Projective Geometry and Camera Models

Computer Vision

CS 543 / ECE 549

University of Illinois

Derek Hoiem

HWs

- HW 1 back today
  - Top segmentation scores:
    - (0.618, 0.659) – Zigang Xiao: multiscale filters
    - (0.623, 0.653) – Fu Ouyang: oriented filters, + morphological operators to close gaps
  - Common mistakes
    - Forgetting to convert to double when computing Laplacian pyramids
    - Mistakes in creating oriented filters
    - Incorrectly applied non-max suppression
  - Solutions are posted

- HW 2 due next Tues

- HW 3 should be out Thursday

Think about your final projects

- Strongly encouraged to work in groups of 2-4 (but if you have a good reason to work by self, could be ok)

- Projects don’t need to be of publishable originality but should evince independent effort to learn about a new topic, try something new, or apply to an application of interest

- Proposals will be due after Spring Break

- Project ideas from Cinda Hereen
  1. **Classroom attendance**: I teach in Siebel 1404, and I'd like to be able to track attendance by taking photos of the room (students who opt out can just put their heads down). Extra facts in the problem: a) I have photo rosters; b) i'm willing to use a tagging system wherein students can verify our assertions; c) the results don't have to be exact.
  2. **Medication tracking:** I would like for a person to be able to chart his/her medicine consumption by taking a photo of the med bottles. Extra facts: a. it has to work on a mobile device (limited computation), b. it could be a matching problem--could photograph known meds and use the photos as the labels.
  3. **Thermometer reading:** In a related problem, I'd like to be able to take a picture of a thermometer (digital or analogue) and record it's reading.

One last note on registration

- Thin-plate splines: combines global affine warp with smooth local deformation

$$E_{TPS}(f) = \sum_{a=1}^{K} ||y_a - f(v_a)||^2 + \lambda \int\int \left[(\frac{\partial^2 f}{\partial x^2})^2 + 2(\frac{\partial^2 f}{\partial x \partial y})^2 + (\frac{\partial^2 f}{\partial y^2})^2 \right] dxdy$$

Diff of predicted vs. actual position

Smoothness cost for local warps

$$f(v_a, d, w) = v_a \cdot d + \phi(v_a) \cdot w$$

Affine warp

Local deformation according to distance from control points

- Robust non-rigid point matching: http://noodle.med.yale.edu/~chui/tps-rpm.html (includes code, demo, paper)
  - Thin-plate spline registration with robustness to outliers

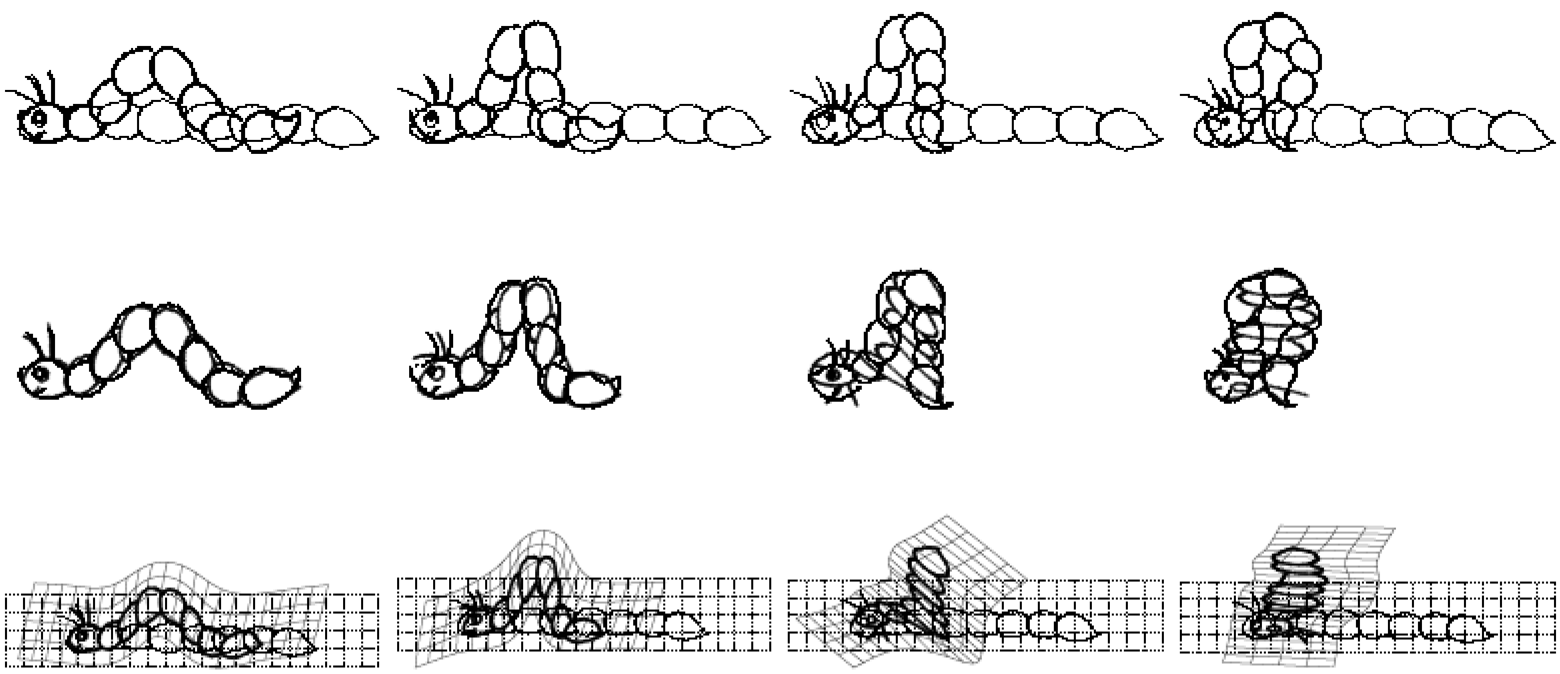
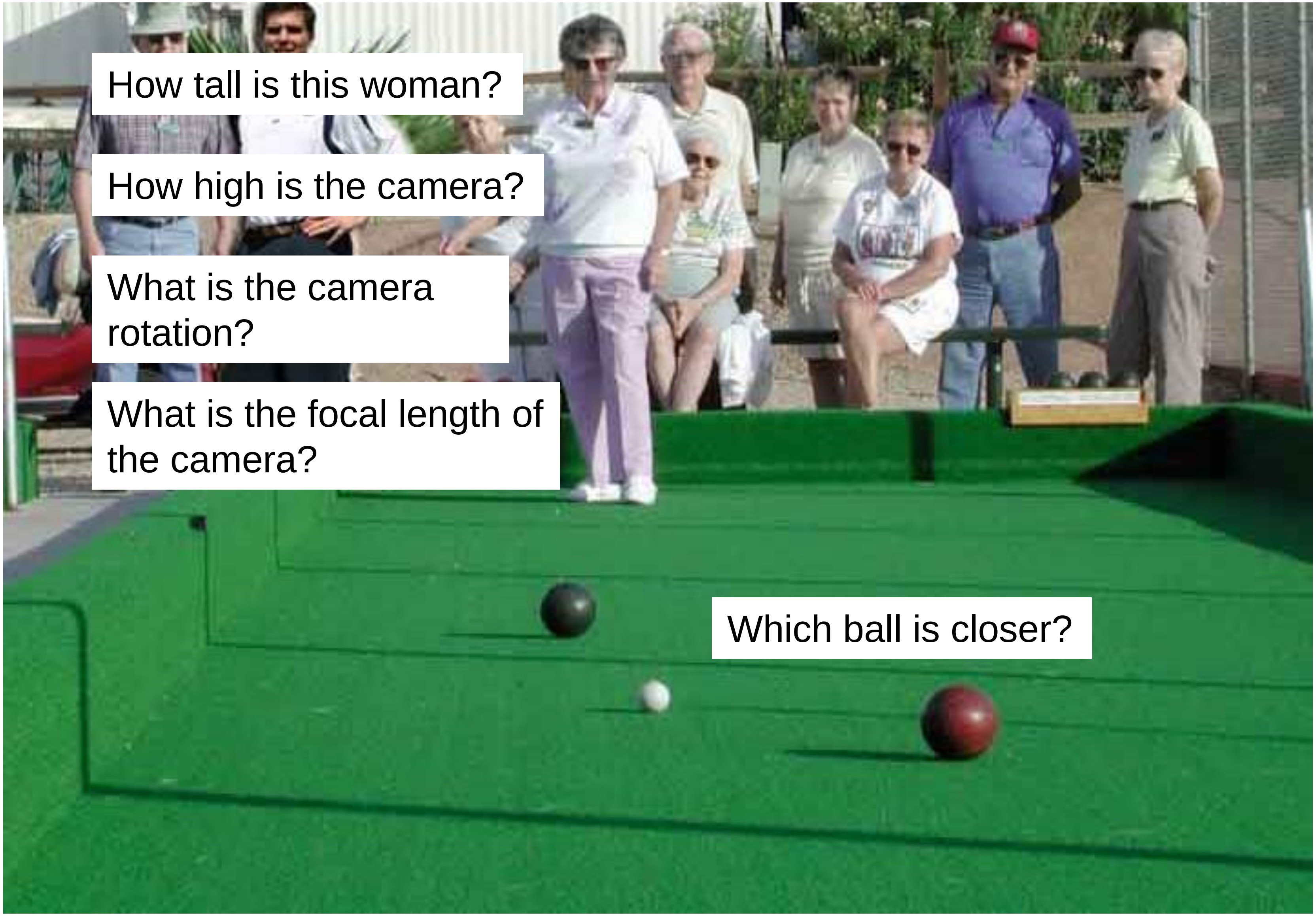


Fig. 12. **Large Deformation—Caterpillar Example.** From left to right, matching frame 1 to frame 5, 7, 11 and 12. **Top:** Original location. **Middle:** matched result. **Bottom:** deformation found.

Next two classes: Single-view Geometry

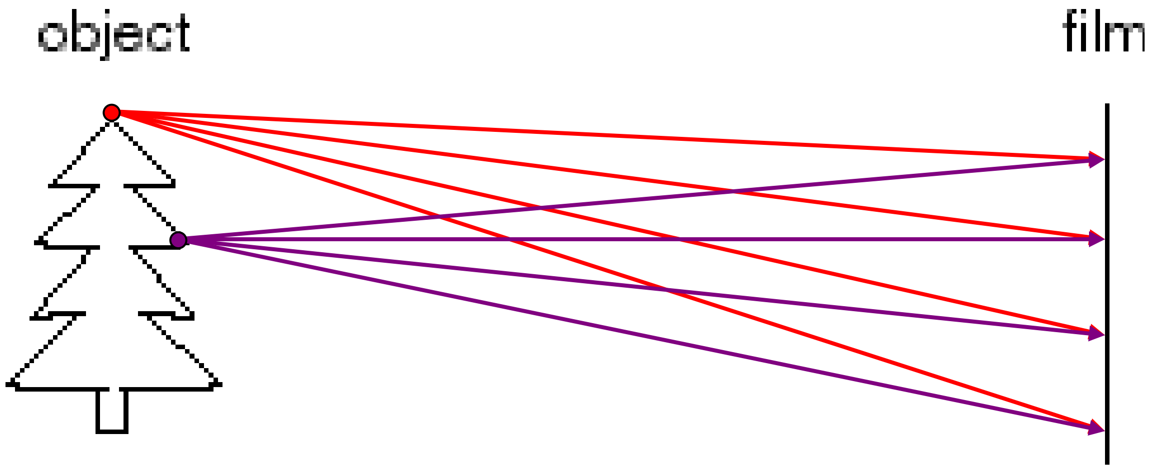


Today's class

Mapping between image and world coordinates

– Pinhole camera model
– Projective geometry
    - Vanishing points and lines
– Projection matrix

Image formation

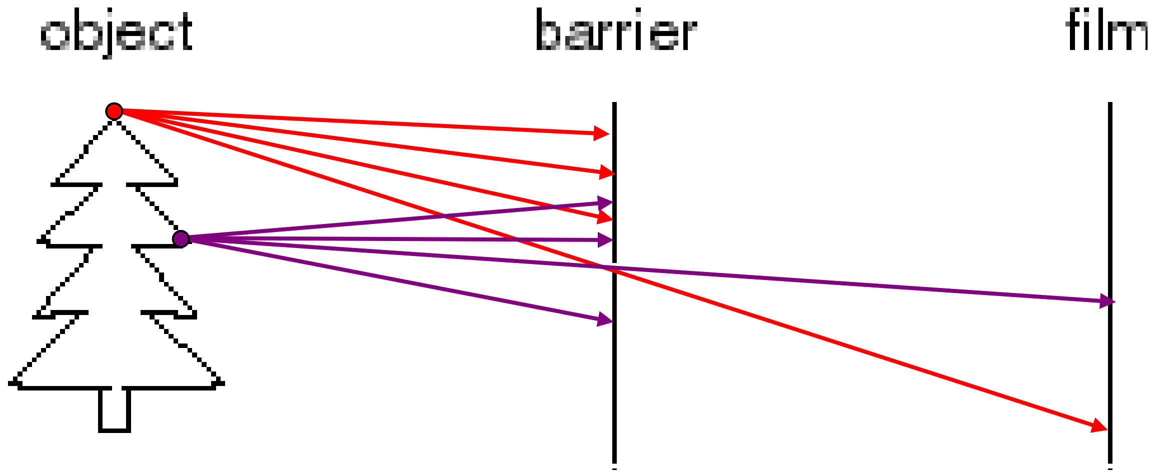


Let's design a camera

- Idea 1: put a piece of film in front of an object
- Do we get a reasonable image?

Slide source: Seitz

Pinhole camera

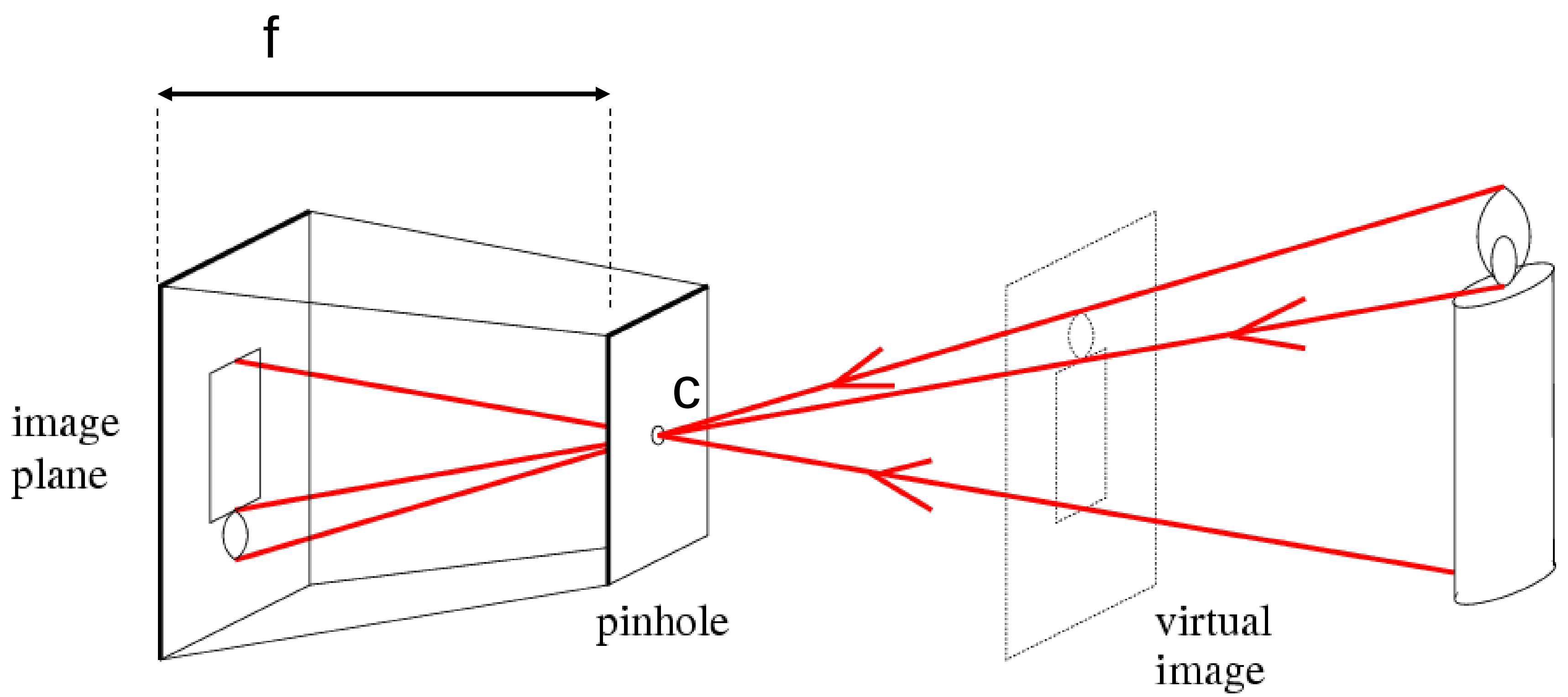


Idea 2: add a barrier to block off most of the rays

- This reduces blurring
- The opening known as the **aperture**

Slide source: Seitz

Pinhole camera



f = focal length
c = center of the camera

Figure from Forsyth

Camera obscura: the pre-camera

- First idea: Mo-Ti, China (470BC to 390BC)

- First built: Alhacen, Iraq/Egypt (965 to 1039AD)

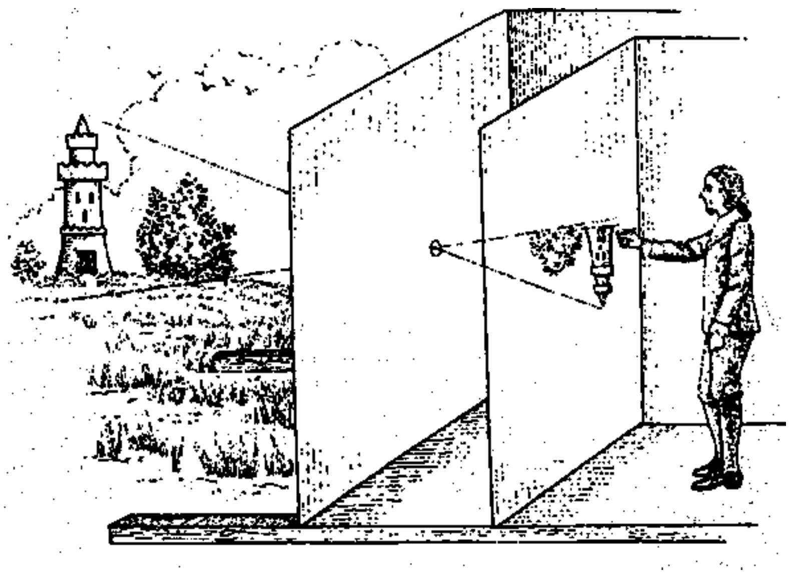
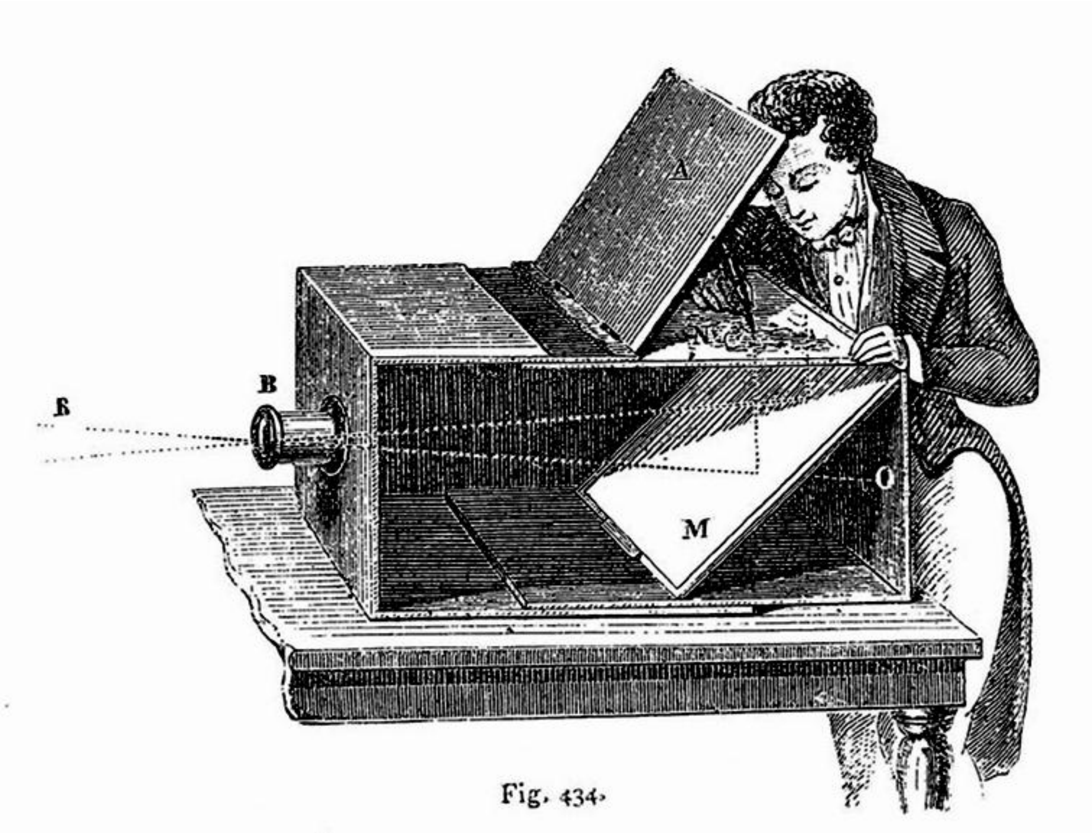


Illustration of Camera Obscura

Freestanding camera obscura at UNC Chapel Hill

Photo by Seth Ilys

Camera Obscura used for Tracing



Lens Based Camera Obscura, 1568

First Photograph

Oldest surviving photograph

- Took 8 hours on pewter plate

Joseph Niepce, 1826

Photograph of the first photograph

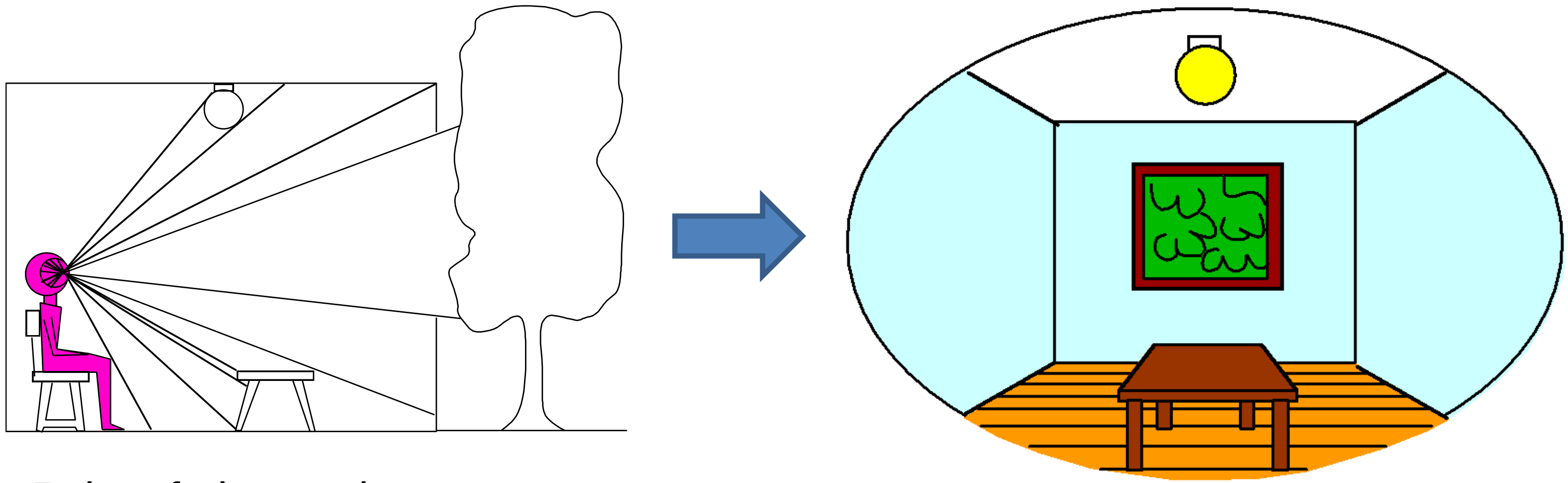
Stored at UT Austin

Niepce later teamed up with Daguerre, who eventually created Daguerrotypes

Dimensionality Reduction Machine (3D to 2D)

3D world

2D image



Point of observation

Slide source: Seitz

Projection can be tricky…

Slide source: Seitz

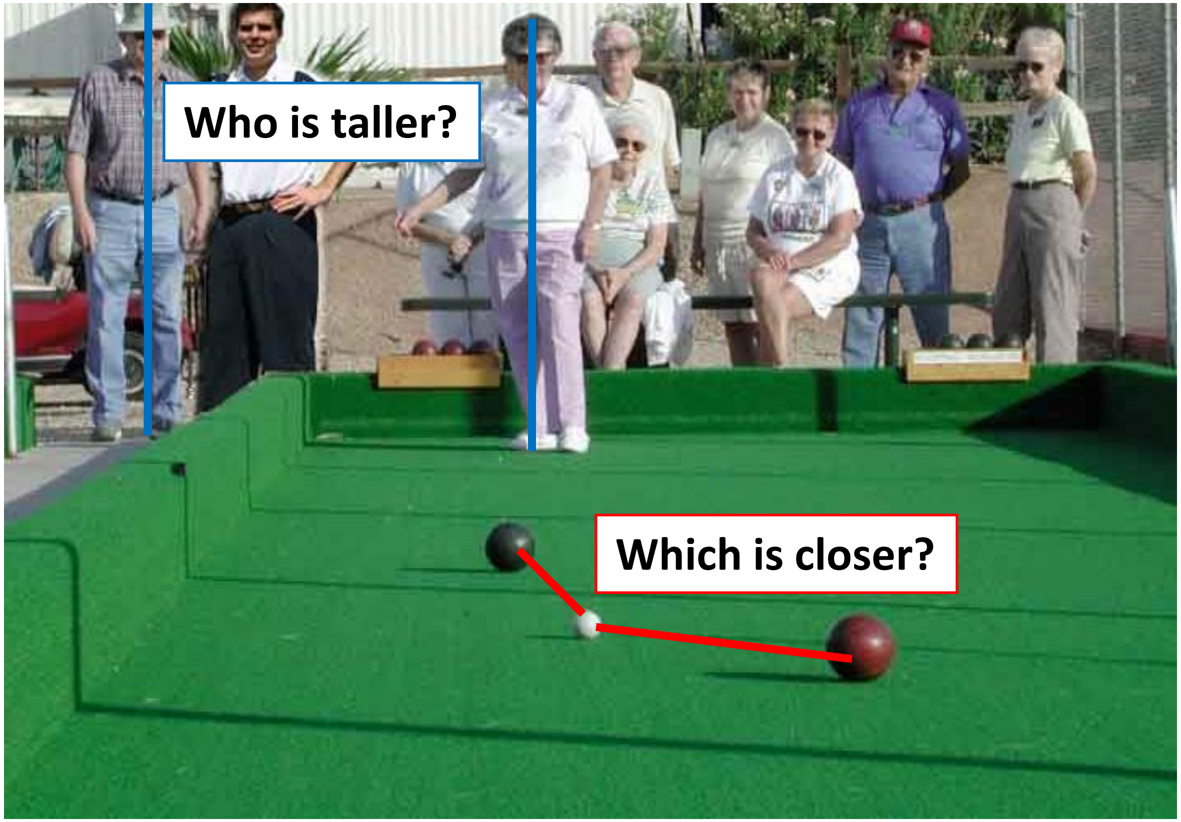
Projection can be tricky…

Making of 3D sidewalk art: http://www.youtube.com/watch?v=3SNYtd0Ayt0

Projective Geometry

What is lost?

- Length



Length is not preserved

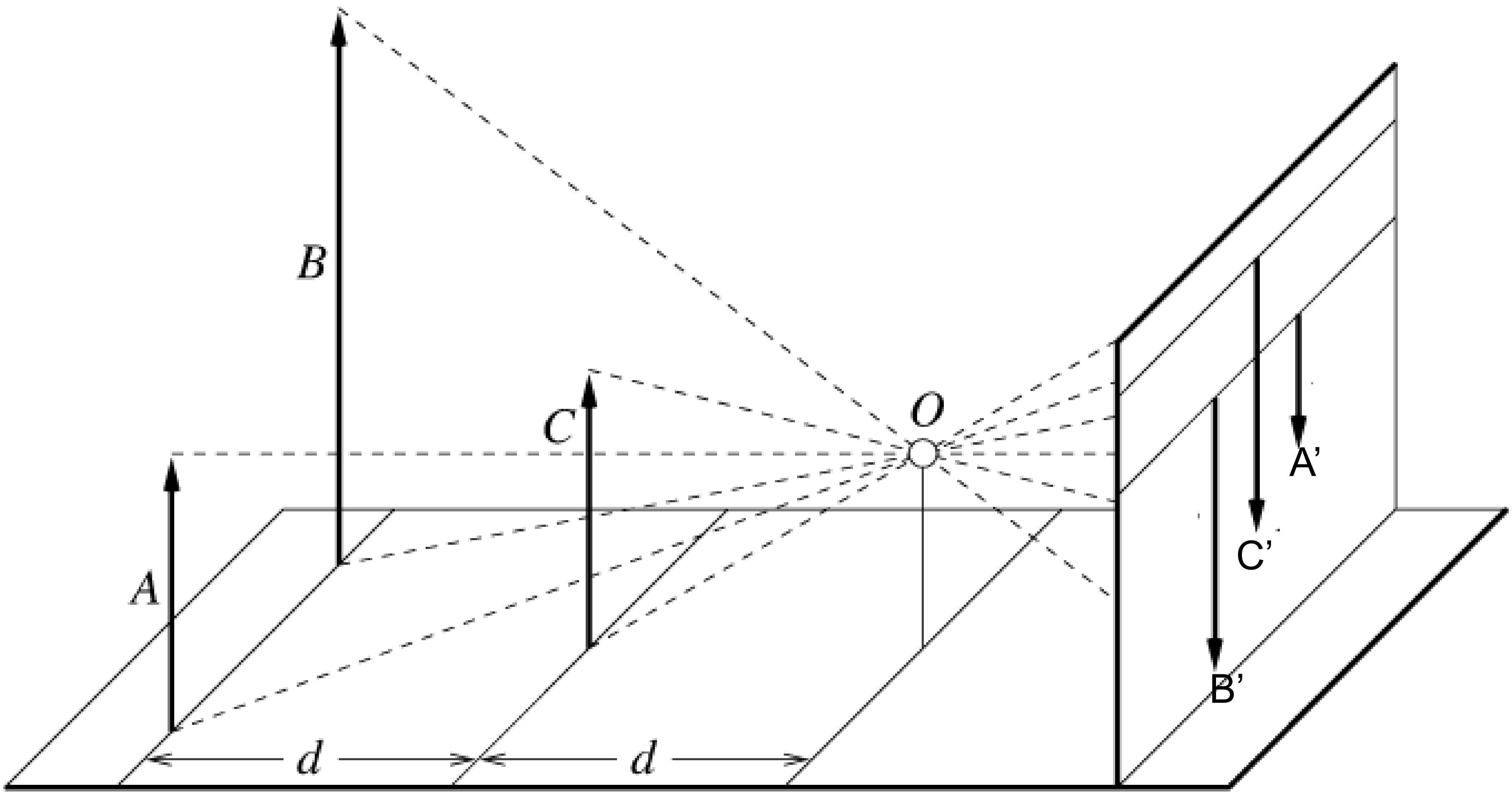
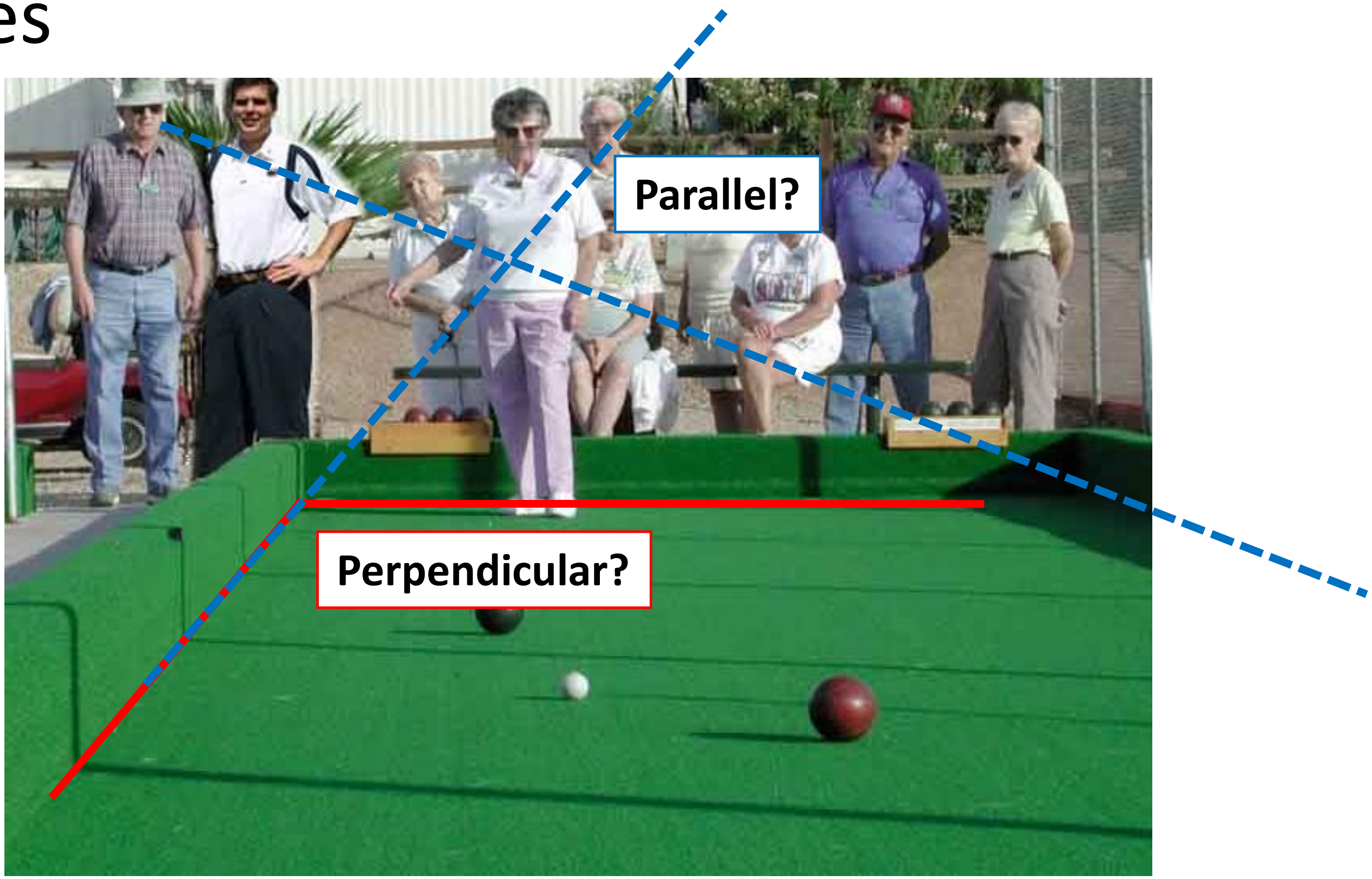


Figure by David Forsyth

Projective Geometry

What is lost?

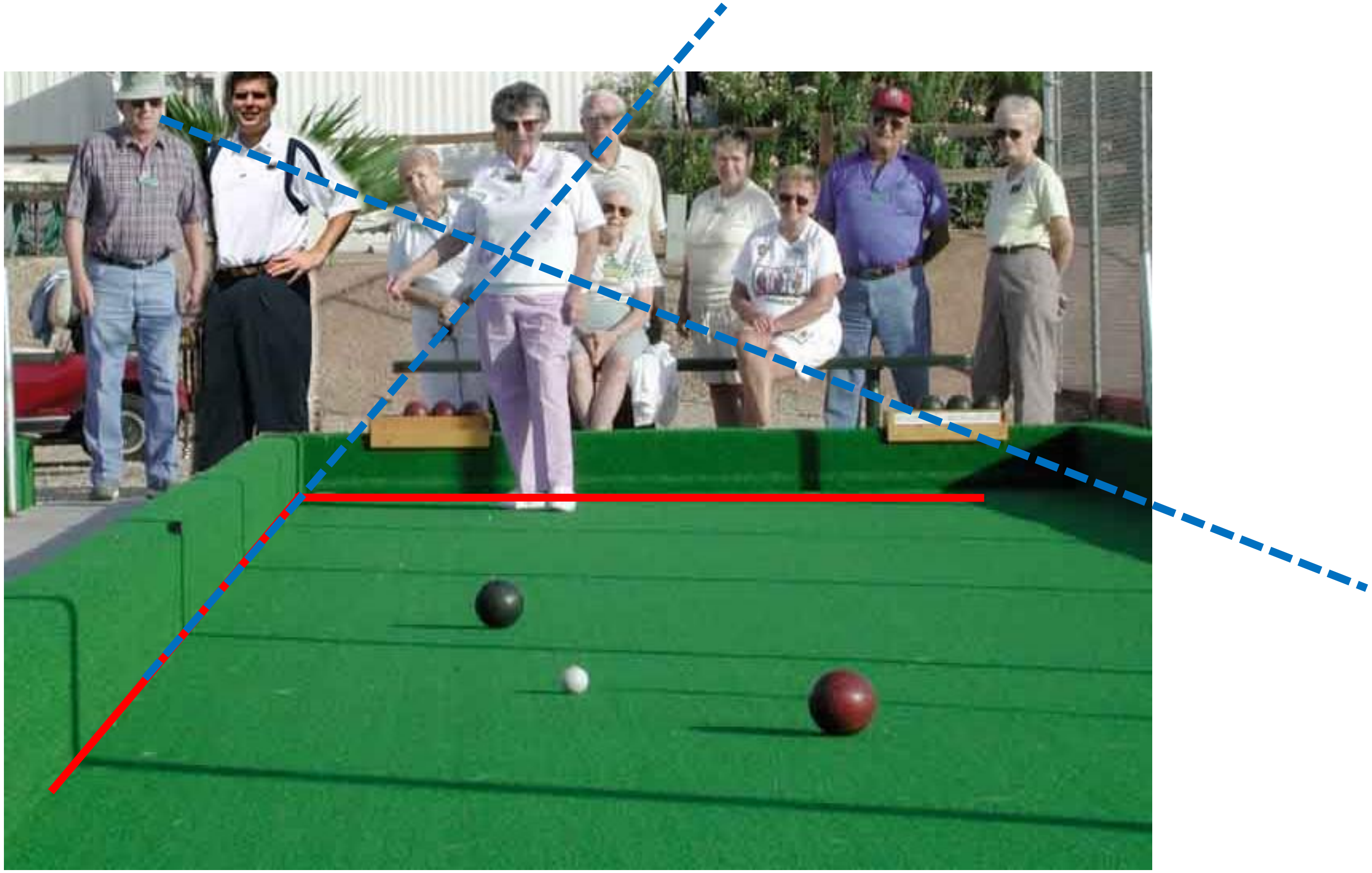
- Length
- Angles



Projective Geometry

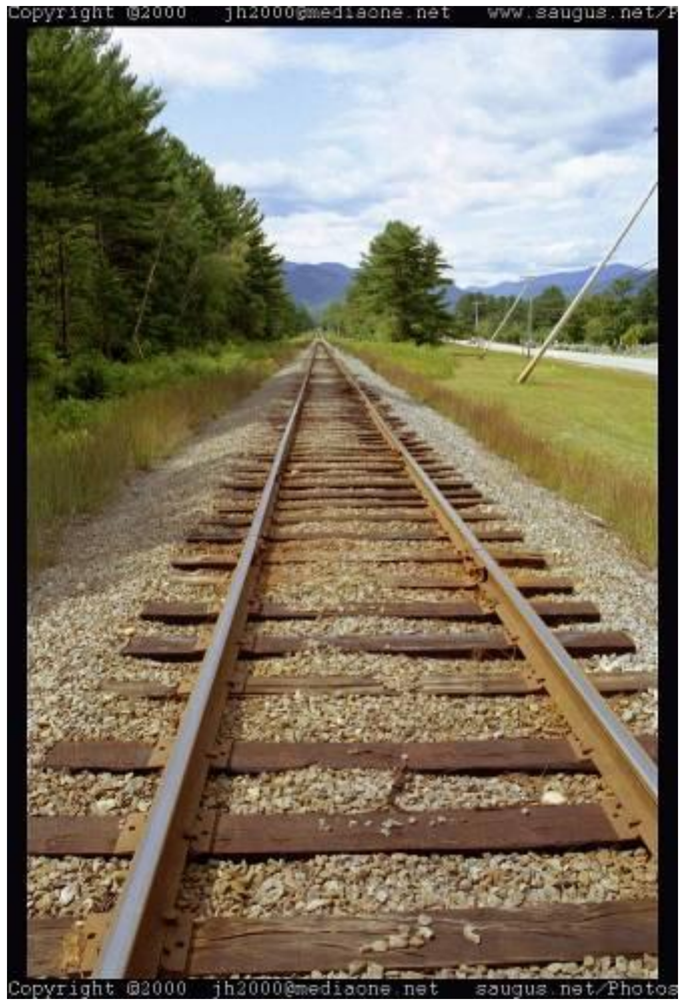
What is preserved?

- Straight lines are still straight

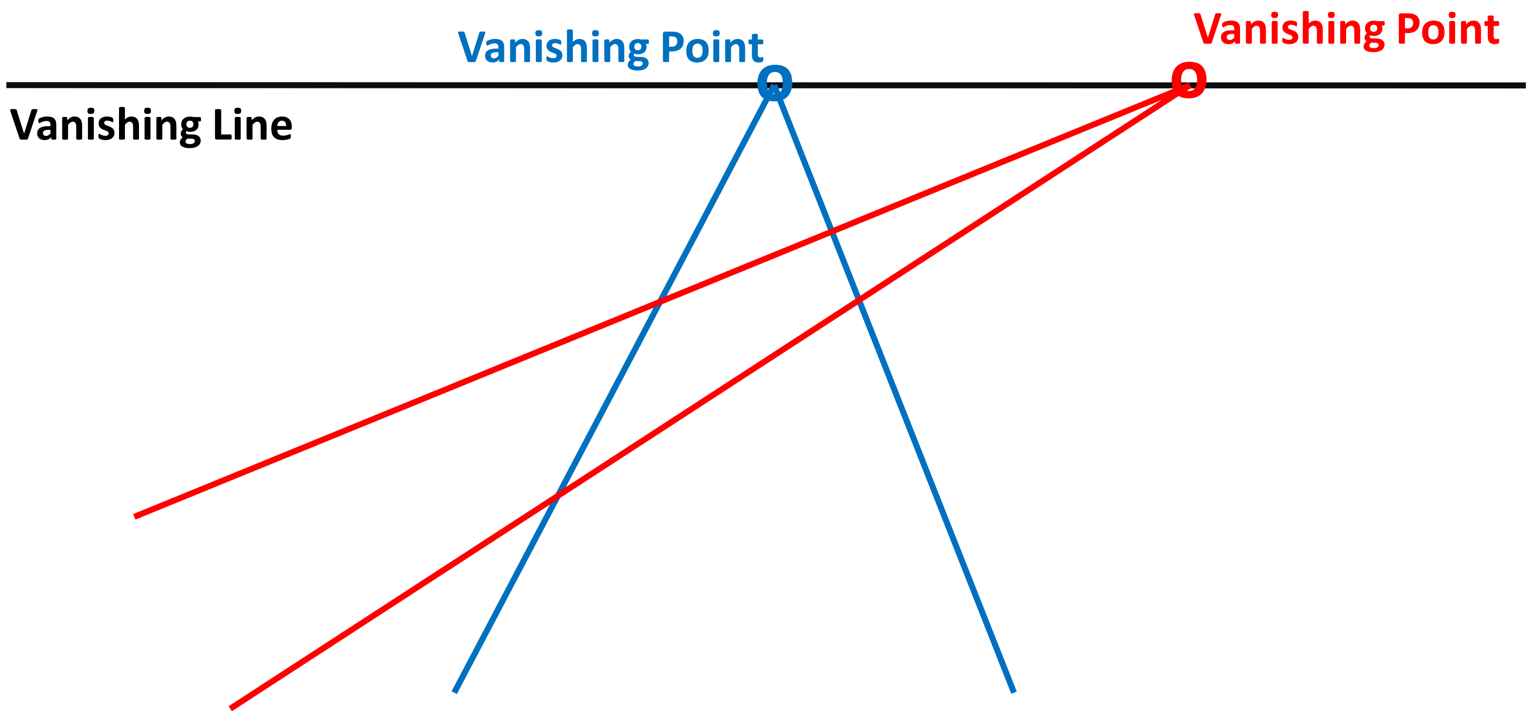


Vanishing points and lines

Parallel lines in the world intersect in the image at a “vanishing point”

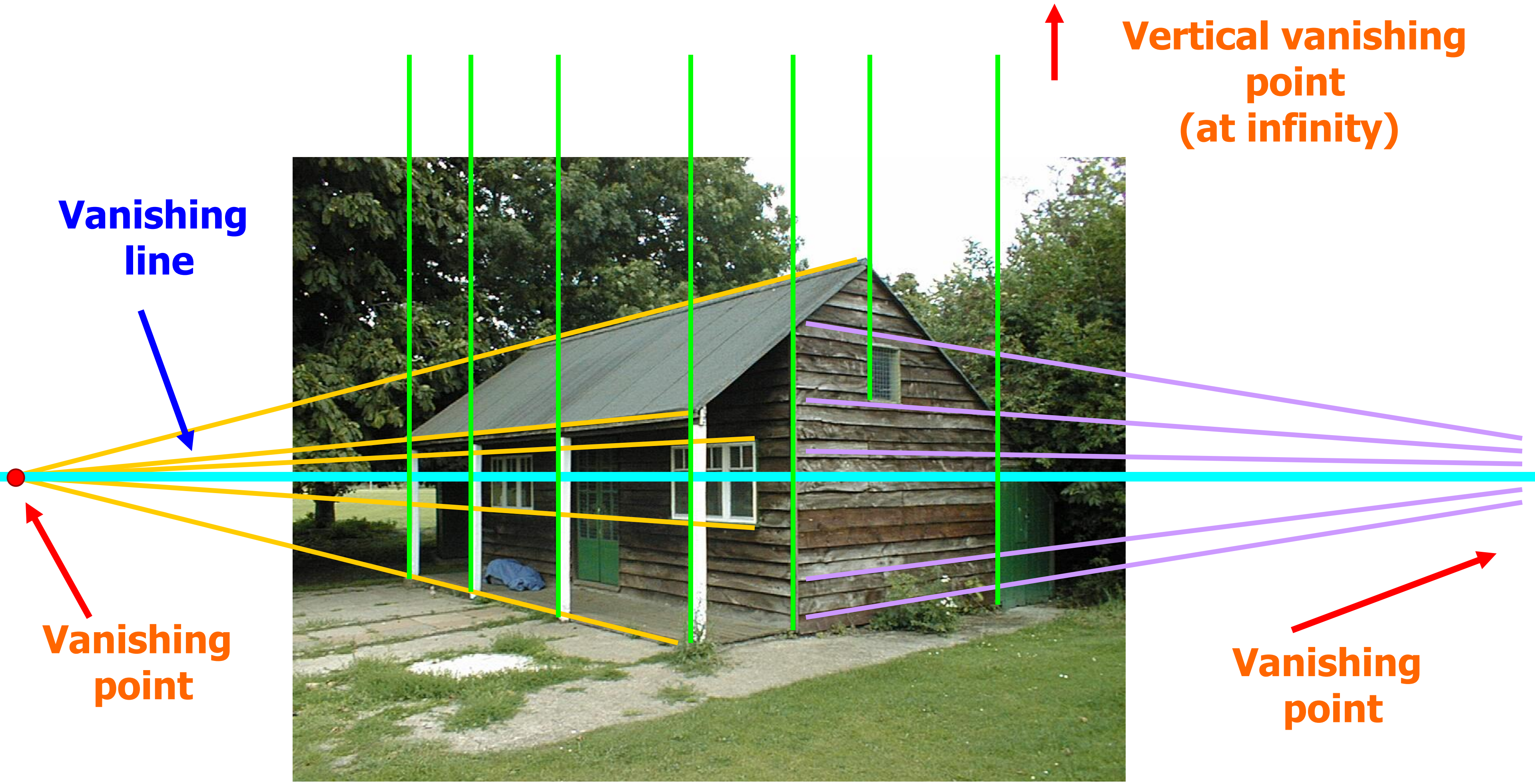


Vanishing points and lines



- The projections of parallel 3D lines intersect at a **vanishing point**
- The projection of parallel 3D planes intersect at a **vanishing line**
- If a set of parallel 3D lines are also parallel to a particular plane, their vanishing point will lie on the vanishing line of the plane
- Not all lines that intersect are parallel
- Vanishing point <-> 3D direction of a line
- Vanishing line <-> 3D orientation of a surface

Vanishing points and lines

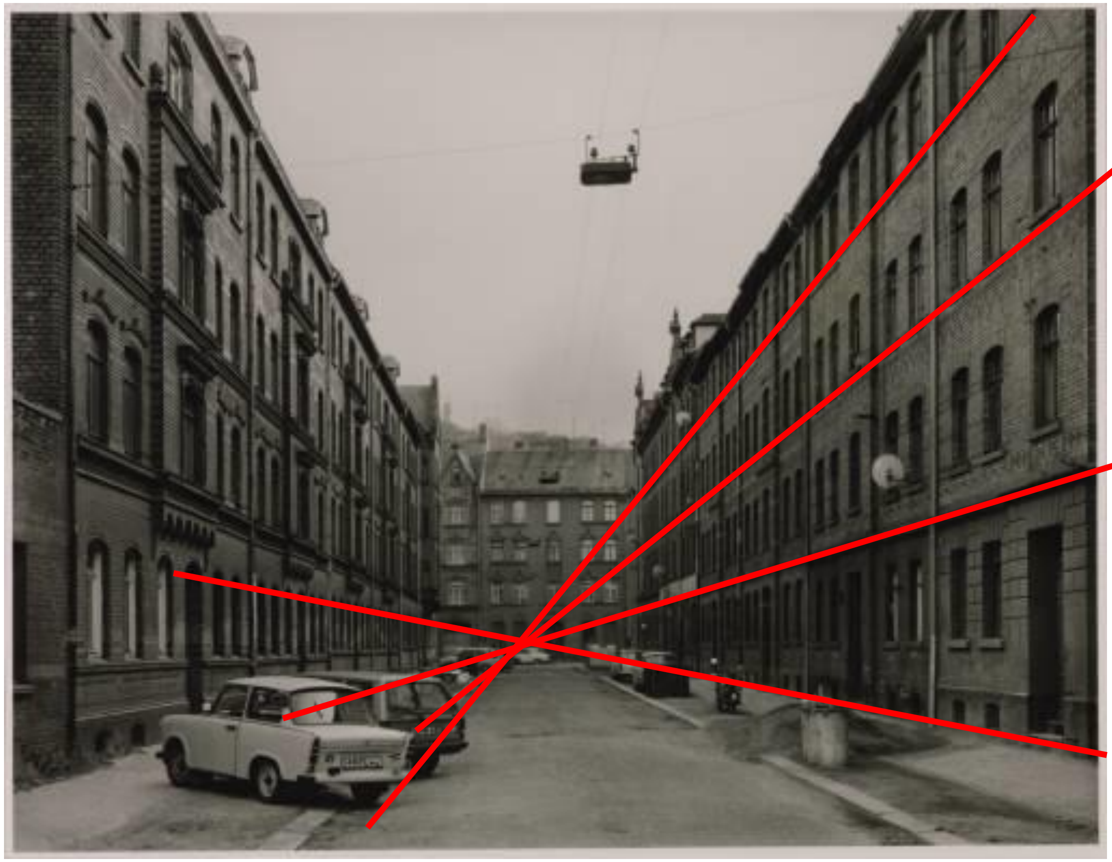


Slide from Efros, Photo from Criminisi

Vanishing points and lines

Photo from online Tate collection

Note on estimating vanishing points



Use multiple lines for better accuracy

… but lines will not intersect at exactly the same point in practice

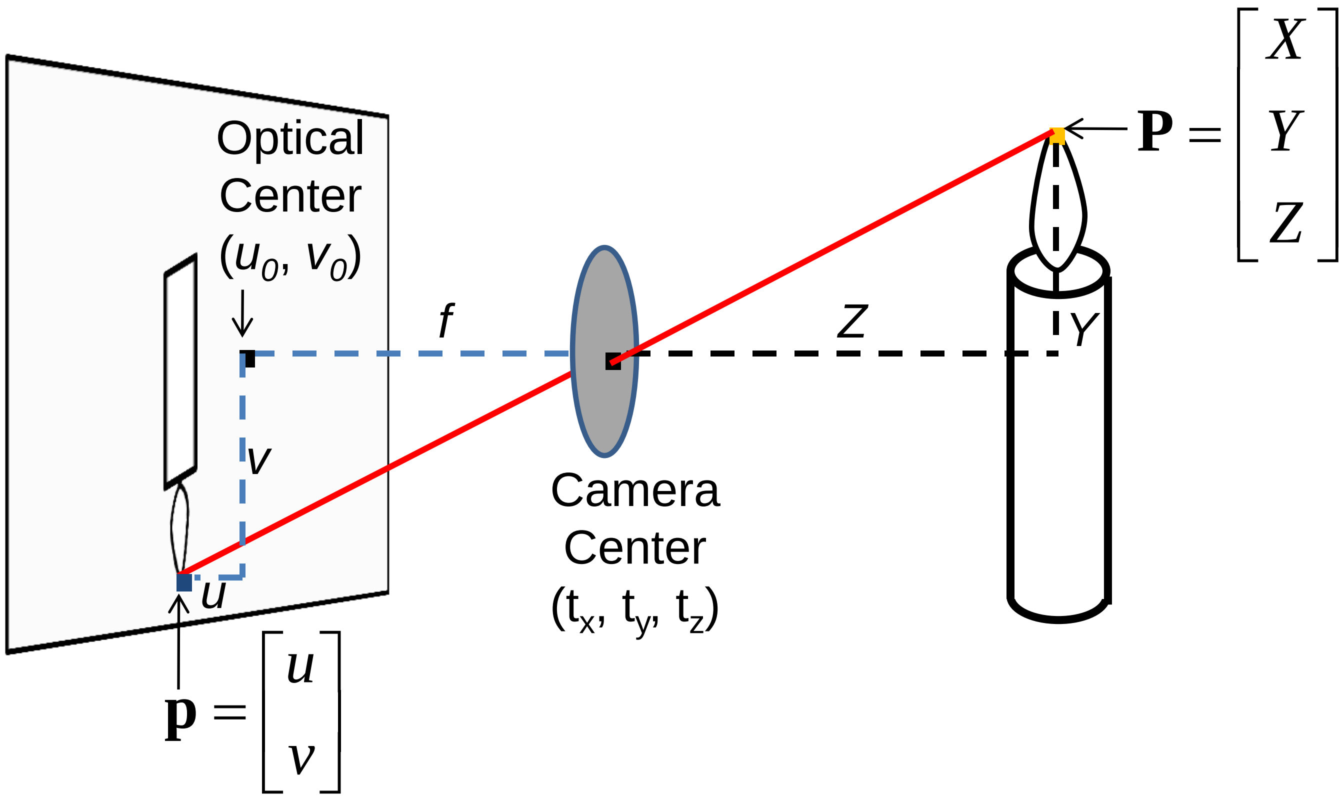
One solution: take mean of intersecting pairs

… bad idea!

Instead, minimize angular differences

Vanishing objects

Projection: world coordinates→image coordinates



Homogeneous coordinates

Conversion

Converting to *homogeneous* coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Homogeneous coordinates

Invariant to scaling

$$k\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} kx \\ ky \\ kw \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{kx}{kw} \\ \frac{ky}{kw} \end{bmatrix} = \begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \end{bmatrix}$$

Homogeneous Coordinates | Cartesian Coordinates

Point in Cartesian is ray in Homogeneous

Basic geometry in homogeneous coordinates

- Line equation: ax + by + c = 0

$$line_i = \begin{bmatrix} a_i \\ b_i \\ c_i \end{bmatrix}$$

- Append 1 to pixel coordinate to get homogeneous coordinate

$$p_i = \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix}$$

- Line given by cross product of two points

$$line_{ij} = p_i \times p_j$$

- Intersection of two lines given by cross product of the lines

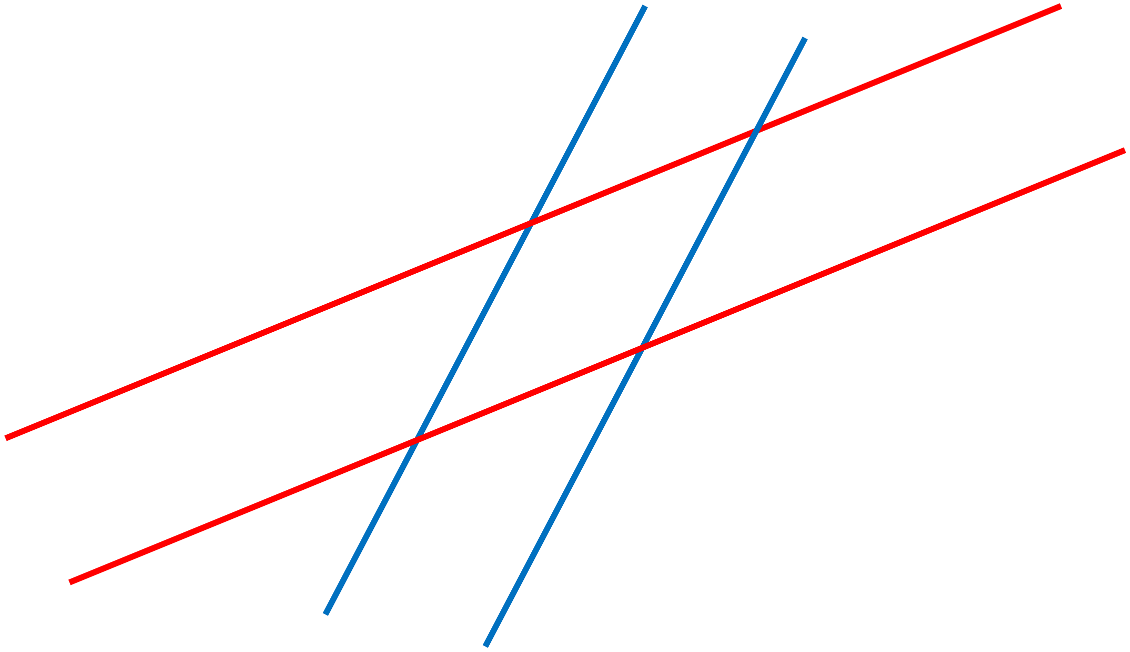
$$q_{ij} = line_i \times line_j$$

Another problem solved by homogeneous coordinates

Intersection of parallel lines

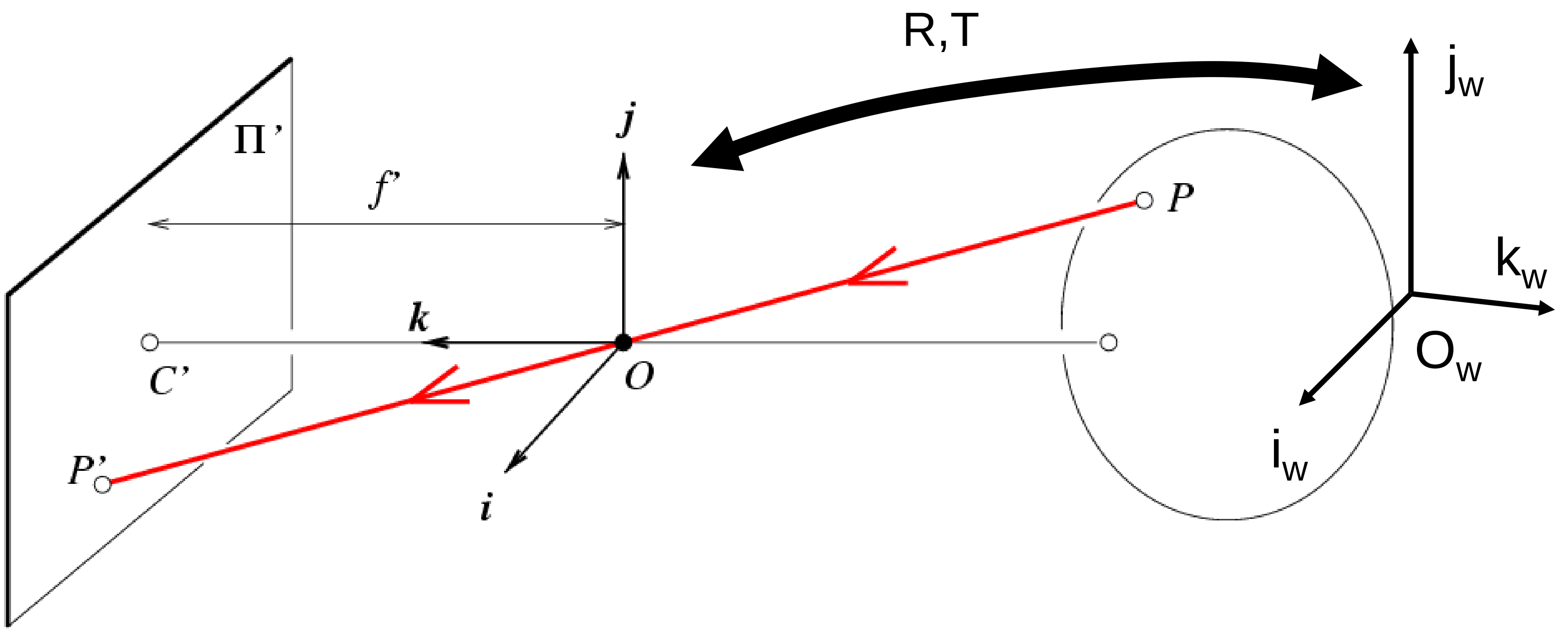
Cartesian: (Inf, Inf)
Homogeneous: (1, 1, 0)

Cartesian: (Inf, Inf)
Homogeneous: (1, 2, 0)



Slide Credit: Saverese

Projection matrix



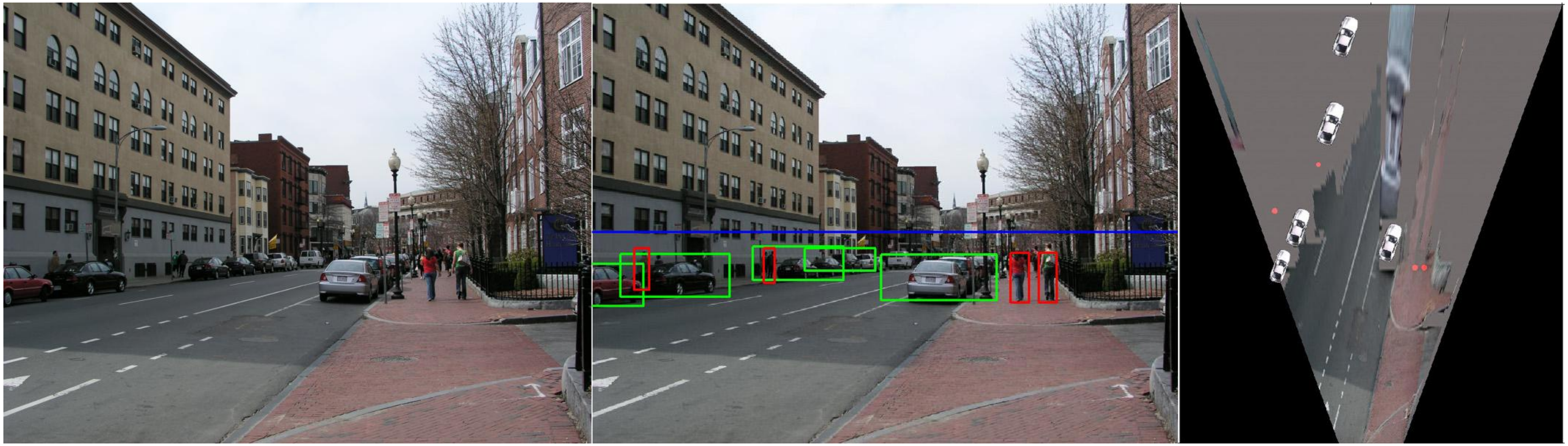
$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

x: Image Coordinates: (u,v,1)
K: Intrinsic Matrix (3x3)
R: Rotation (3x3)
t: Translation (3x1)
X: World Coordinates: (X,Y,Z,1)

Interlude: when have I used this stuff?

When have I used this stuff?

Object Recognition (CVPR 2006)

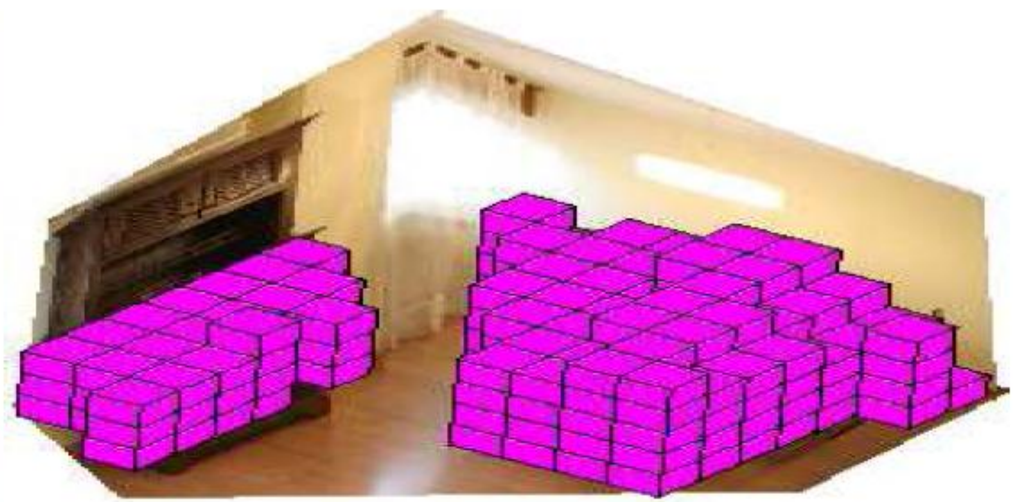


When have I used this stuff?

Single-view reconstruction (SIGGRAPH 2005)

When have I used this stuff?

Getting spatial layout in indoor scenes (ICCV 2009)



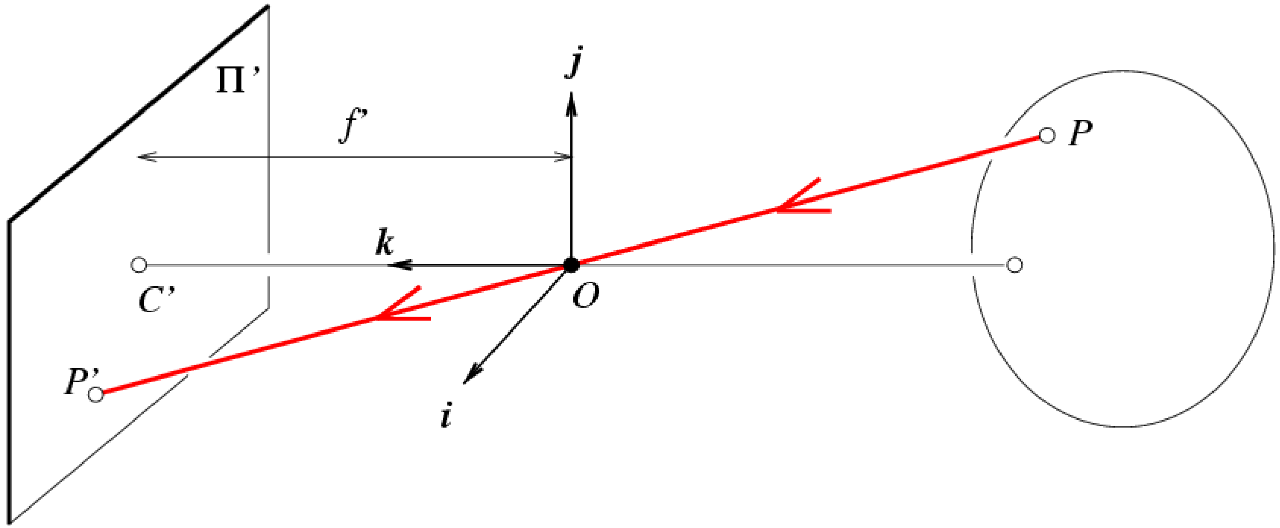
When have I used this stuff?

Inserting synthetic objects into images: http://vimeo.com/28962540


GOLFER


GOLFER

Projection matrix



Intrinsic Assumptions

- Unit aspect ratio
- Optical center at (0,0)
- No skew

Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

K

$$\mathbf{x} = \mathbf{K}\left[\mathbf{I} \quad \mathbf{0}\right]\mathbf{X} \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide Credit: Saverese

Remove assumption: known optical center

Intrinsic Assumptions

- Unit aspect ratio
- No skew

Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \quad \Rightarrow \quad w\begin{bmatrix}u\\v\\1\end{bmatrix} = \begin{bmatrix}f & 0 & u_0 & 0\\0 & f & v_0 & 0\\0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

Remove assumption: square pixels

Intrinsic Assumptions
- No skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Remove assumption: non-skewed pixels

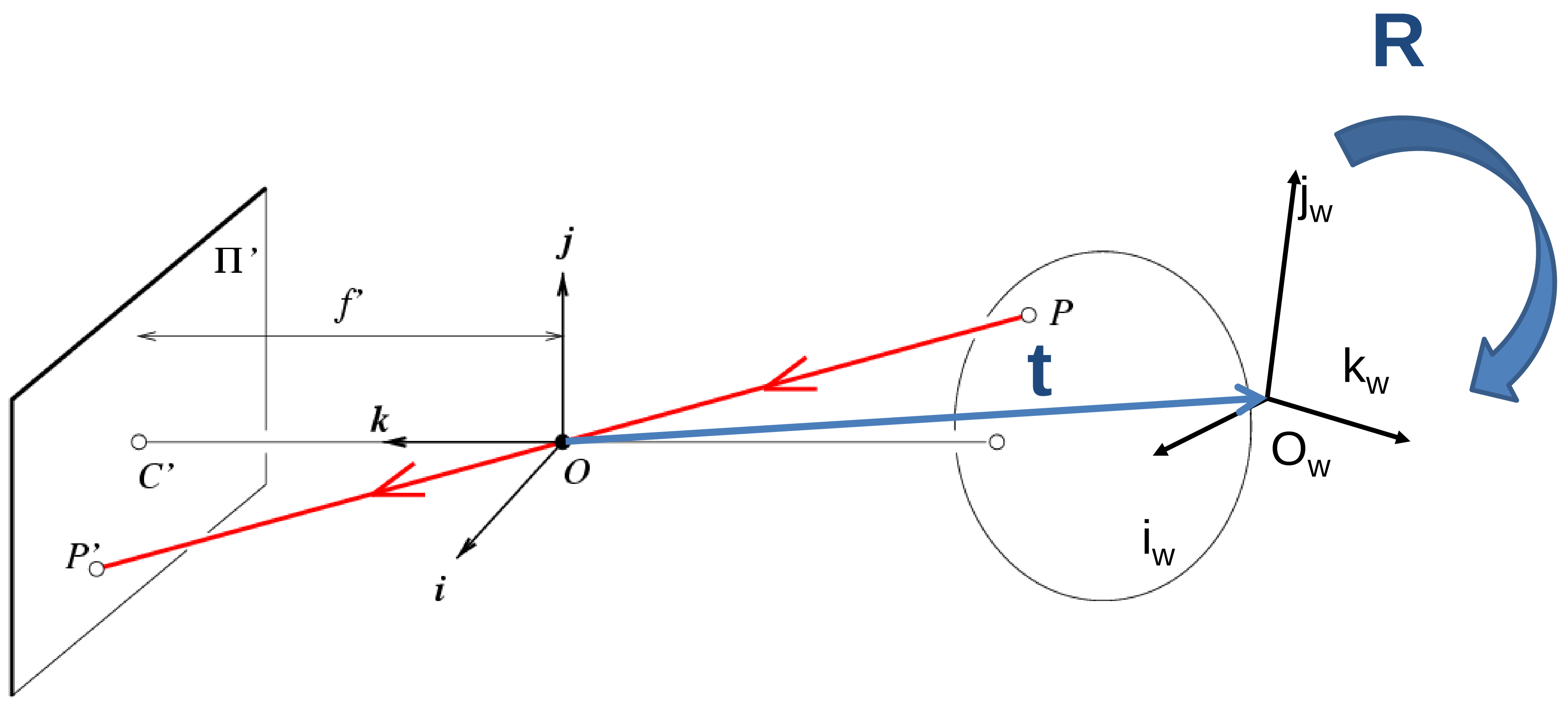
Intrinsic Assumptions

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix}u \\ v \\ 1\end{bmatrix} = \begin{bmatrix}\alpha & s & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x \\ y \\ z \\ 1\end{bmatrix}$$

Note: different books use different notation for parameters

Oriented and Translated Camera



Allow camera translation

Intrinsic Assumptions

Extrinsic Assumptions

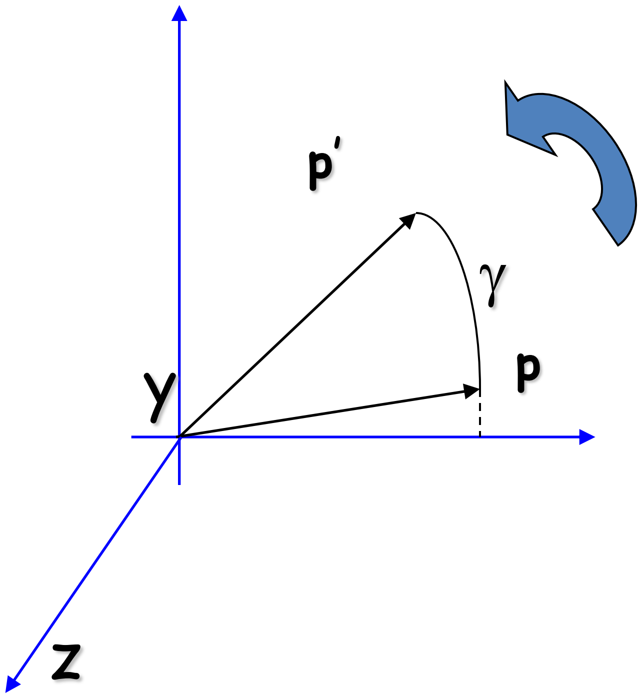
- No rotation

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{t}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix}u \\ v \\ 1\end{bmatrix} = \begin{bmatrix}\alpha & 0 & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z\end{bmatrix}\begin{bmatrix}x \\ y \\ z \\ 1\end{bmatrix}$$

Slide Credit: Saverese

3D Rotation of Points

Rotation around the coordinate axes, counter-clockwise:



$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Allow camera rotation

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Degrees of freedom

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

5 6

$$w\begin{bmatrix}u\\v\\1\end{bmatrix} = \begin{bmatrix}\alpha & s & u_0\\0 & \beta & v_0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}r_{11} & r_{12} & r_{13} & t_x\\r_{21} & r_{22} & r_{23} & t_y\\r_{31} & r_{32} & r_{33} & t_z\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

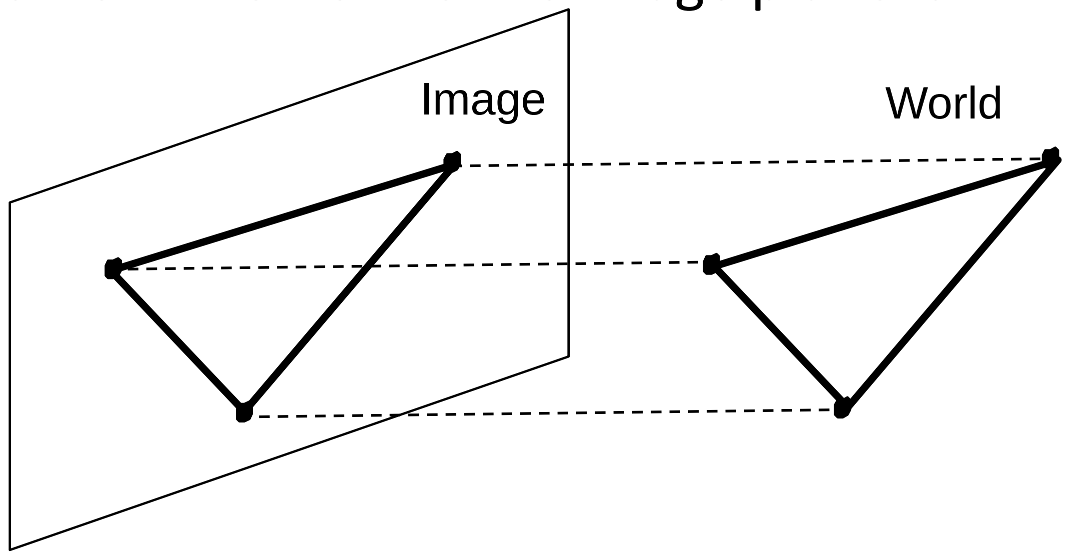
Vanishing Point = Projection from Infinity

$$\mathbf{p} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix} \Rightarrow \mathbf{p} = \mathbf{KR}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \mathbf{p} = \mathbf{K}\begin{bmatrix} x_R \\ y_R \\ z_R \end{bmatrix}$$

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_R \\ y_R \\ z_R \end{bmatrix} \Rightarrow \quad u = \frac{fx_R}{z_R} + u_0 \qquad v = \frac{fy_R}{z_R} + v_0$$

Orthographic Projection

- Special case of perspective projection
  - Distance from the COP to the image plane is infinite



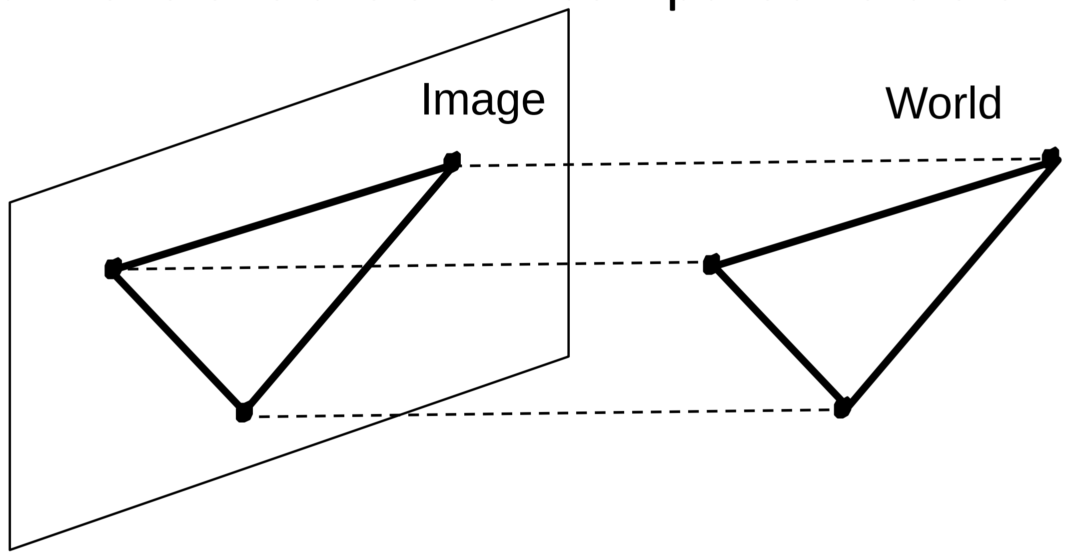
  - Also called "parallel projection"
  - What's the projection matrix?

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide by Steve Seitz

Scaled Orthographic Projection

- Special case of perspective projection
  - Object dimensions are small compared to distance to camera



  - Also called "weak perspective"
  - What's the projection matrix?

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 0 & s \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide by Steve Seitz

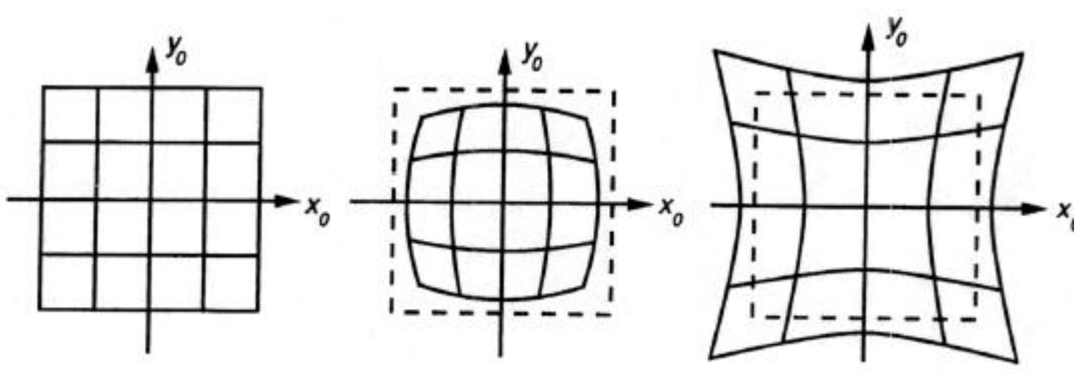
Example

Far field: object appearance doesn't change as objects translate

Near field: object appearance changes as objects translate

Beyond Pinholes: Radial Distortion

- Common in wide-angle lenses or for special applications (e.g., security)
- Creates non-linear terms in projection
- Usually handled by through solving for non-linear terms and then correcting image

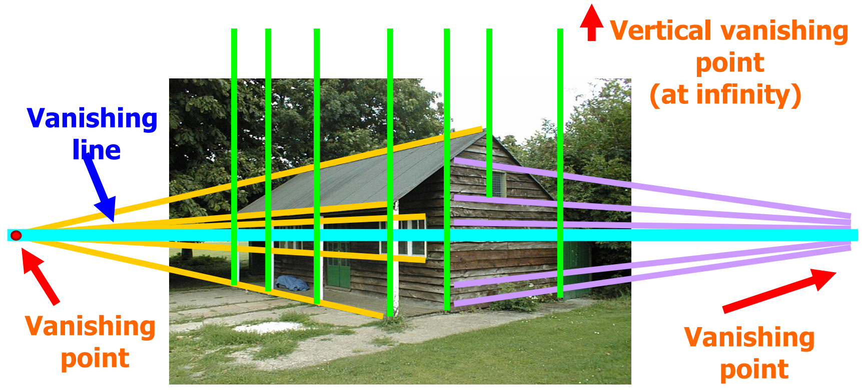


No Distortion | Barrel Distortion | Pincushion Distortion

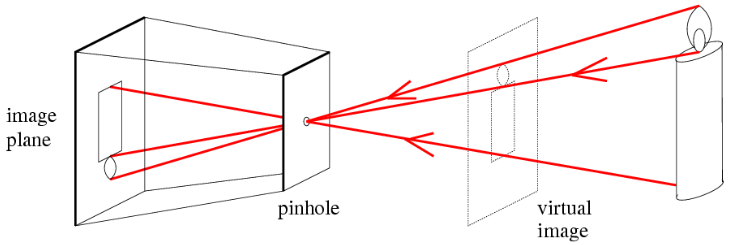
Corrected Barrel Distortion

Image from Martin Habbecke

Things to remember



- Vanishing points and vanishing lines



- Pinhole camera model and camera projection matrix

$$\mathbf{x} = \mathbf{K}[\mathbf{R} \quad \mathbf{t}]\mathbf{X}$$

- Homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Next class

- Applications of camera model and projective geometry
  - Recovering the camera intrinsic and extrinsic parameters from an image
  - Recovering size in the world
  - Projecting from one plane to another