Single-view Metrology and Camera Calibration

Computer Vision

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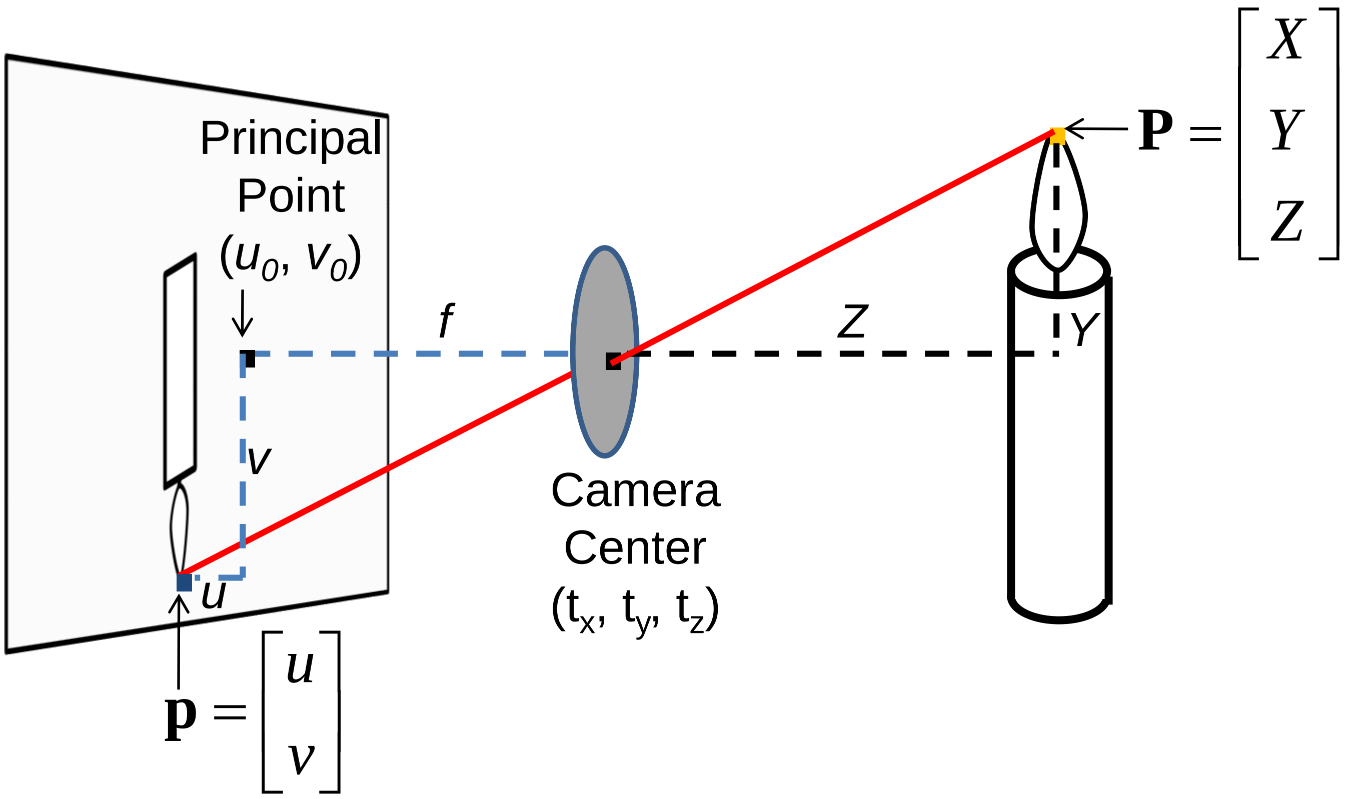
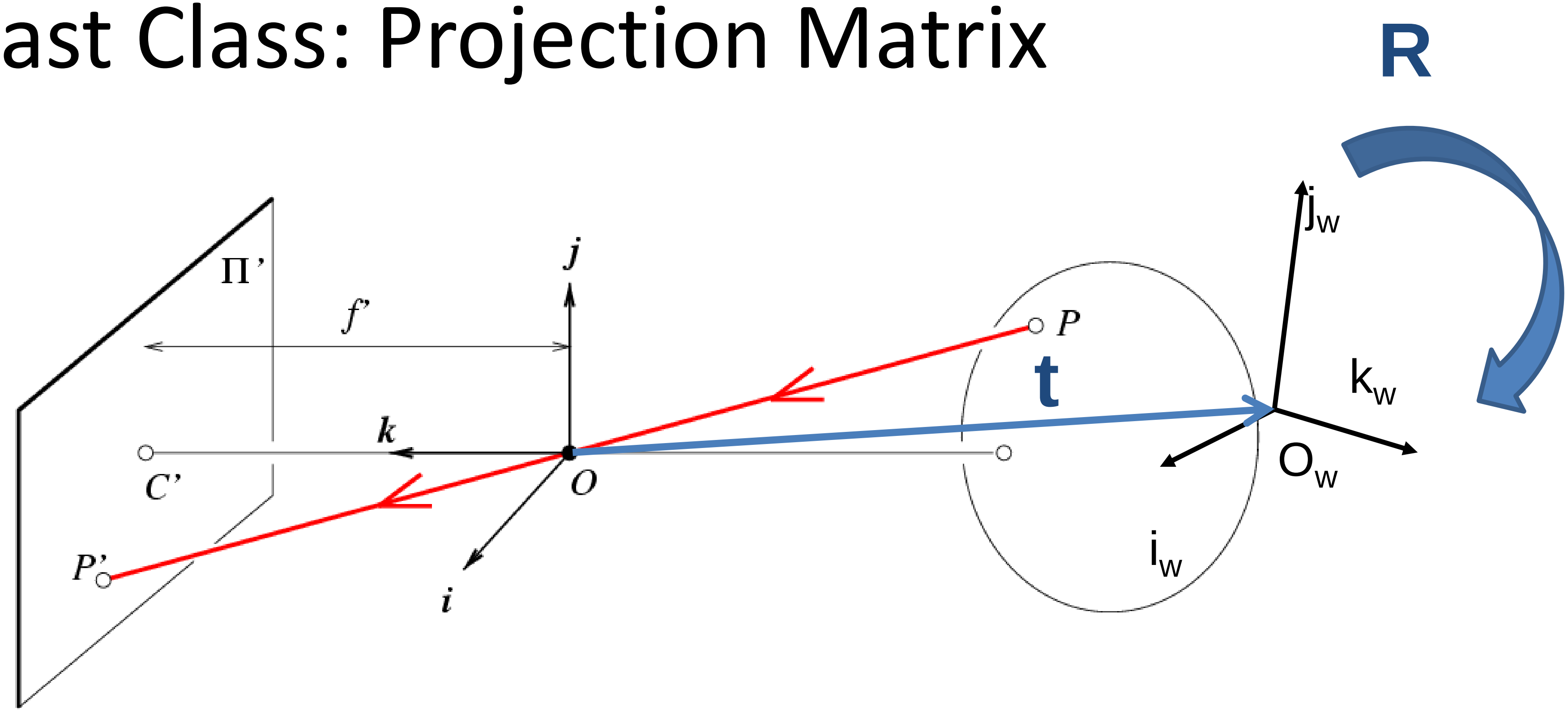
Last Class: Pinhole Camera

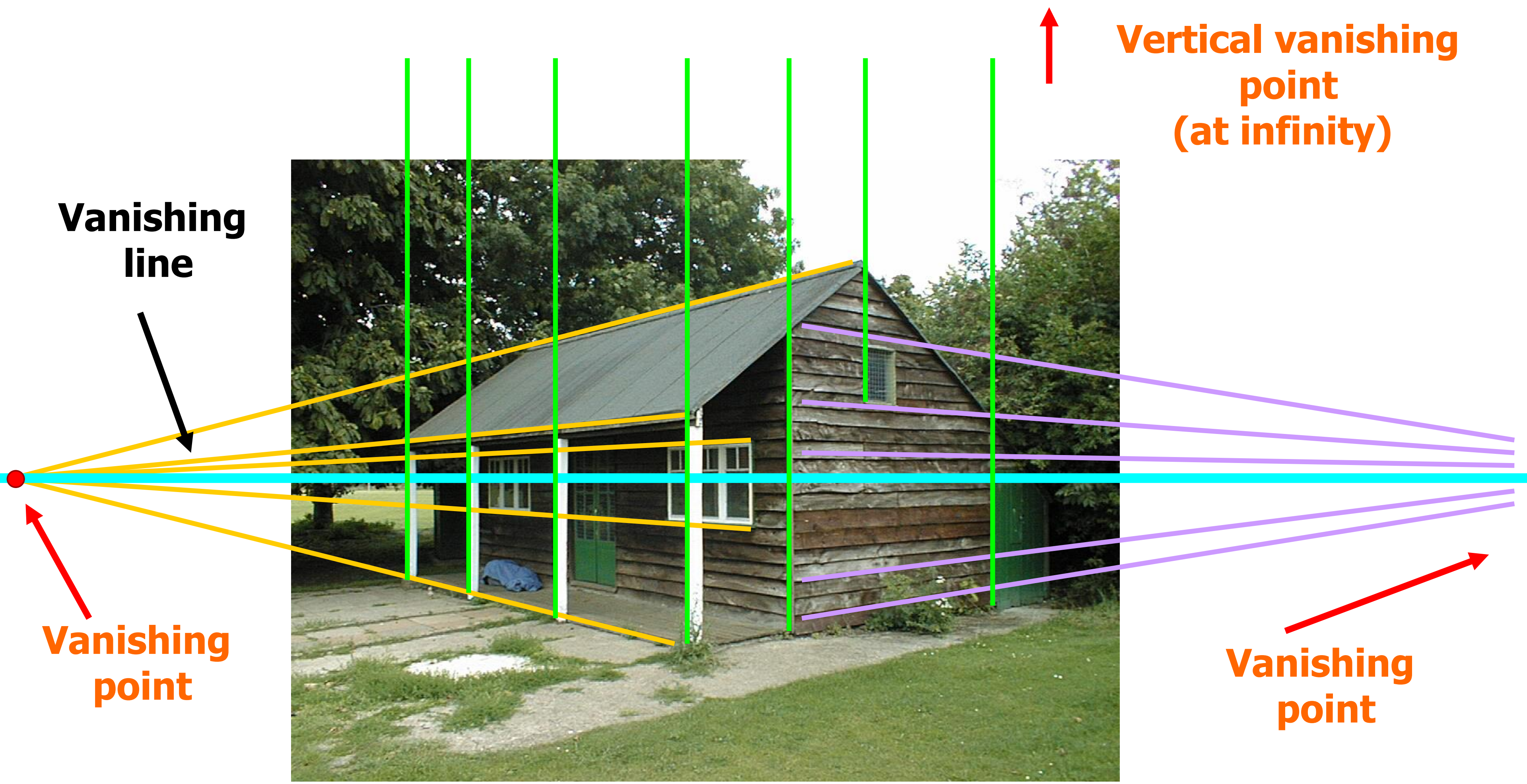


Last Class: Projection Matrix



$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & s & u_0 \\ 0 & \alpha f & v_0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Last class: Vanishing Points



Slide from Efros, Photo from Criminisi

This class

- How can we calibrate the camera?

- How can we measure the size of objects in the world from an image?

- What about other camera properties: focal length, field of view, depth of field, aperture, f-number?

How to calibrate the camera?

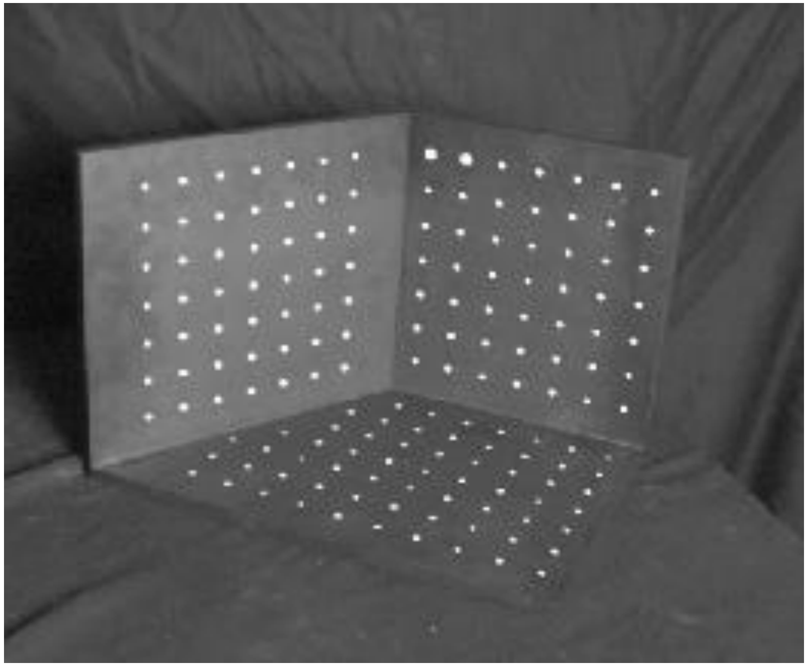
$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$\begin{bmatrix} wu \\ wv \\ w \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Calibrating the Camera

Method 1: Use an object (calibration grid) with known geometry

- Correspond image points to 3d points
- Get least squares solution (or non-linear solution)



$$\begin{bmatrix} wu \\ wv \\ w \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Linear method

- Solve using linear least squares

$$\begin{bmatrix} wu \\ wv \\ w \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 & -u_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 & -v_1 \\ & & & & & & \vdots & & & & & \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -u_nX_n & -u_nY_n & -u_nZ_n & -u_n \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -v_nX_n & -v_nY_n & -v_nZ_n & -v_n \end{bmatrix} \begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \\ m_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

Ax=0 form

Calibration with linear method

- Advantages
  - Easy to formulate and solve
  - Provides initialization for non-linear methods

- Disadvantages
  - Doesn’t directly give you camera parameters
  - Doesn’t model radial distortion
  - Can’t impose constraints, such as known focal length
  - Doesn’t minimize projection error

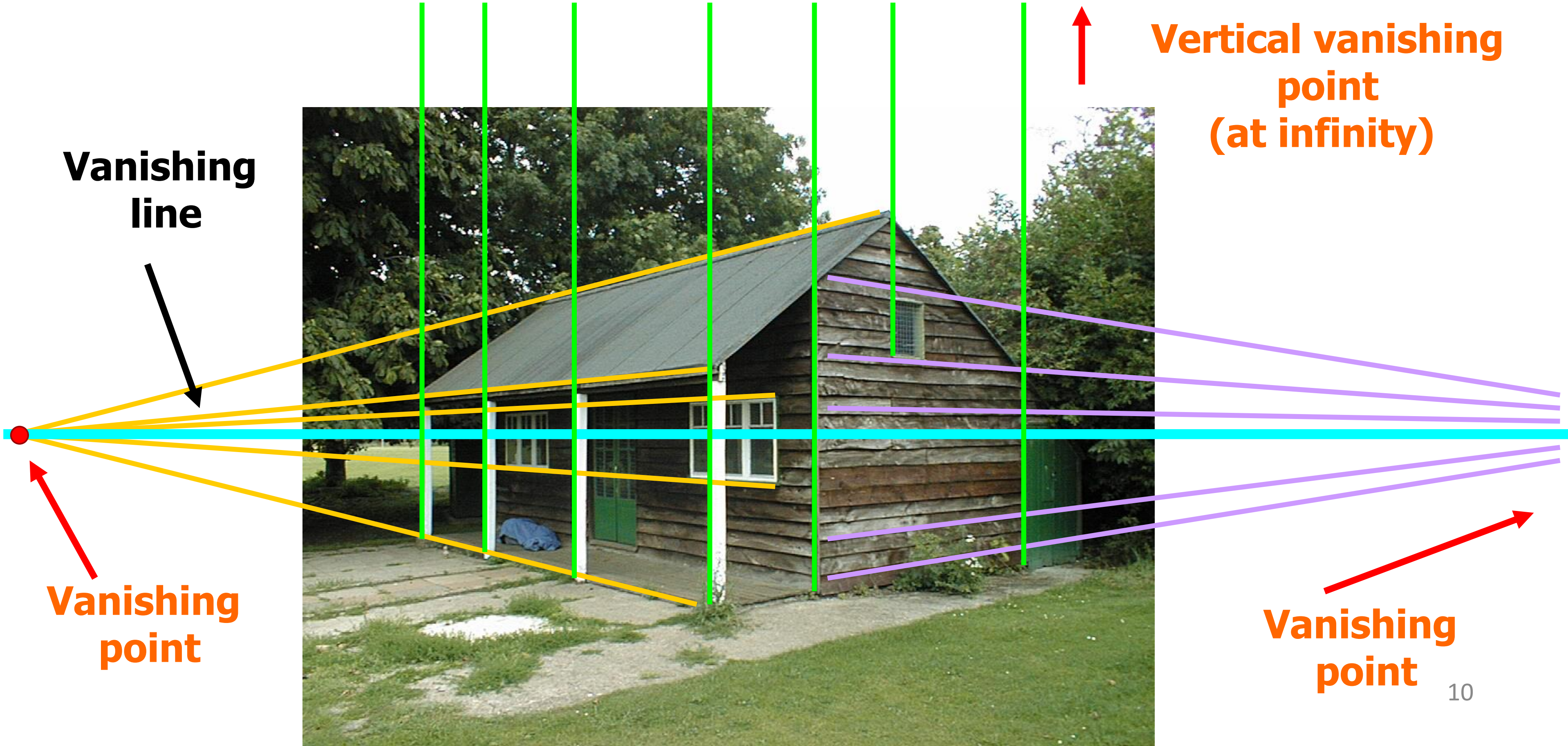
- Non-linear methods are preferred
  - Define error as difference between projected points and measured points
  - Minimize error using Newton’s method or other non-linear optimization

Can solve for explicit camera parameters:
http://ksimek.github.io/2012/08/14/decompose/

Calibrating the Camera

Method 2: Use vanishing points

– Find vanishing points corresponding to orthogonal directions



Calibration by orthogonal vanishing points

- Intrinsic camera matrix
  - Use orthogonality as a constraint
  - Model K with only f, u_0, v_0

$$\mathbf{p}_i = \mathbf{KRX}_i$$

For vanishing points

$$\mathbf{X}_i^T \mathbf{X}_j = 0$$

- What if you don't have three finite vanishing points?
  - Two finite VP: solve f, get valid u_0, v_0 closest to image center
  - One finite VP: u_0, v_0 is at vanishing point; can't solve for f

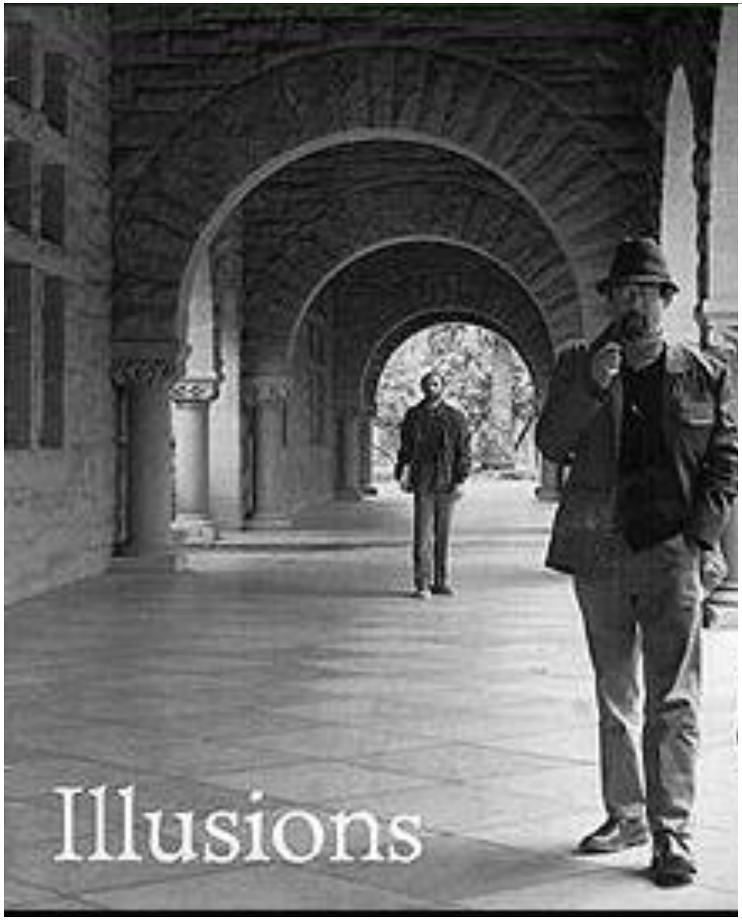
Calibration by vanishing points

- Intrinsic camera matrix

$$\mathbf{p}_i = \mathbf{KRX}_i$$

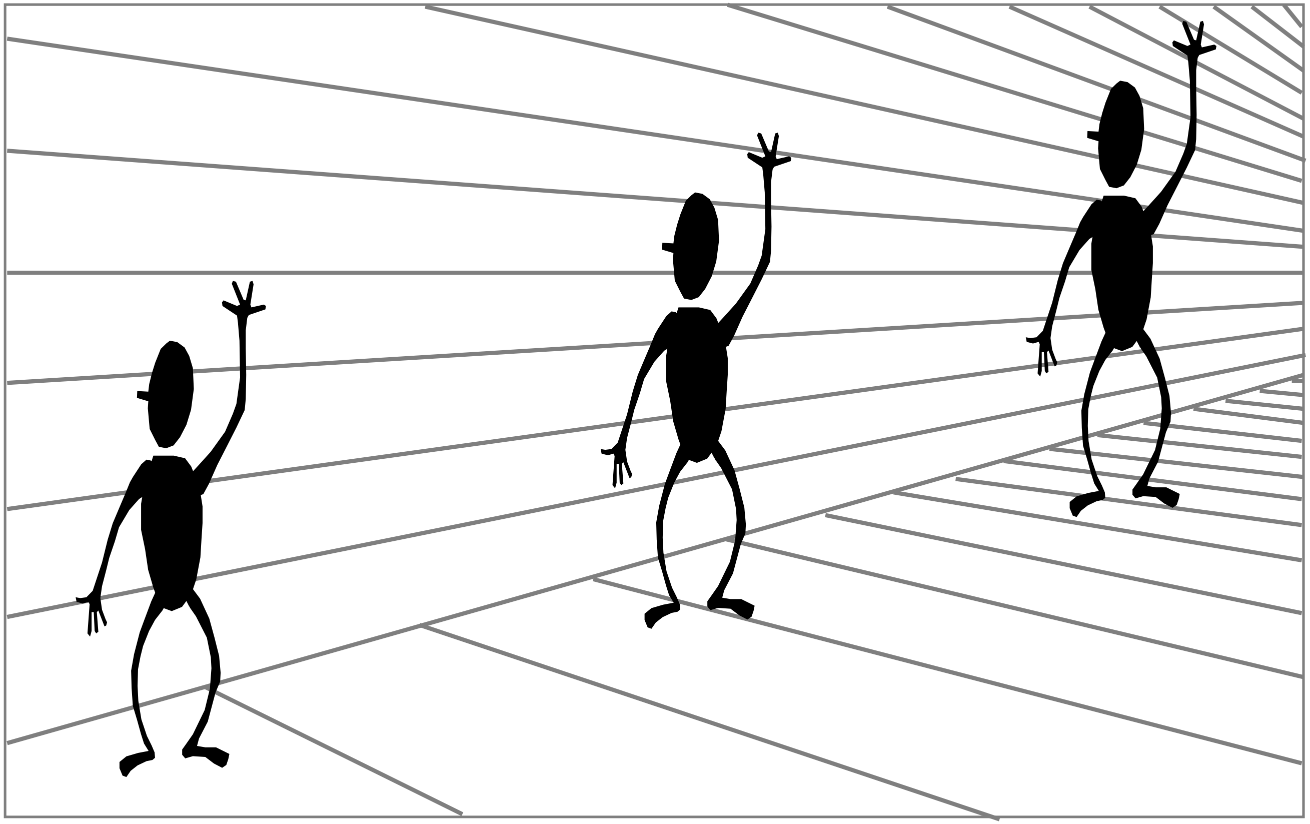
- Rotation matrix
  - Set directions of vanishing points
    - e.g., $\mathbf{X}_1 = [1, 0, 0]$
  - Each VP provides one column of **R**
  - Special properties of **R**
    - $\text{inv}(\mathbf{R}) = \mathbf{R}^T$
    - Each row and column of **R** has unit length

How can we measure the size of 3D objects from an image?



Slide by Steve Seitz

Perspective cues



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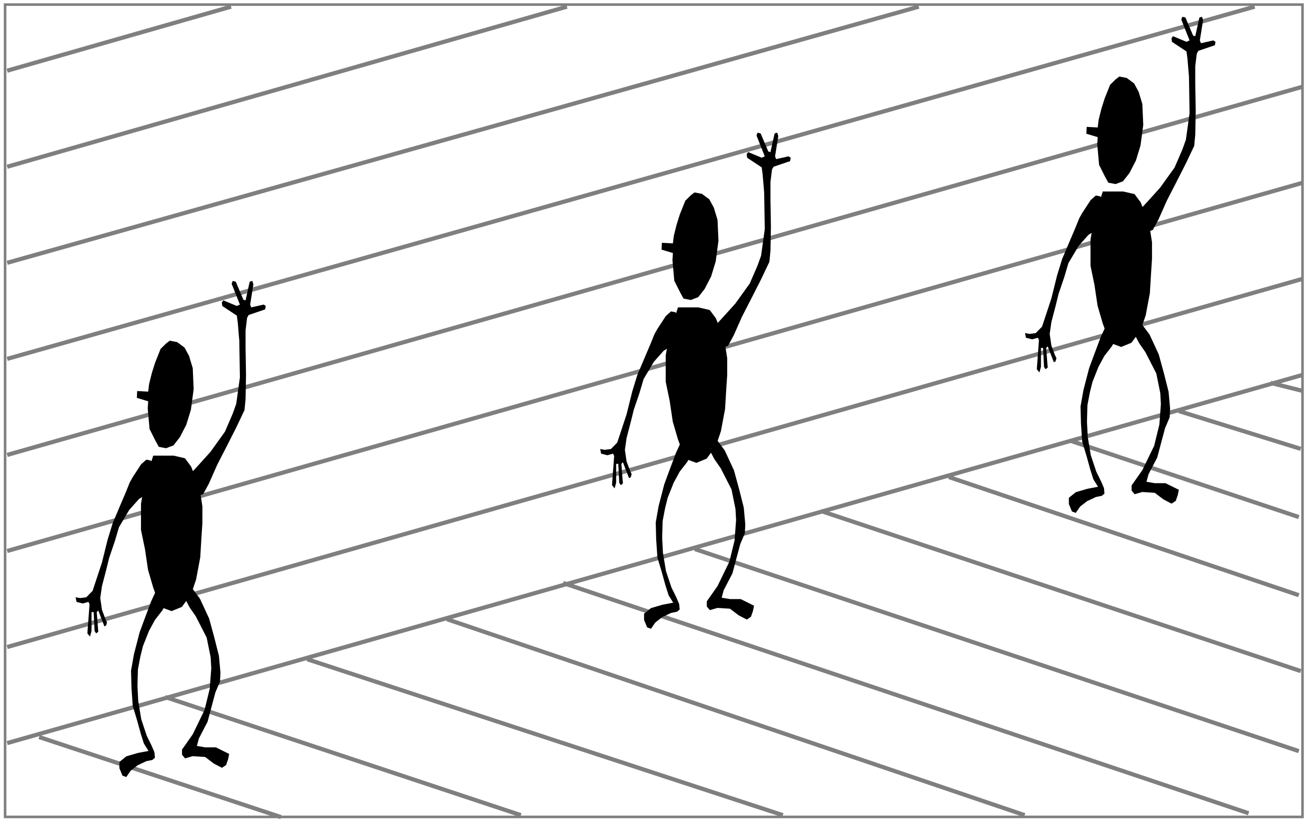
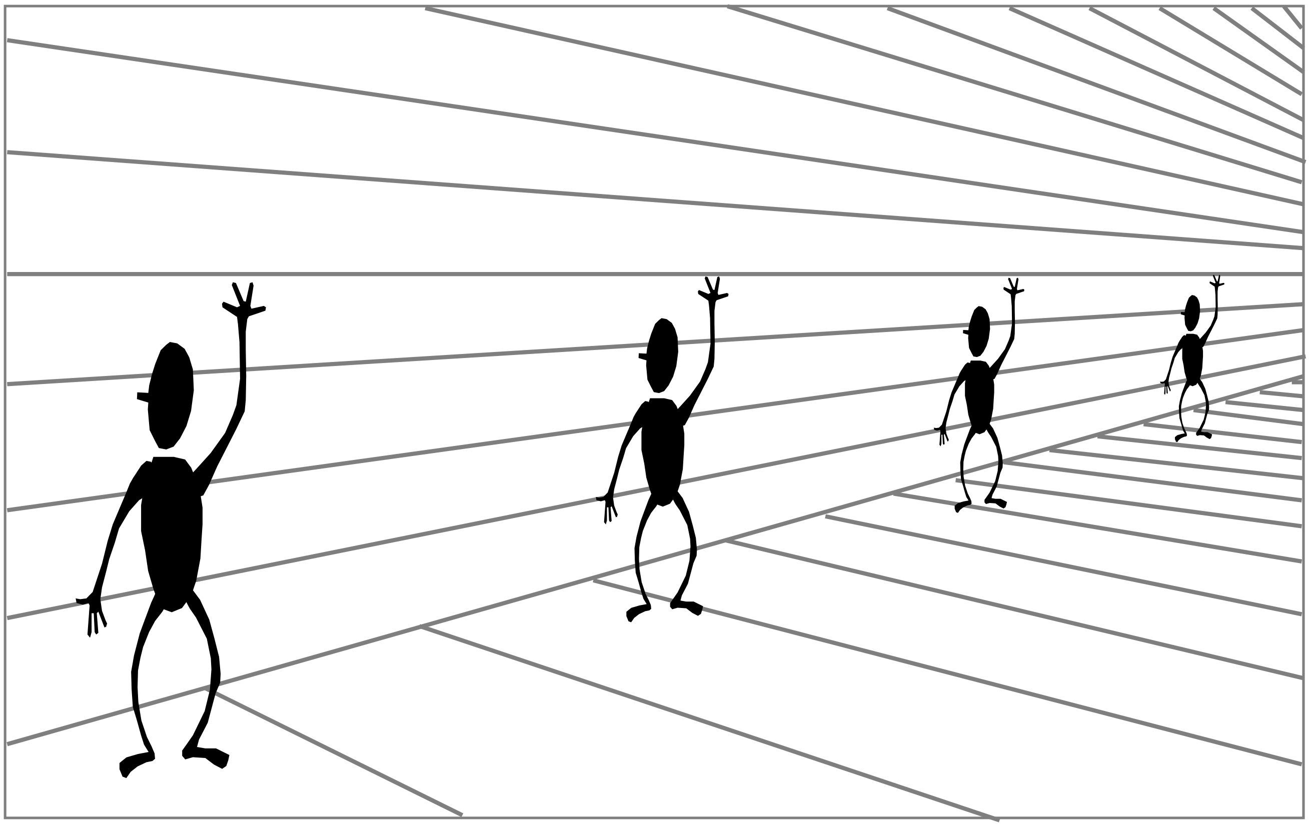
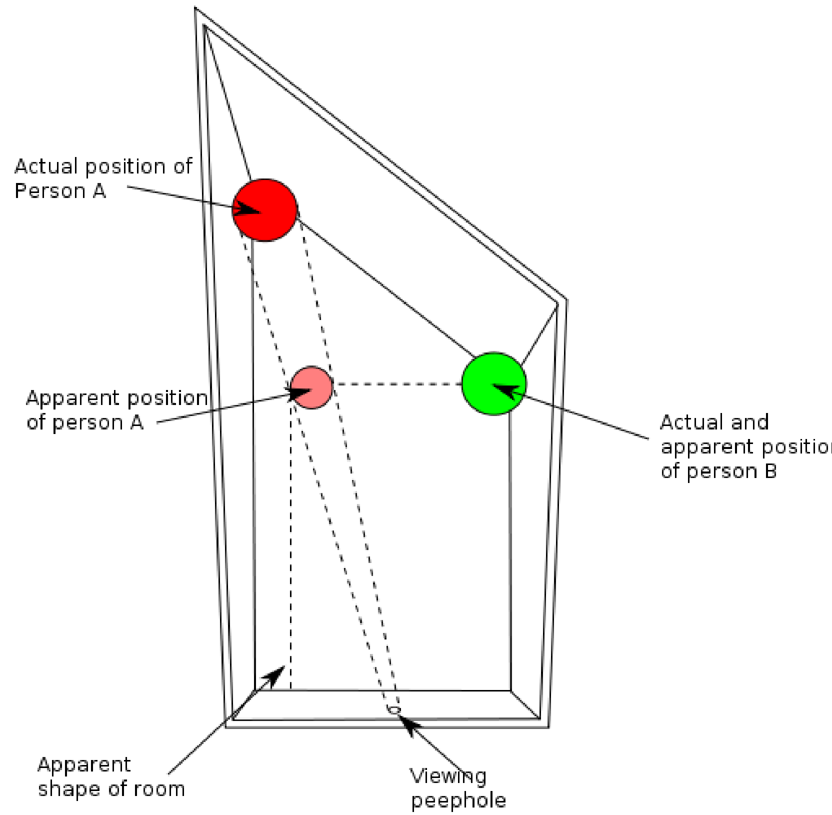
Perspective cues



Perspective cues



Ames Room



Actual position of Person A
Apparent position of person A
Actual and apparent position of person B
Apparent shape of room
Viewing peephole

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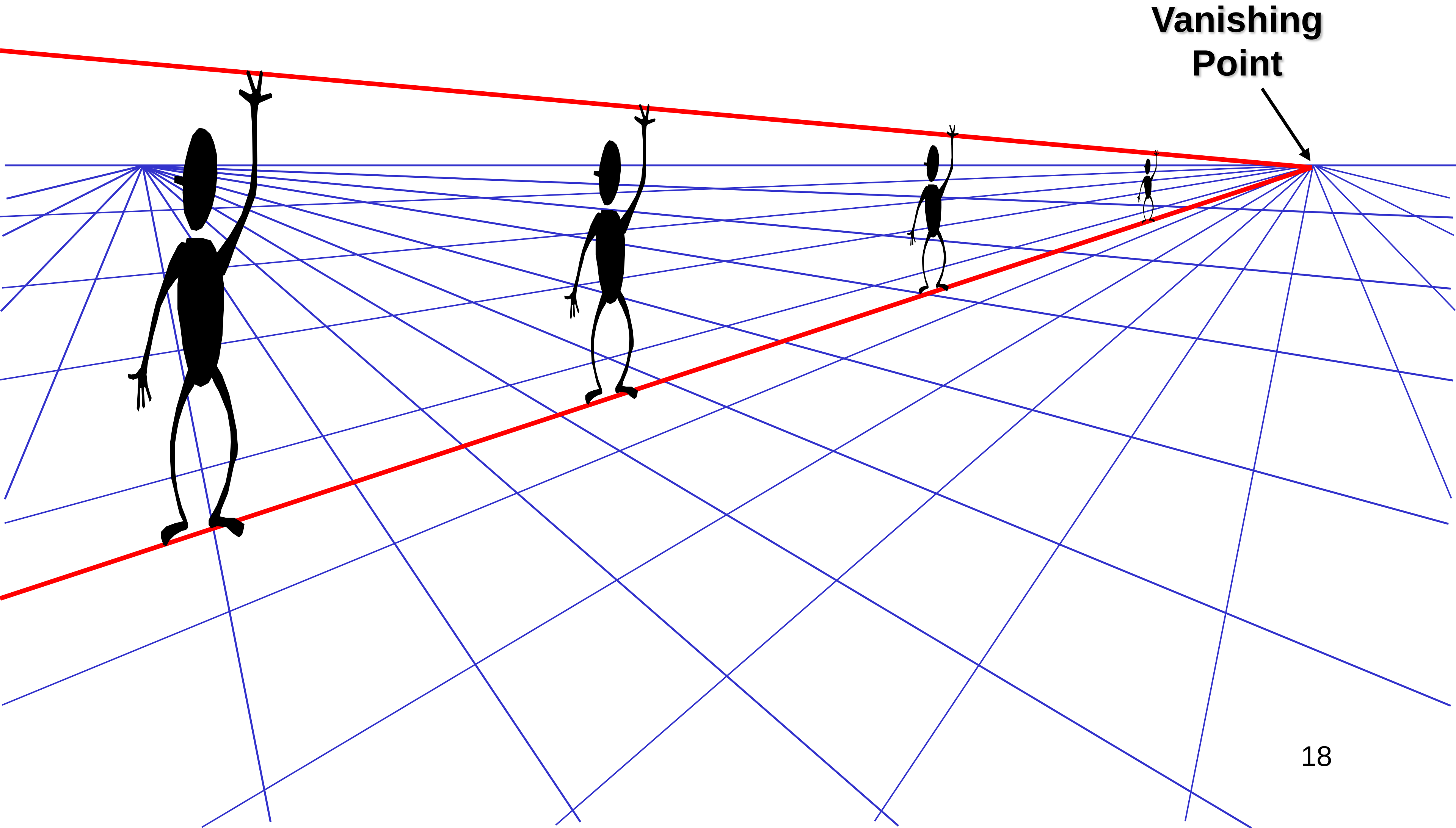
Comparing heights



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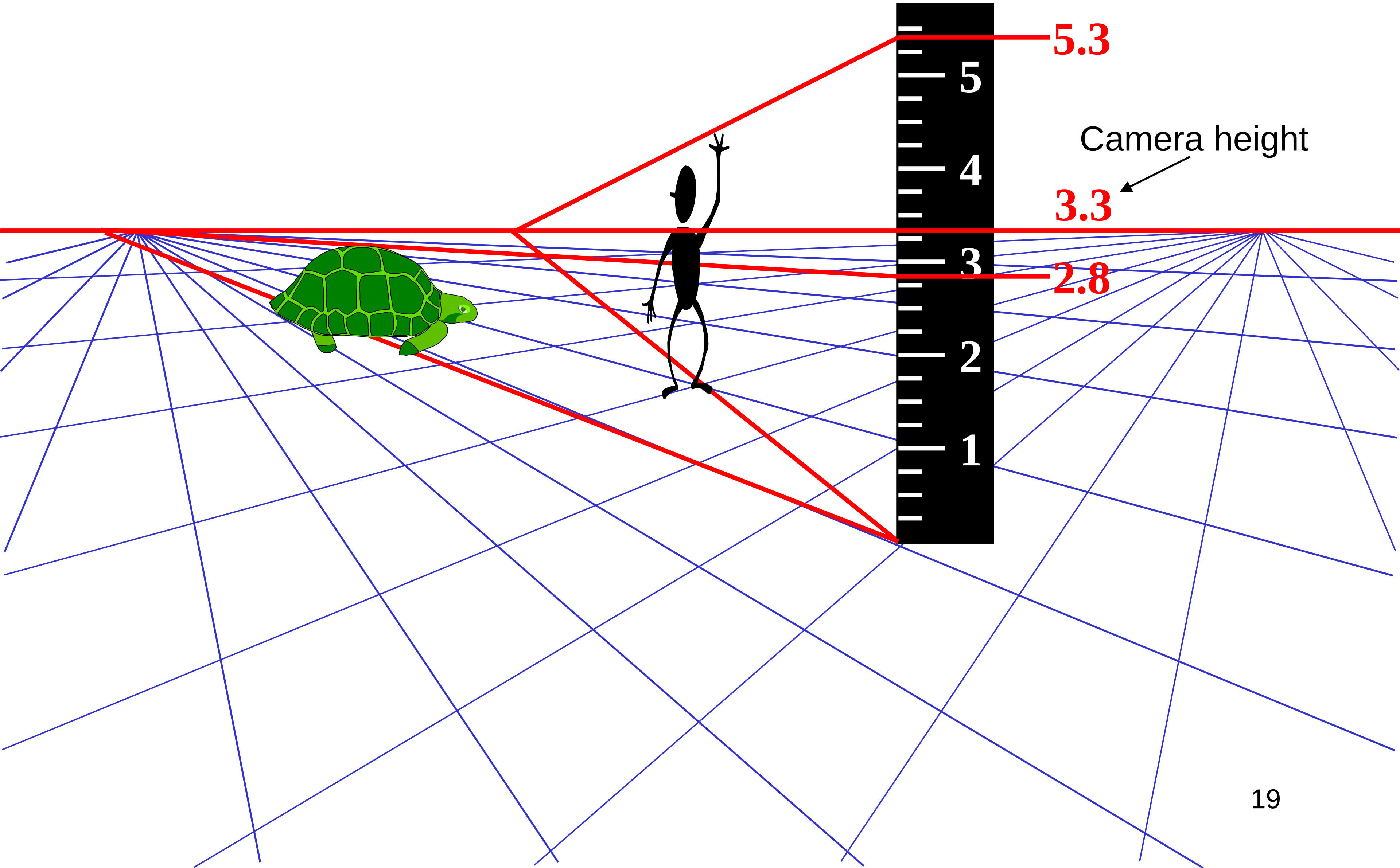
Measuring height



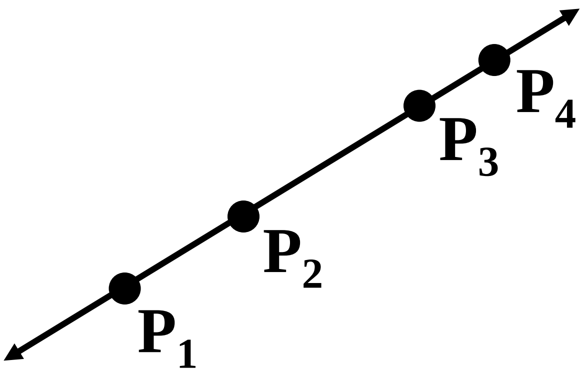
Which is higher – the camera or the man in the parachute?

The cross ratio

A Projective Invariant

- Something that does not change under projective transformations (including perspective projection)

The cross-ratio of 4 collinear points



$$\frac{\|\mathbf{P}_3 - \mathbf{P}_1\| \, \|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_3 - \mathbf{P}_2\| \, \|\mathbf{P}_4 - \mathbf{P}_1\|}$$

$$\mathbf{P}_i = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

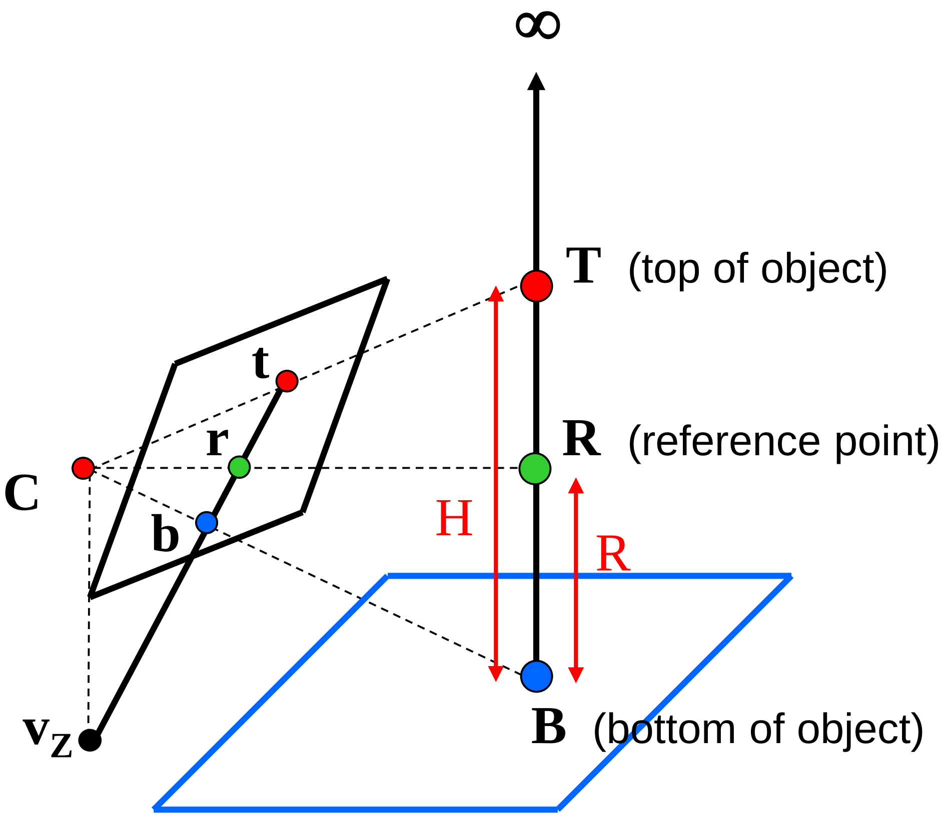
Can permute the point ordering

$$\frac{\|\mathbf{P}_1 - \mathbf{P}_3\| \, \|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_1 - \mathbf{P}_2\| \, \|\mathbf{P}_4 - \mathbf{P}_3\|}$$

- 4! = 24 different orders (but only 6 distinct values)

This is the fundamental invariant of projective geometry

Measuring height



$$\frac{\|\mathbf{B}-\mathbf{T}\|\,\|\infty-\mathbf{R}\|}{\|\mathbf{B}-\mathbf{R}\|\,\|\infty-\mathbf{T}\|} = \frac{H}{R}$$

scene cross ratio

$$\frac{\|\mathbf{b}-\mathbf{t}\|\,\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{b}-\mathbf{r}\|\,\|\mathbf{v}_Z-\mathbf{t}\|} = \frac{H}{R}$$

image cross ratio

scene points represented as $\mathbf{P} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$ image points as $\mathbf{p} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

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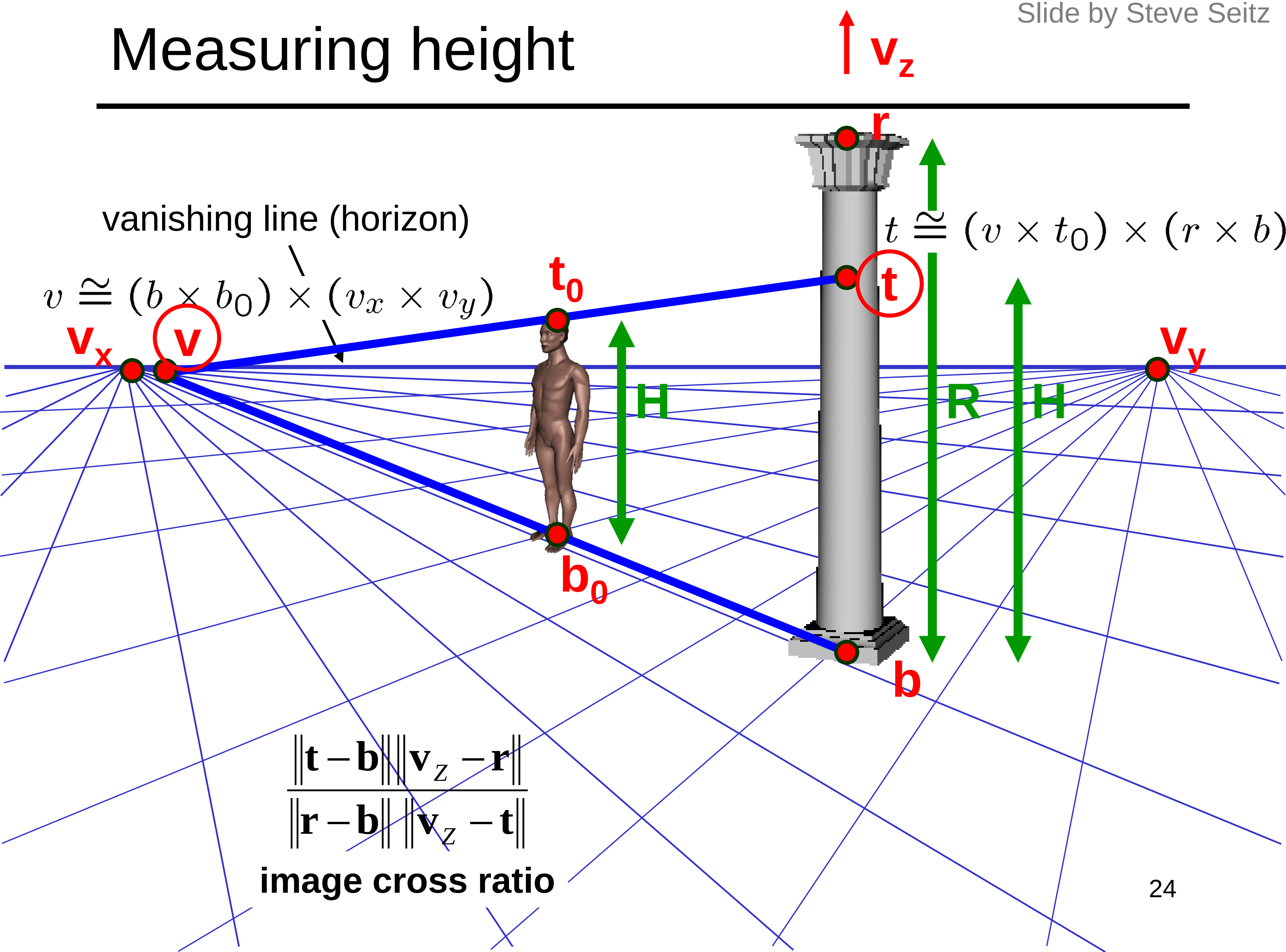
Measuring height



$$\frac{\|\mathbf{t}-\mathbf{b}\| \|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\| \|\mathbf{v}_Z-\mathbf{t}\|}$$

image cross ratio

Measuring height

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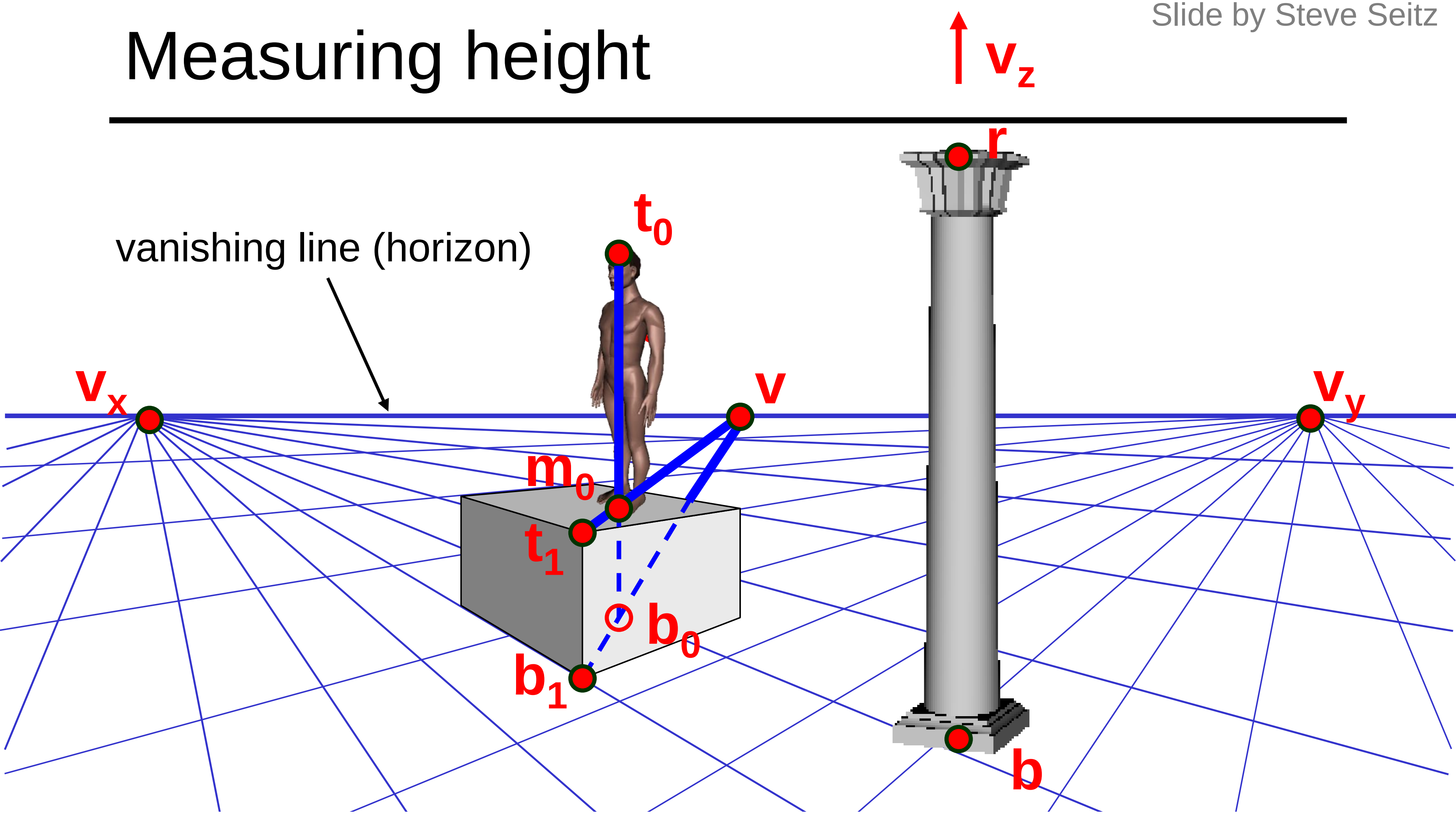
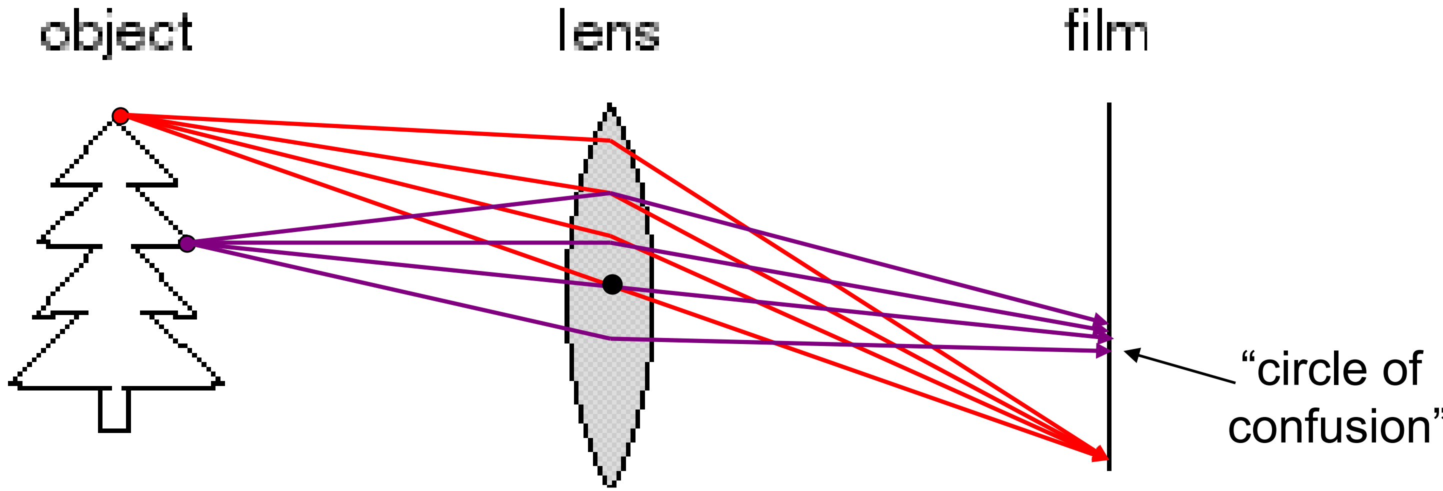


What if the point on the ground plane $\mathbf{b_0}$ is not known?

- Here the guy is standing on the box, height of box is known
- Use one side of the box to help find $\mathbf{b_0}$ as shown above

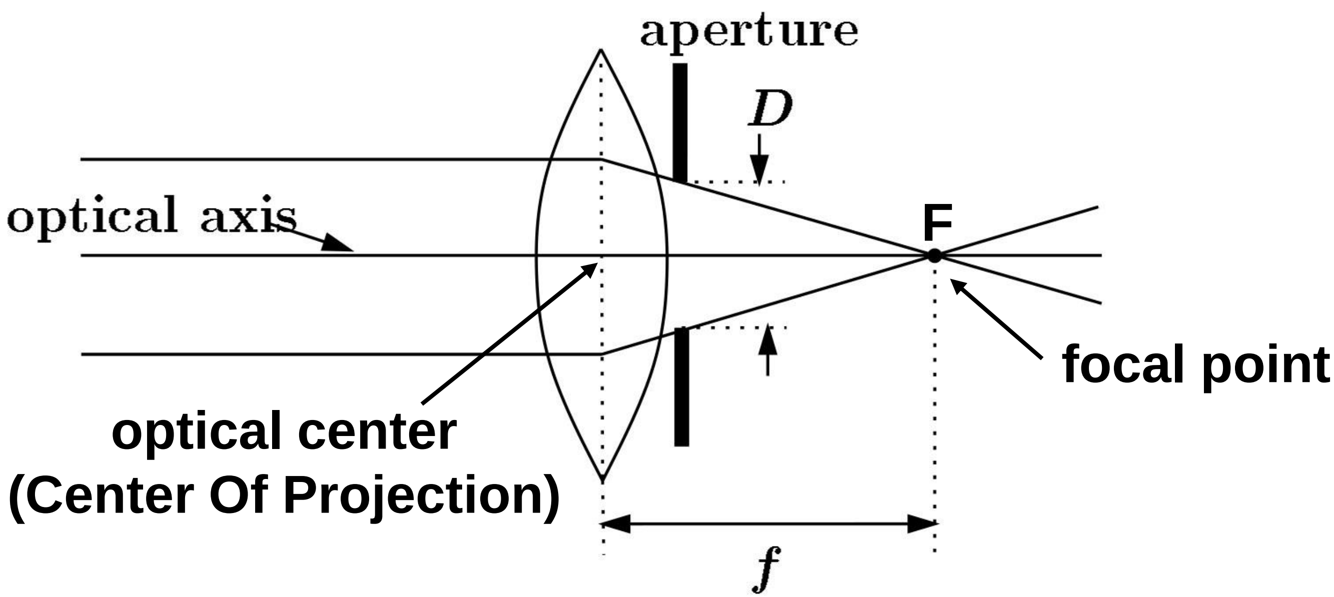
What about focus, aperture, DOF, FOV, etc?

Adding a lens



- A lens focuses light onto the film
  - There is a specific distance at which objects are “in focus”
    - other points project to a “circle of confusion” in the image
  - Changing the shape of the lens changes this distance

Focal length, aperture, depth of field

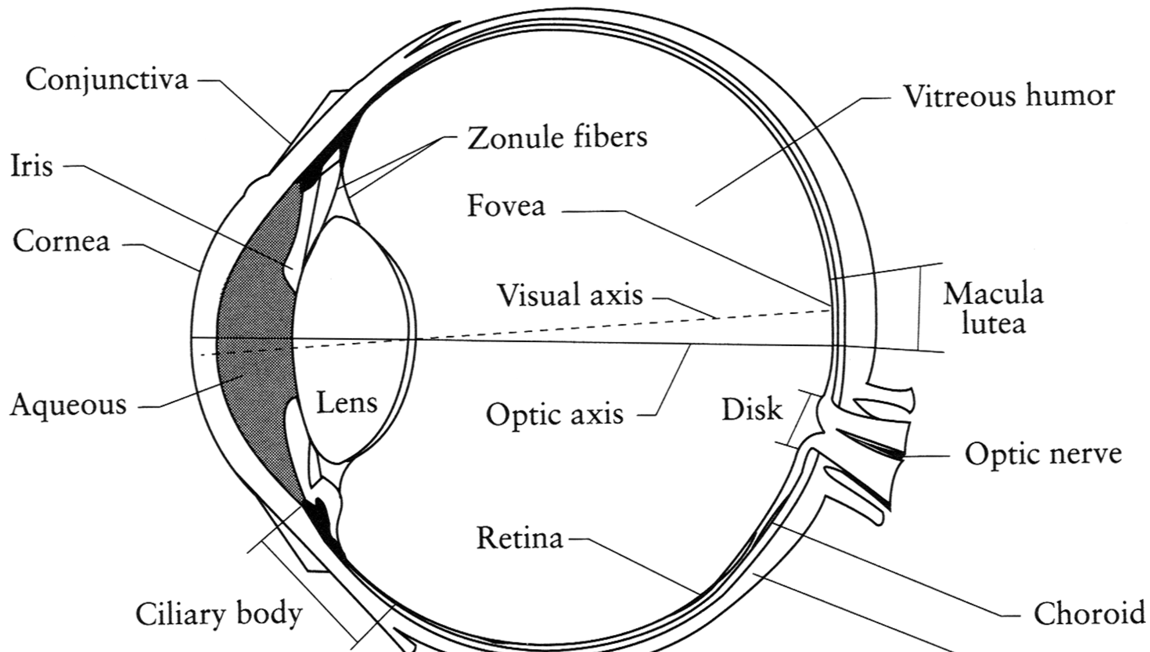


A lens focuses parallel rays onto a single focal point

- focal point at a distance *f* beyond the plane of the lens
- Aperture of diameter D restricts the range of rays

Slide source: Seitz

The eye



- The human eye is a camera
  - **Iris** - colored annulus with radial muscles
  - **Pupil** (aperture) - the hole whose size is controlled by the iris
  - **Retina** (film): photoreceptor cells (rods and cones)

Slide source: Seitz

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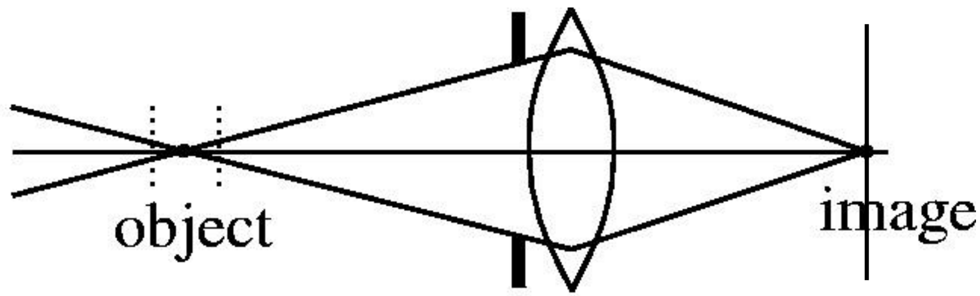
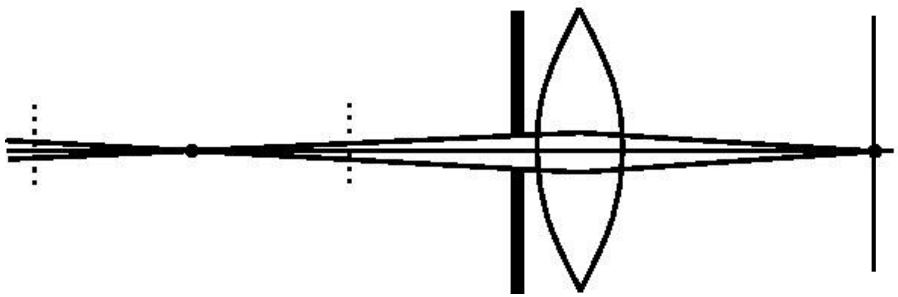
Depth of field



f / 5.6



f / 32

Changing the aperture size or focal length affects depth of field

Flower images from Wikipedia http://en.wikipedia.org/wiki/Depth_of_field

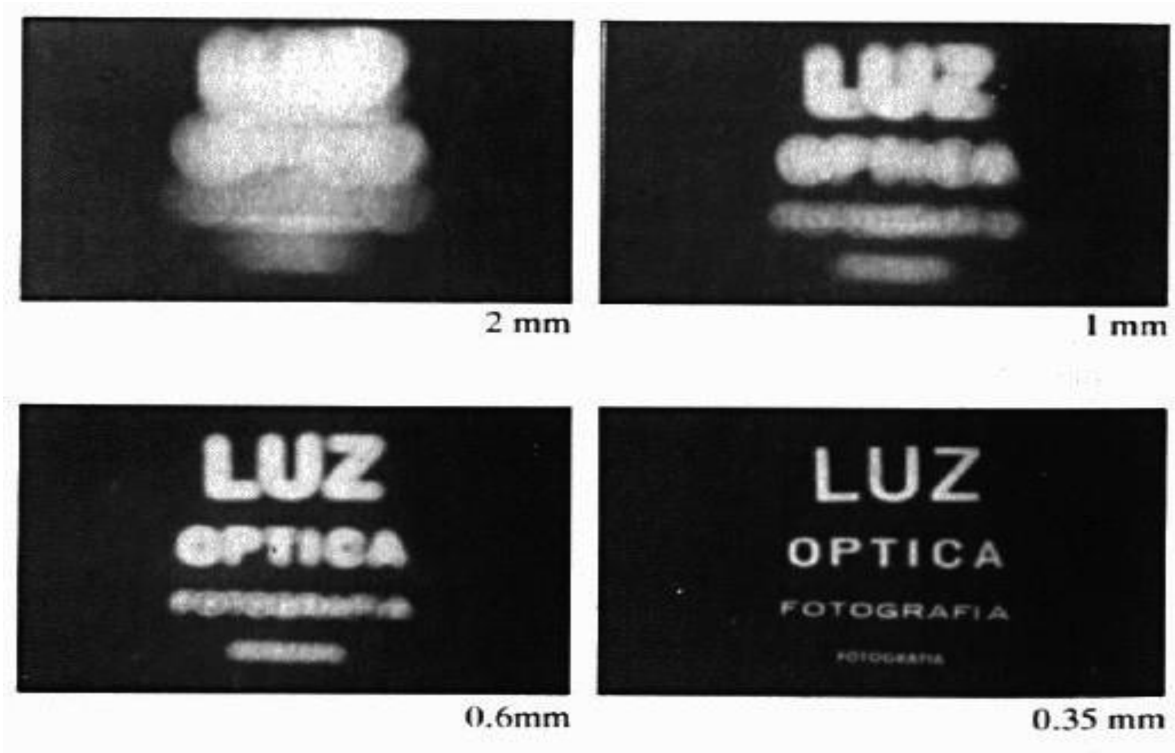
Slide from Efros

Varying the aperture

Large aperture = small DOF

Small aperture = large DOF

Shrinking the aperture



- Why not make the aperture as small as possible?
  - Less light gets through
  - Diffraction effects

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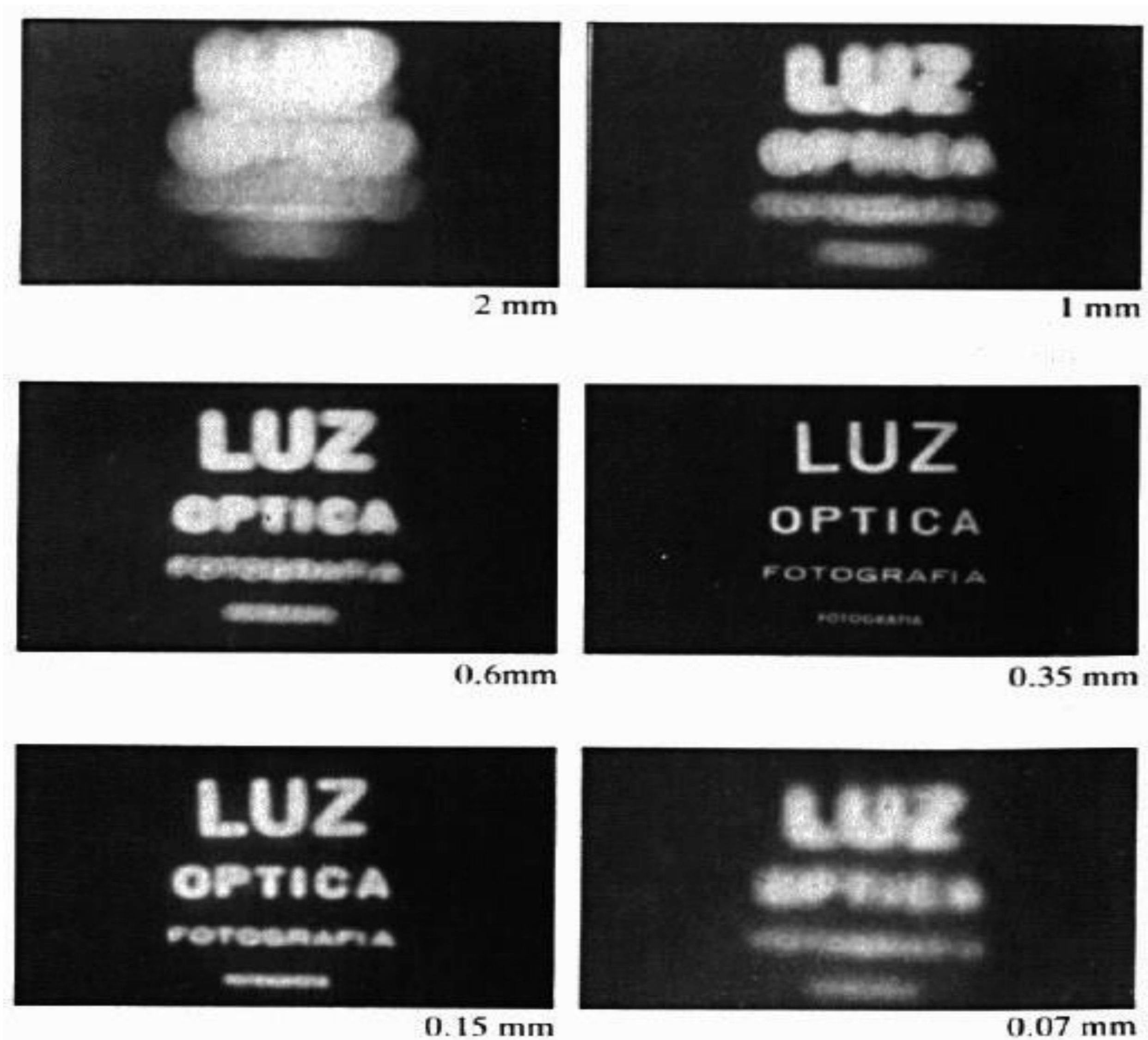
Shrinking the aperture



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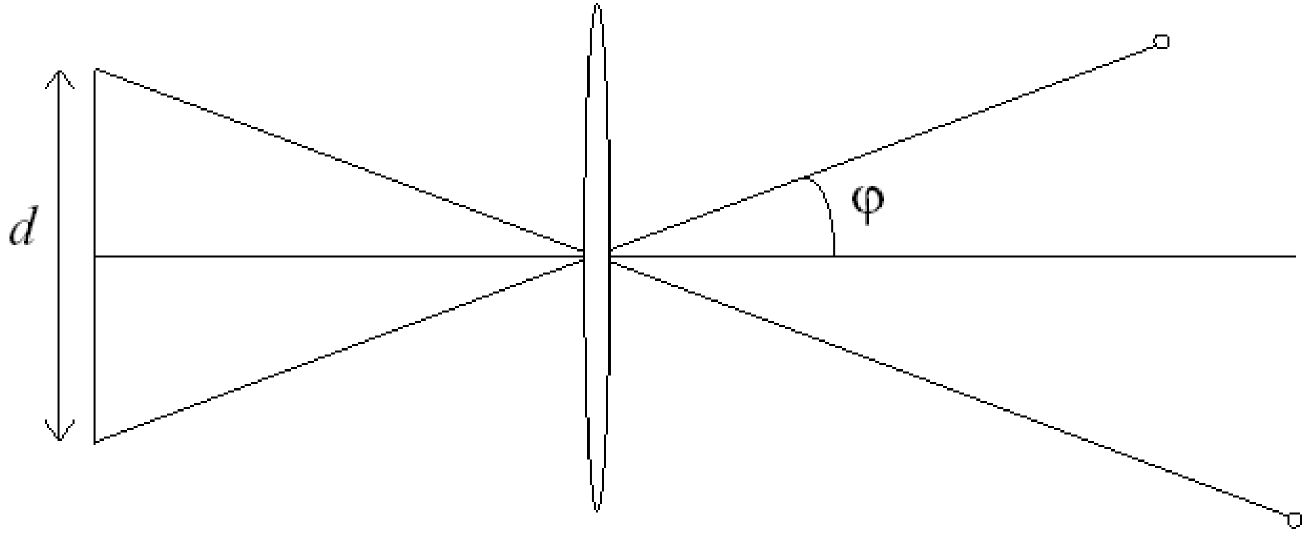
Relation between field of view and focal length

Field of view (angle width)

Film/Sensor Width

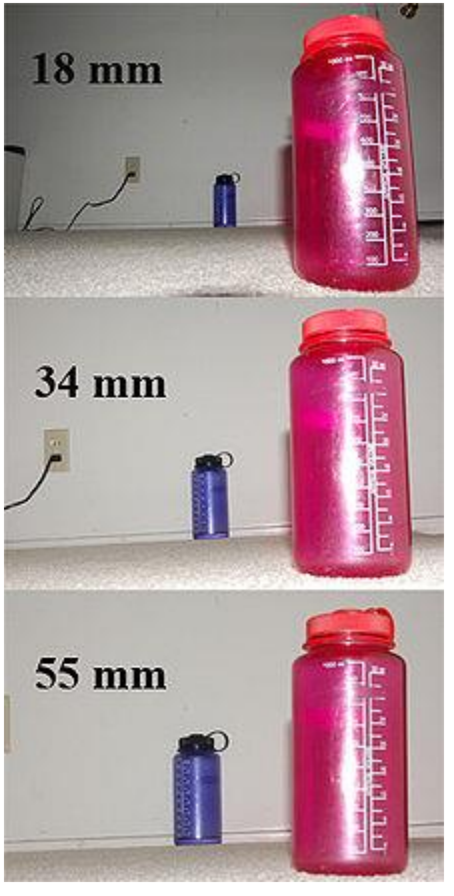
$$fov = \tan^{-1} \frac{d}{2f}$$

Focal length



Dolly Zoom or "Vertigo Effect"

http://www.youtube.com/watch?v=NB4bikrNzMk

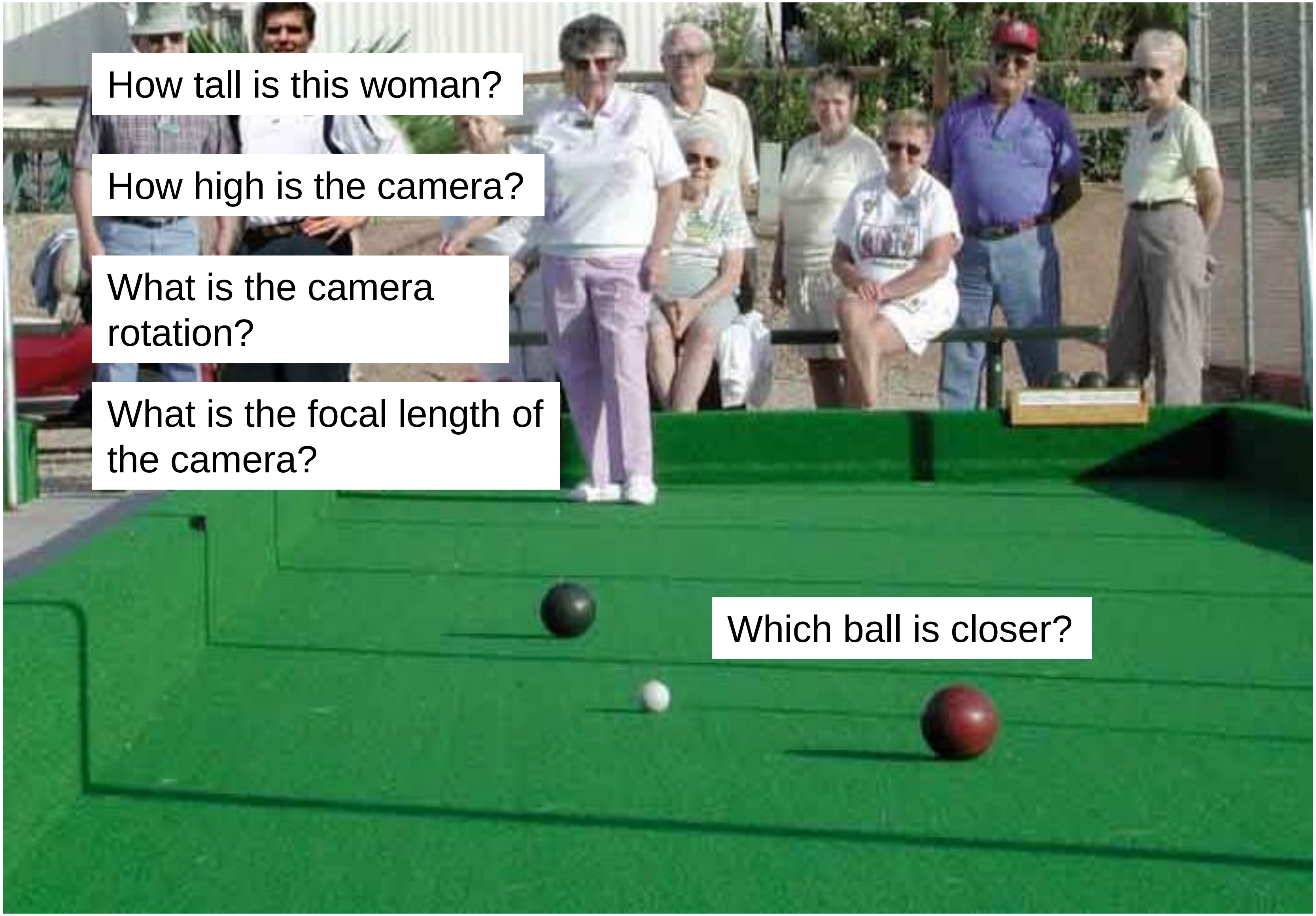


How is this done?

Zoom in while moving away

http://en.wikipedia.org/wiki/Focal_length

Review


How tall is this woman?
How high is the camera?
What is the camera rotation?
What is the focal length of the camera?
Which ball is closer?

Next class

- Image stitching