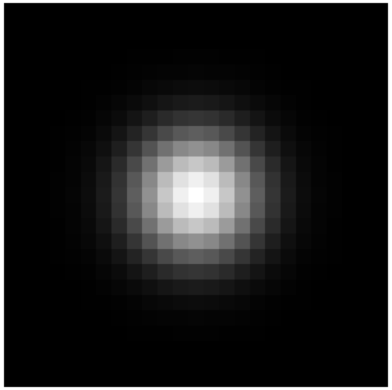
- Commutative: $a * b = b * a$
  - Conceptually no difference between filter and signal
- Associative: $a * (b * c) = (a * b) * c$
  - Often apply several filters one after another: $(((a * b_1) * b_2) * b_3)$
  - This is equivalent to applying one filter: a * $(b_1 * b_2 * b_3)$
- Distributes over addition: $a * (b + c) = (a * b) + (a * c)$
- Scalars factor out: $ka * b = a * kb = k (a * b)$
- Identity: unit impulse e = [0, 0, 1, 0, 0],
  $a * e = a$

Source: S. Lazebnik

Important filter: Gaussian

- Spatially-weighted average





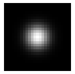
0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

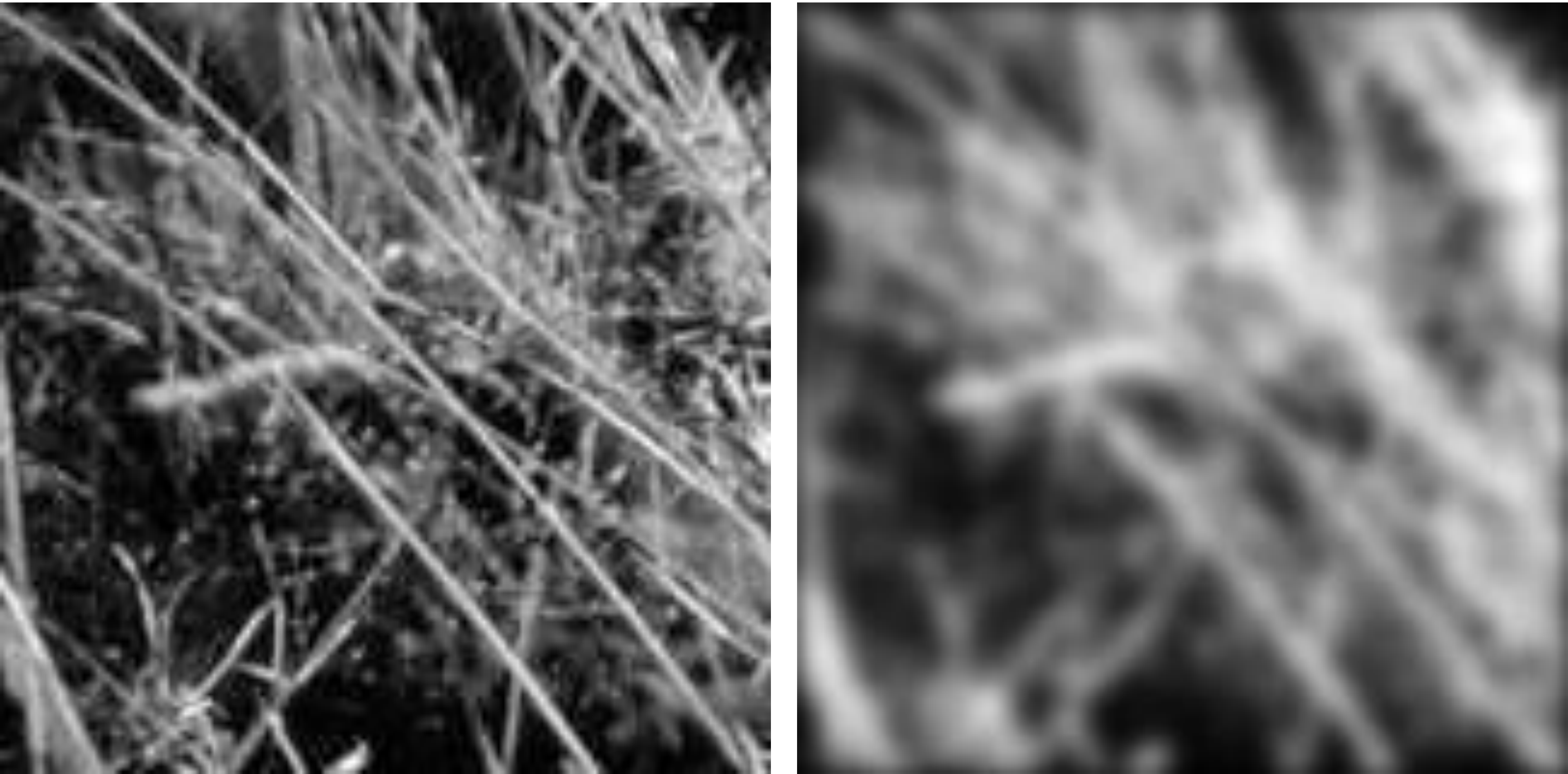
5 x 5, σ = 1

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

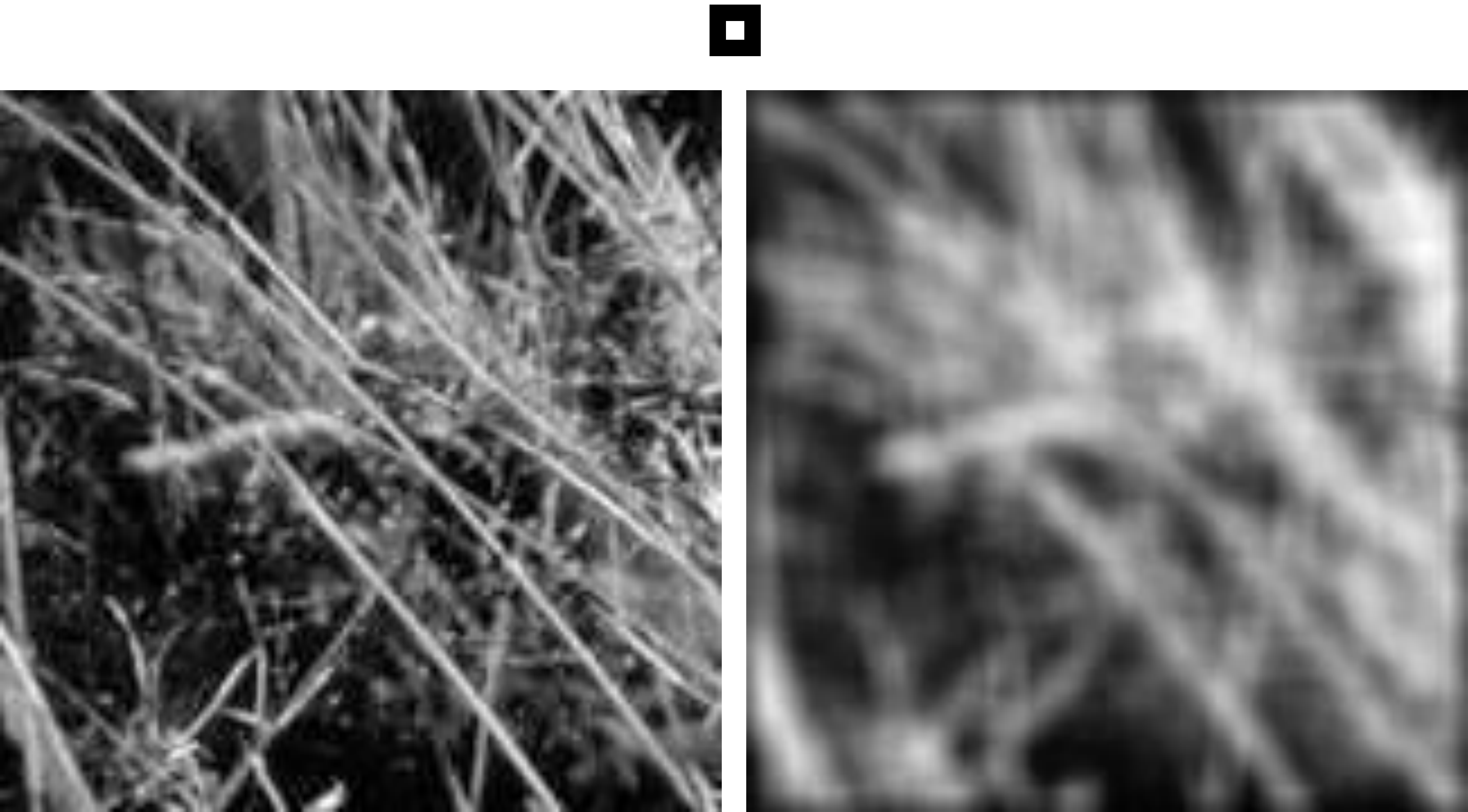
Slide credit: Christopher Rasmussen

Smoothing with Gaussian filter





Smoothing with box filter



Gaussian filters

- Remove "high-frequency" components from the image (low-pass filter)
  - Images become more smooth
- Convolution with self is another Gaussian
  - So can smooth with small-width kernel, repeat, and get same result as larger-width kernel would have
  - Convolving two times with Gaussian kernel of width σ is same as convolving once with kernel of width $\sigma\sqrt{2}$
- *Separable* kernel
  - Factors into product of two 1D Gaussians

Source: K. Grauman

Separability of the Gaussian filter

$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2 + y^2}{2\sigma^2}}$$

$$= \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{x^2}{2\sigma^2}} \right) \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{y^2}{2\sigma^2}} \right)$$

The 2D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y

In this case, the two functions are the (identical) 1D Gaussian

Source: D. Lowe

Separability example

2D filtering
(center location only)

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 3 & 3 \\ 3 & 5 & 5 \\ 4 & 4 & 6 \end{bmatrix}$$

The filter factors
into a product of 1D
filters:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

Perform filtering
along rows:

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 3 & 3 \\ 3 & 5 & 5 \\ 4 & 4 & 6 \end{bmatrix} = \begin{bmatrix} & 11 & \\ & 18 & \\ & 18 & \end{bmatrix}$$

Followed by filtering
along the remaining column:

Source: K. Grauman

Separability

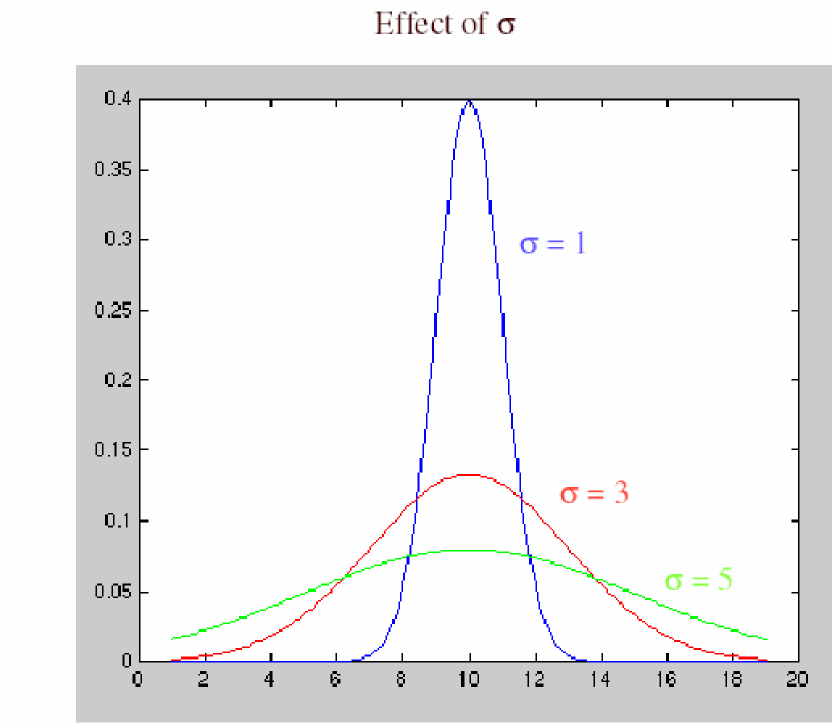
- Why is separability useful in practice?

Some practical matters

Practical matters

How big should the filter be?

- Values at edges should be near zero ← important!
- Rule of thumb for Gaussian: set filter half-width to about 3 σ



Practical matters

- What about near the edge?
  - the filter window falls off the edge of the image
  - need to extrapolate
  - methods:
    - clip filter (black)
    - wrap around
    - copy edge
    - reflect across edge

Source: S. Marschner

Practical matters

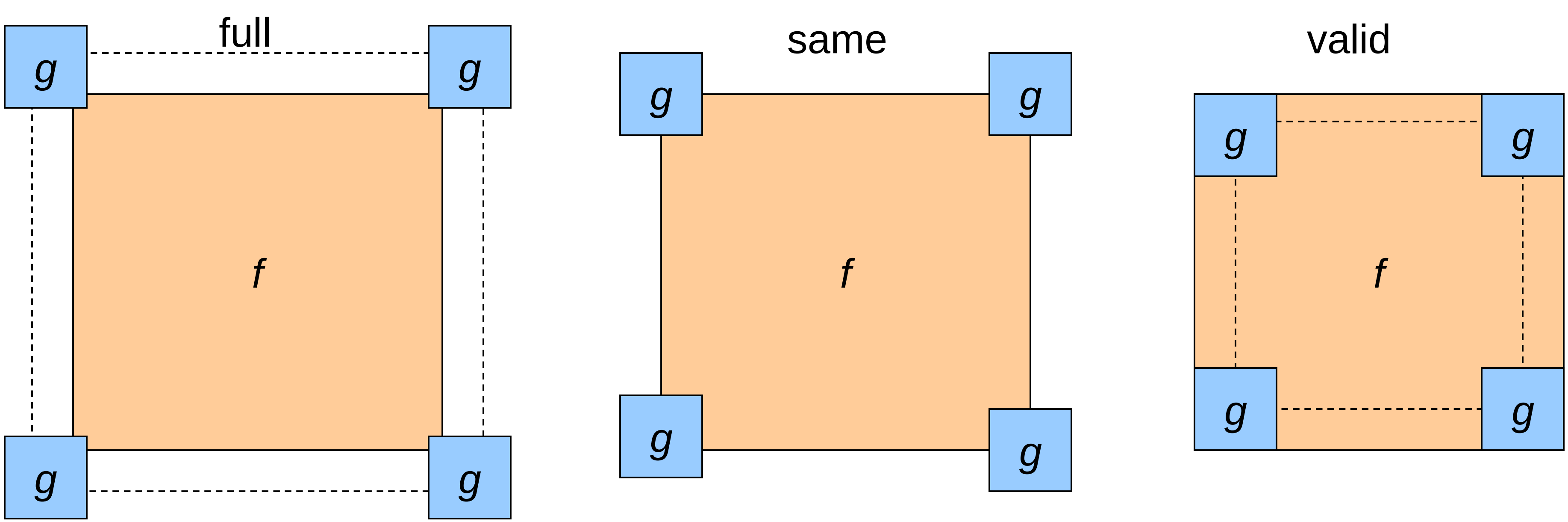
– methods (MATLAB):

- clip filter (black): imfilter(f, g, 0)
- wrap around: imfilter(f, g, ‘circular’)
- copy edge: imfilter(f, g, ‘replicate’)
- reflect across edge: imfilter(f, g, ‘symmetric’)

Source: S. Marschner

Practical matters

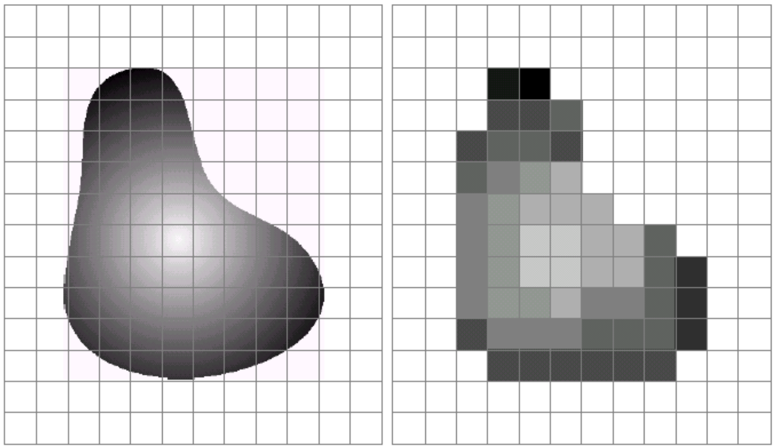
- What is the size of the output?
- MATLAB: filter2(g, f, *shape*)
  - *shape* = 'full': output size is sum of sizes of f and g
  - *shape* = 'same': output size is same as f
  - *shape* = 'valid': output size is difference of sizes of f and g



Source: S. Lazebnik

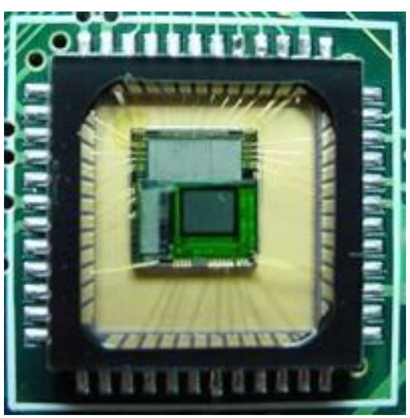
A little more about color…

Digital Color Images



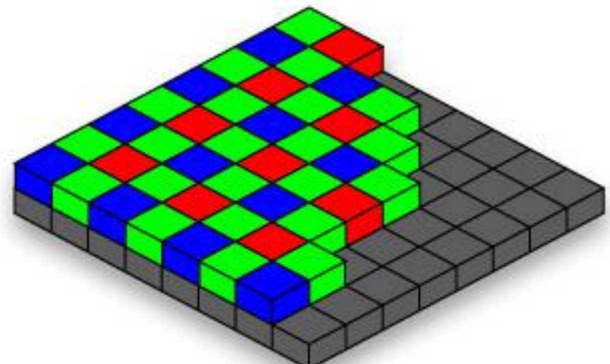
a b

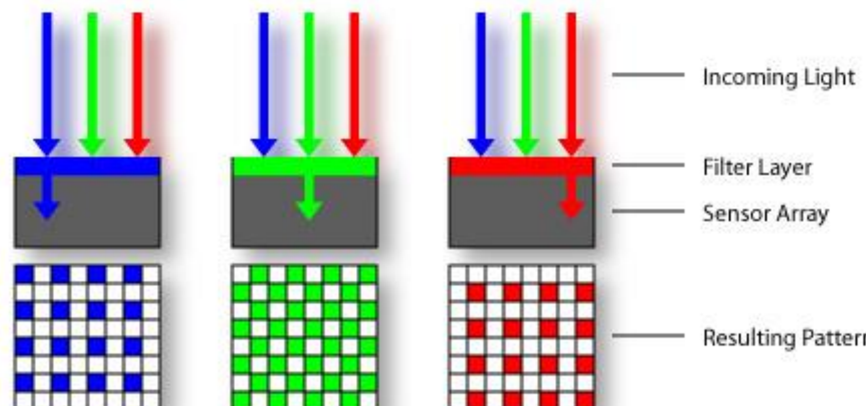
FIGURE 2.17 (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.



CMOS sensor

Bayer Filter

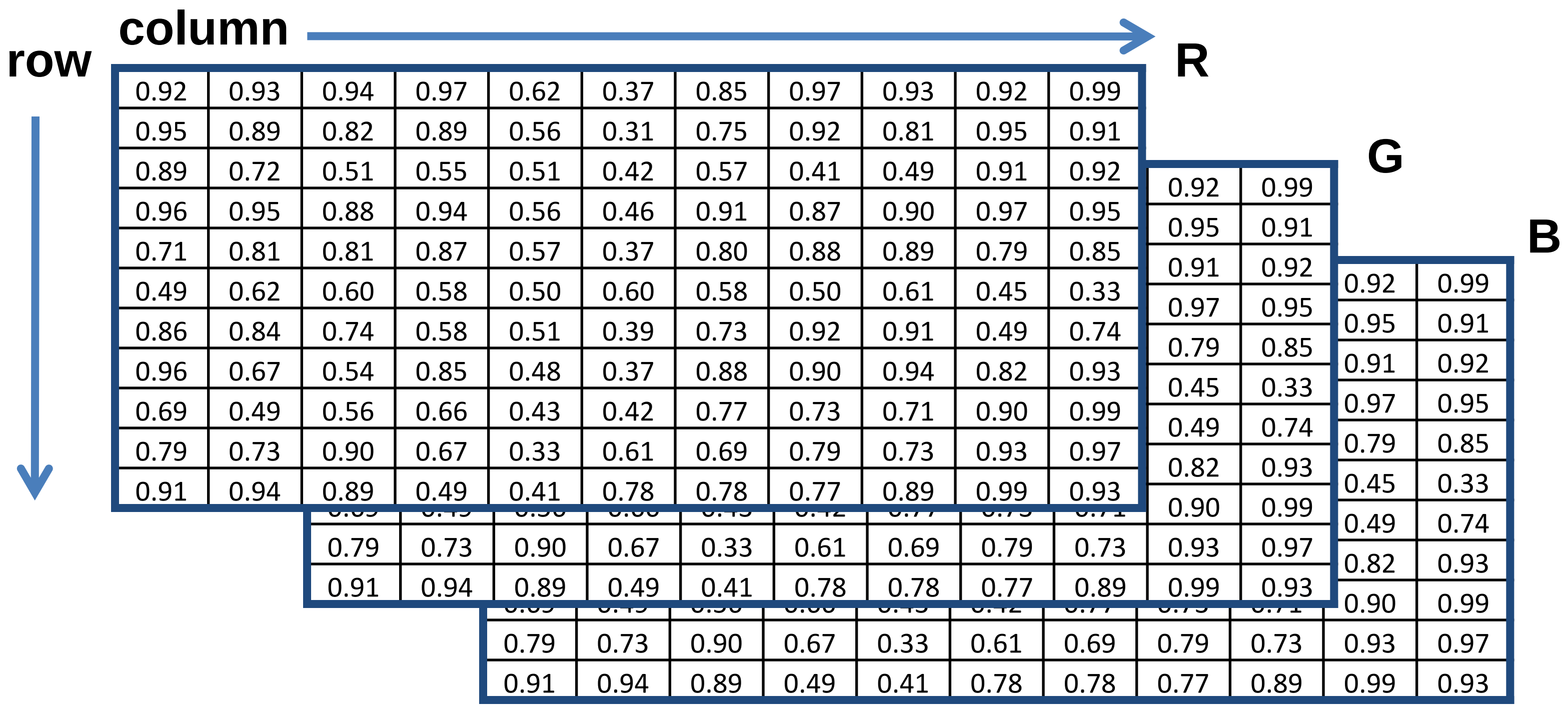




Color Image

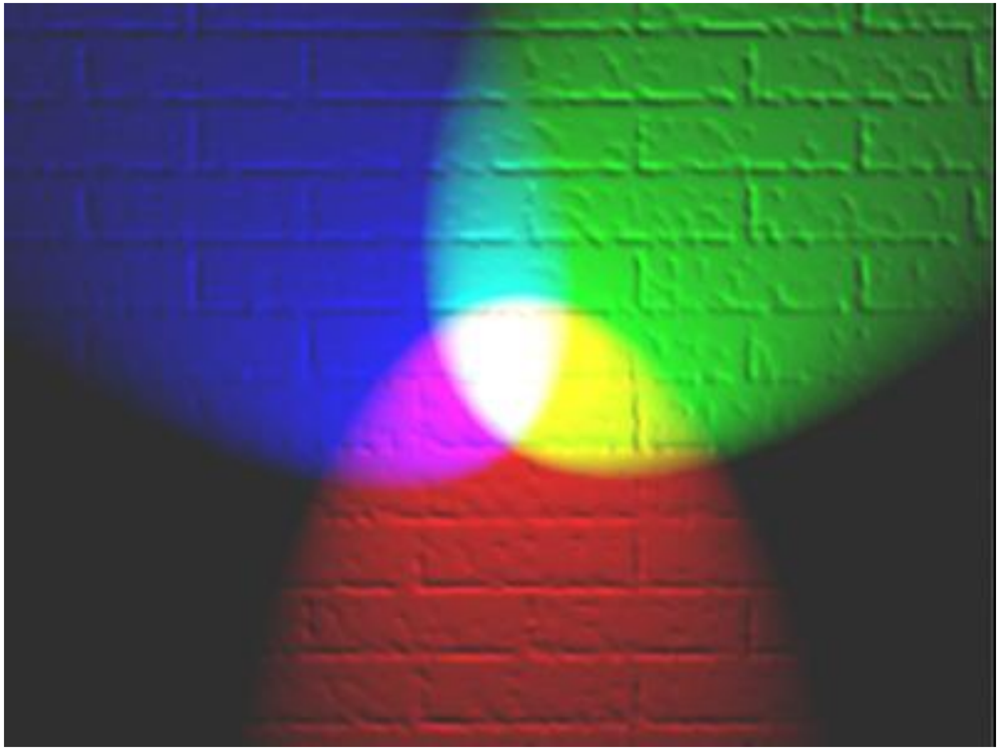
Images in Matlab

- Images represented as a matrix
- Suppose we have a NxM RGB image called "im"
  - im(1,1,1) = top-left pixel value in R-channel
  - im(y, x, b) = y pixels down, x pixels to right in the b^{th} channel
  - im(N, M, 3) = bottom-right pixel in B-channel
- imread(filename) returns a uint8 image (values 0 to 255)
  - Convert to double format (values 0 to 1) with im2double



Color spaces

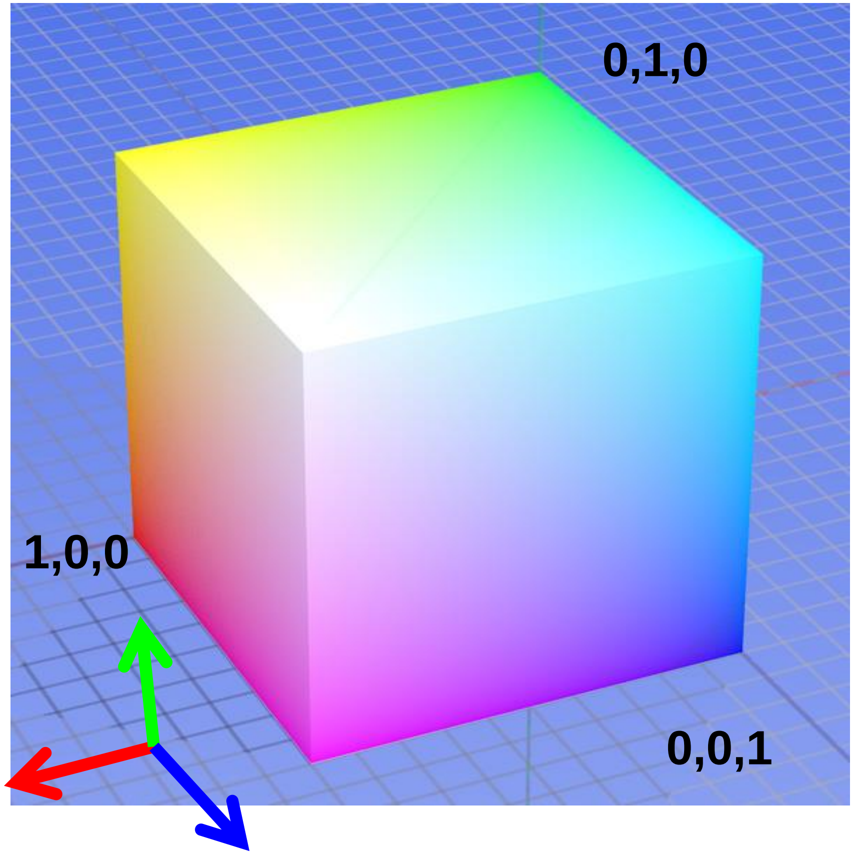
- How can we represent color?



http://en.wikipedia.org/wiki/File:RGB_illumination.jpg

Color spaces: RGB

Default color space



R
(G=0,B=0)

G
(R=0,B=0)

B
(R=0,G=0)

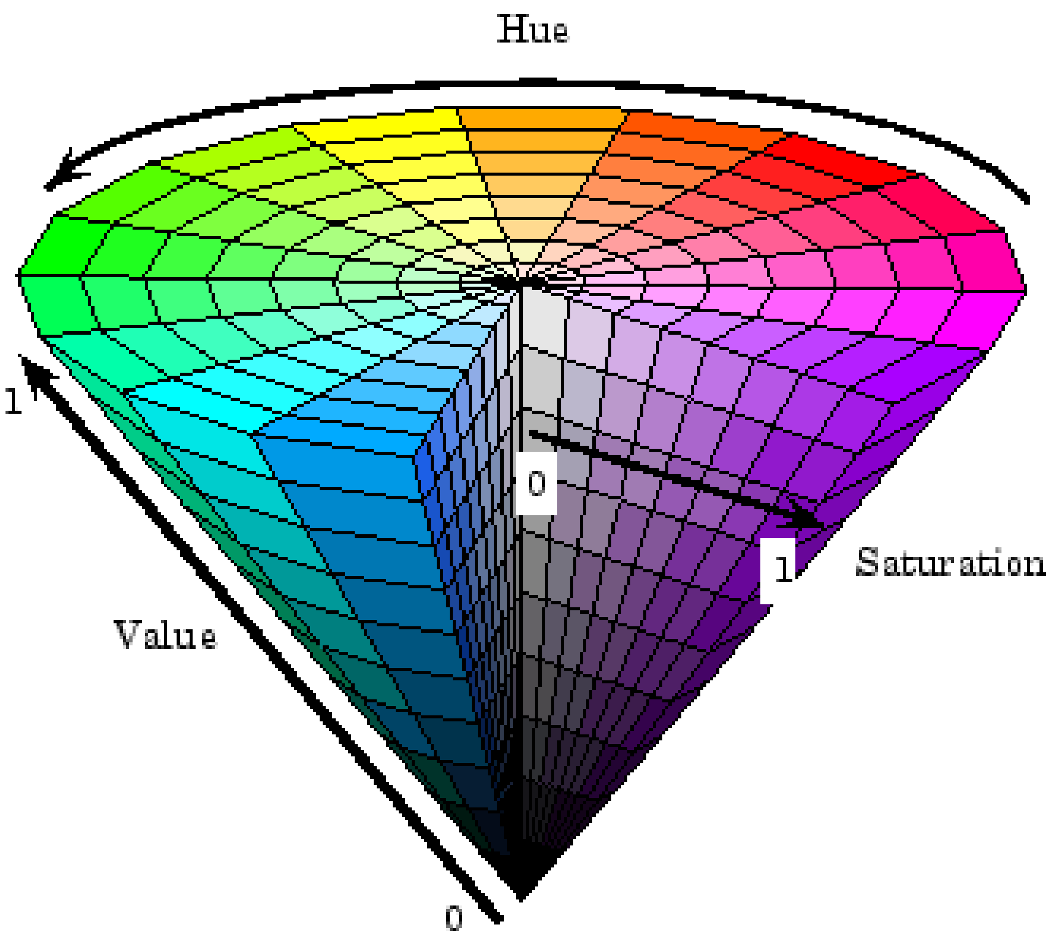
Some drawbacks

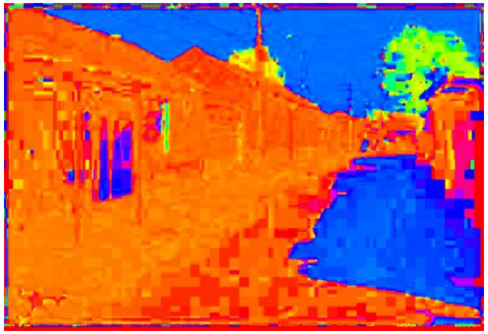
- Strongly correlated channels
- Non-perceptual

Image from: http://en.wikipedia.org/wiki/File:RGB_color_solid_cube.png

Color spaces: HSV

Intuitive color space





H
(S=1,V=1)

S
(H=1,V=1)

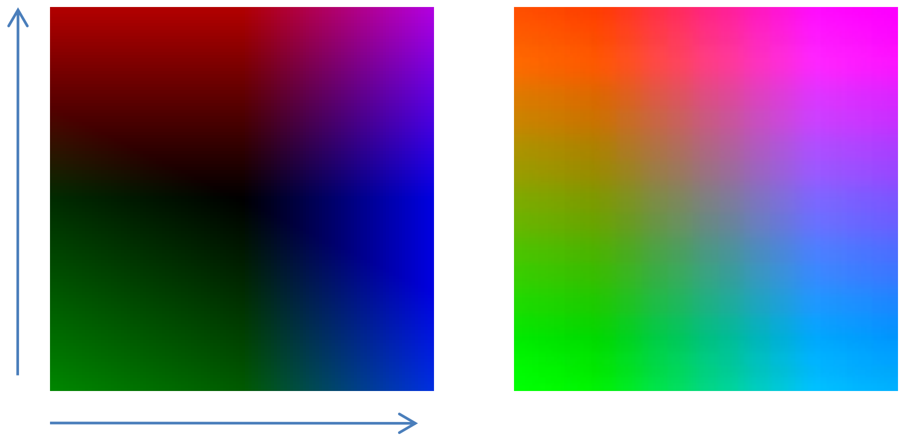
V
(H=1,S=0)

Color spaces: YCbCr

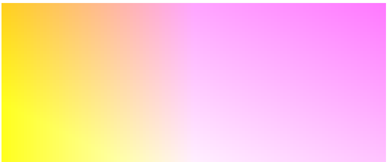
Fast to compute, good for compression, used by TV

Y=0

Y=0.5



Y=1



$$Y' = 16 + \frac{65.738 \cdot R'_D}{256} + \frac{129.057 \cdot G'_D}{256} + \frac{25.064 \cdot B'_D}{256}$$

$$C_B = 128 + \frac{-37.945 \cdot R'_D}{256} - \frac{74.494 \cdot G'_D}{256} + \frac{112.439 \cdot B'_D}{256}$$

$$C_R = 128 + \frac{112.439 \cdot R'_D}{256} - \frac{94.154 \cdot G'_D}{256} - \frac{18.285 \cdot B'_D}{256}$$

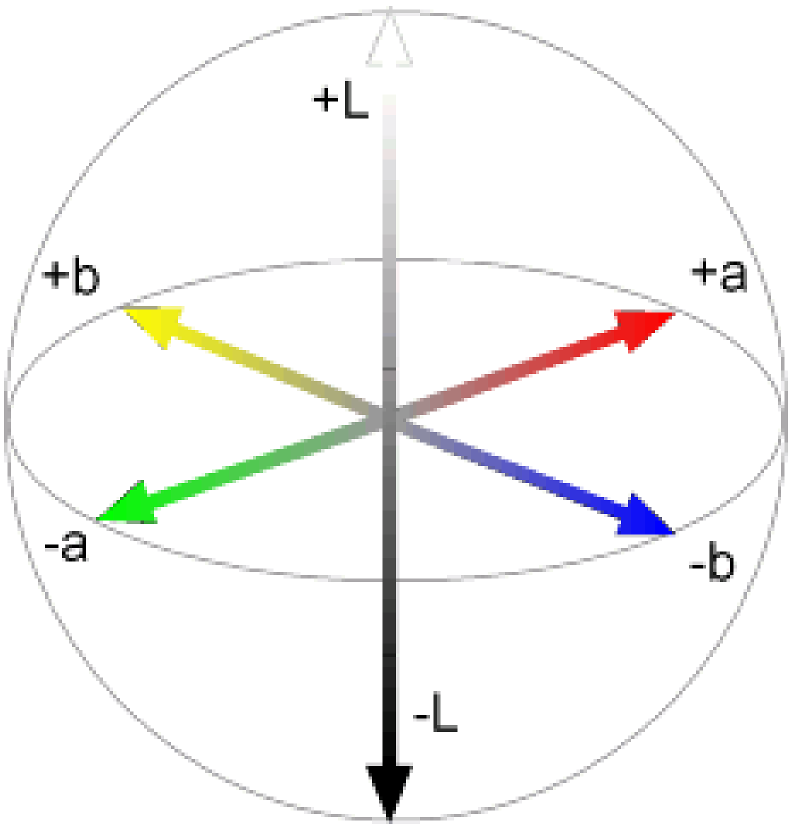
Y
(Cb=0.5,Cr=0.5)

Cb
(Y=0.5,Cr=0.5)

Cr
(Y=0.5,Cb=05)

Color spaces: CIE L*a*b*

"Perceptually uniform" color space



Luminance = brightness
Chrominance = color

L
(a=0,b=0)

a
(L=65,b=0)

b
(L=65,a=0)

Which contains more information?

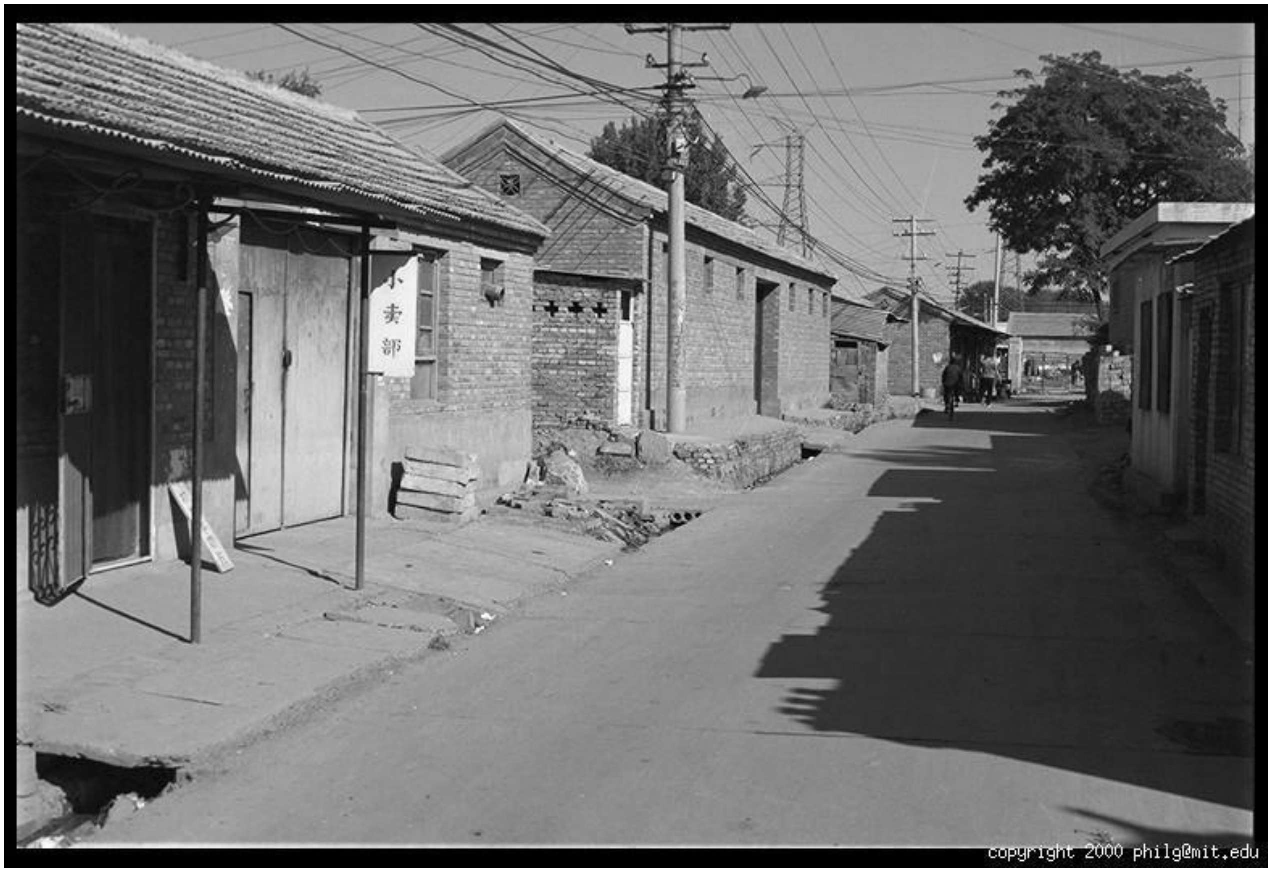
(a) **intensity** (1 channel)

(b) **chrominance** (2 channels)

Most information in intensity

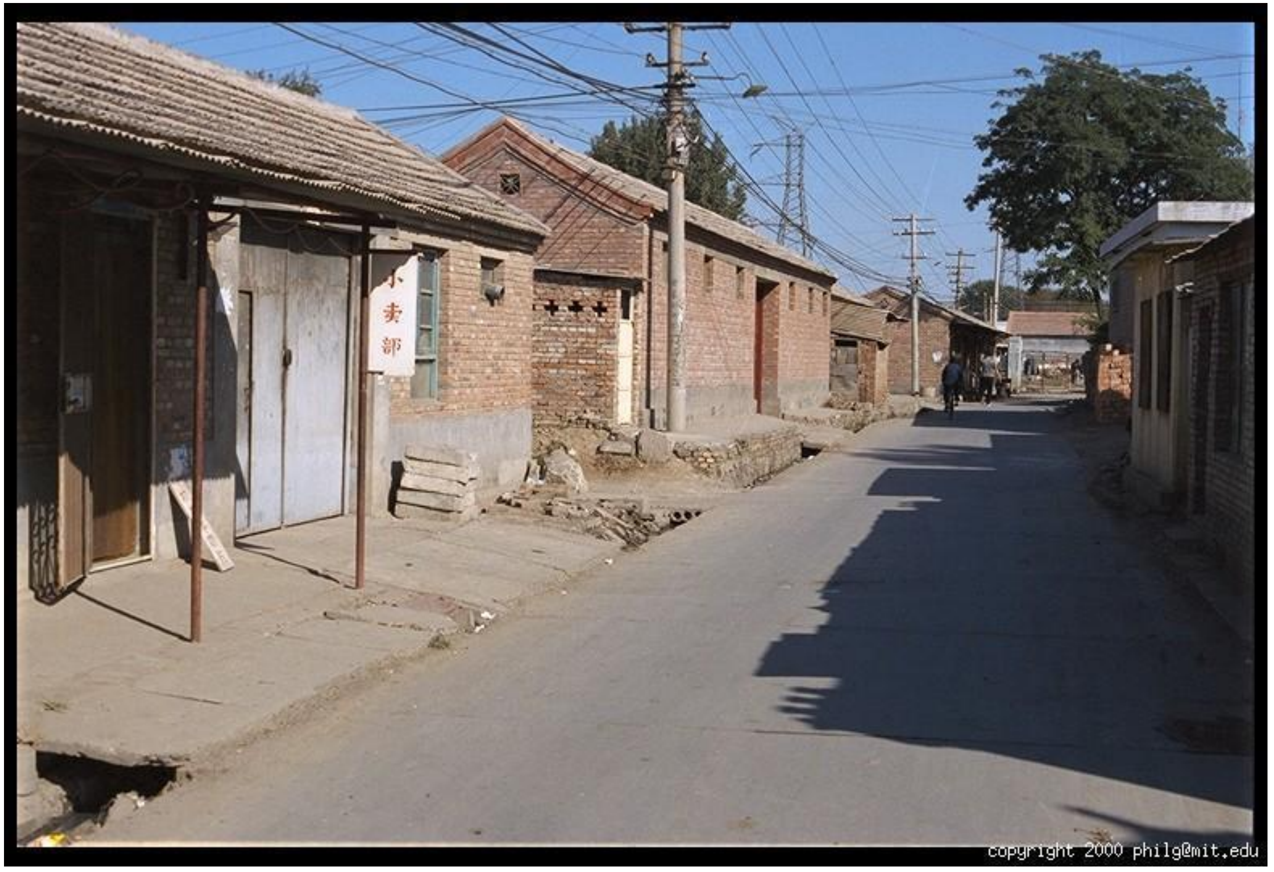
Only color shown – constant intensity

Most information in intensity



Only intensity shown – constant color

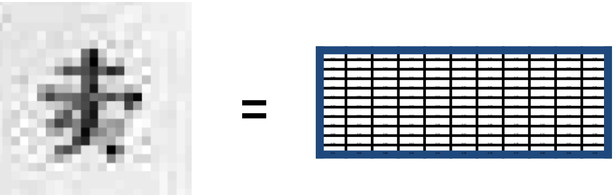
Most information in intensity



Original image

Take-home messages

- Image is a matrix of numbers (light intensities at different orientations)
  - Interpreted mainly through local comparisons



- Linear filtering is sum of dot product at each position
  - Can smooth, sharpen, translate (among many other uses)

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

- Attend to details: filter size, extrapolation, cropping

- Color spaces beyond RGB sometimes useful