Network Games

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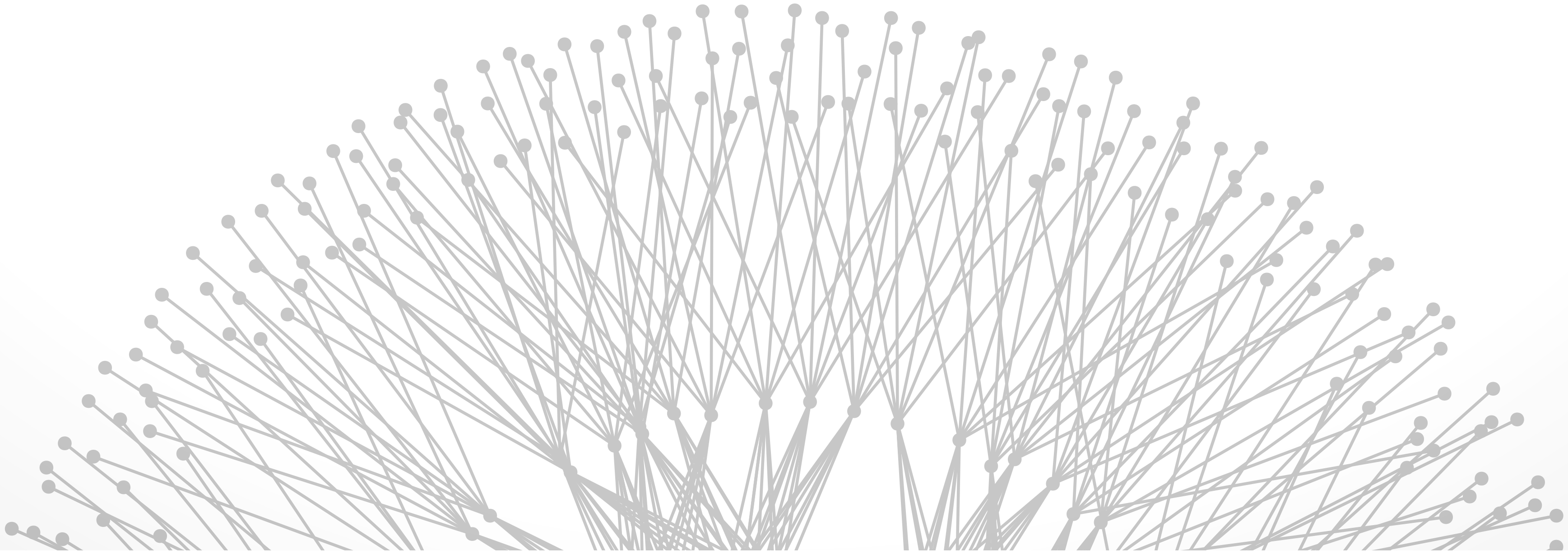
CS 538 March 26 2018



Demo

Game theory basics

Games & networks: a natural fit

Game theory

Studies interaction
between selfish agents

Networking

Enables interaction
between agents

Networks make games happen!

Game theory

Components defining a game

- Two or more players
- Set of strategies for each player
- For each combination of played strategies, a payoff or utility for each player

Prisoner's Dilemma

Blue player strategies

Red player strategies

	Cooperate	Defect
Cooperate	-1, -1	-12, 0
Defect	0, -12	-5, -5

Nash equilibrium

A chosen strategy for each player such that no player can improve its (expected) utility by changing its strategy

- Pure strategy: player picks single deterministic action
- Mixed strategy: player picks random strategy according to some distribution

Can you find a Nash equilibrium?

Blue player strategies

Red player strategies

	Cooperate	Defect
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Prisoner's dilemma Nash eq.

Blue player strategies

Red player strategies

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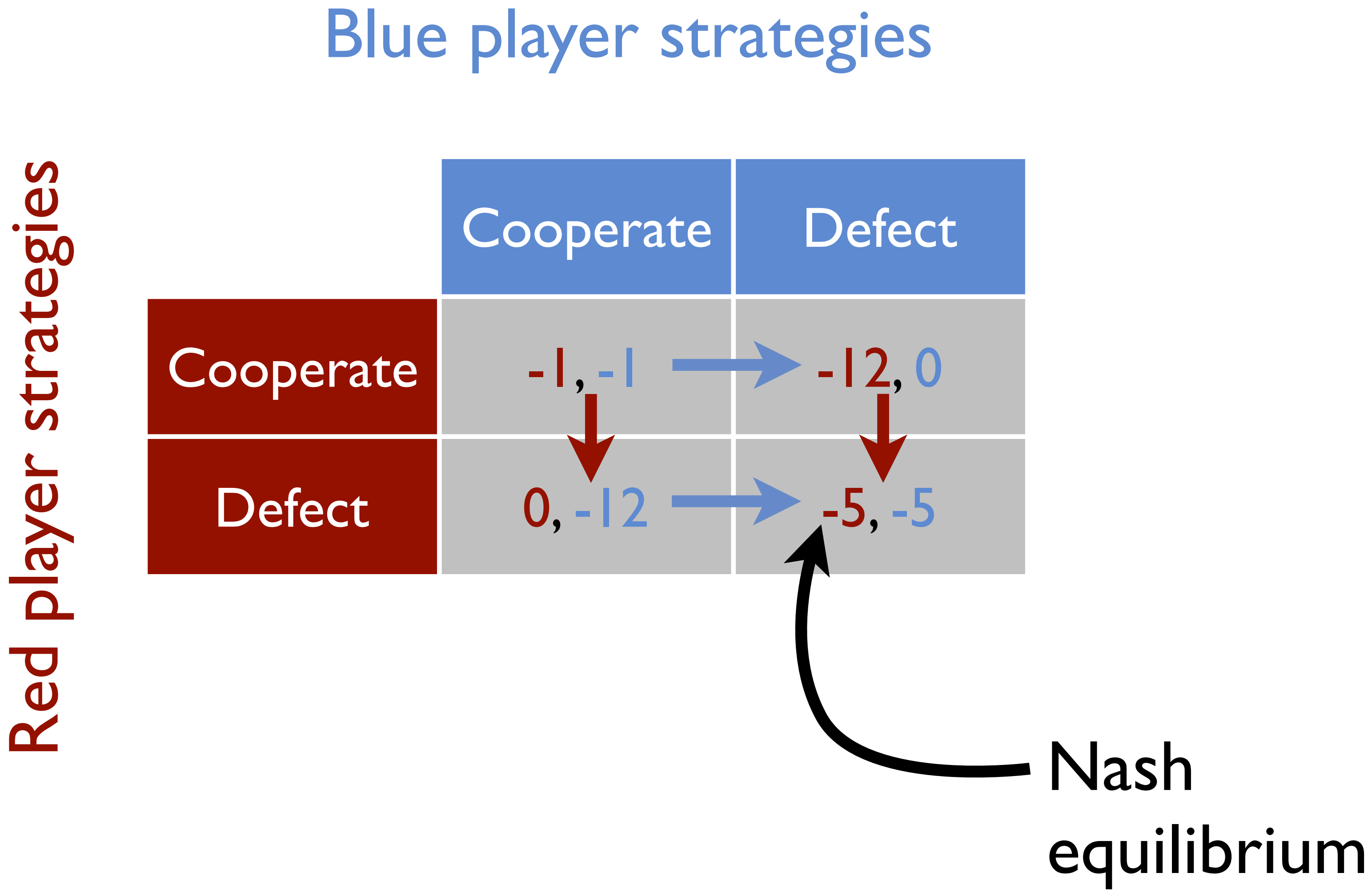
Prisoner's dilemma Nash eq.



Blue player strategies

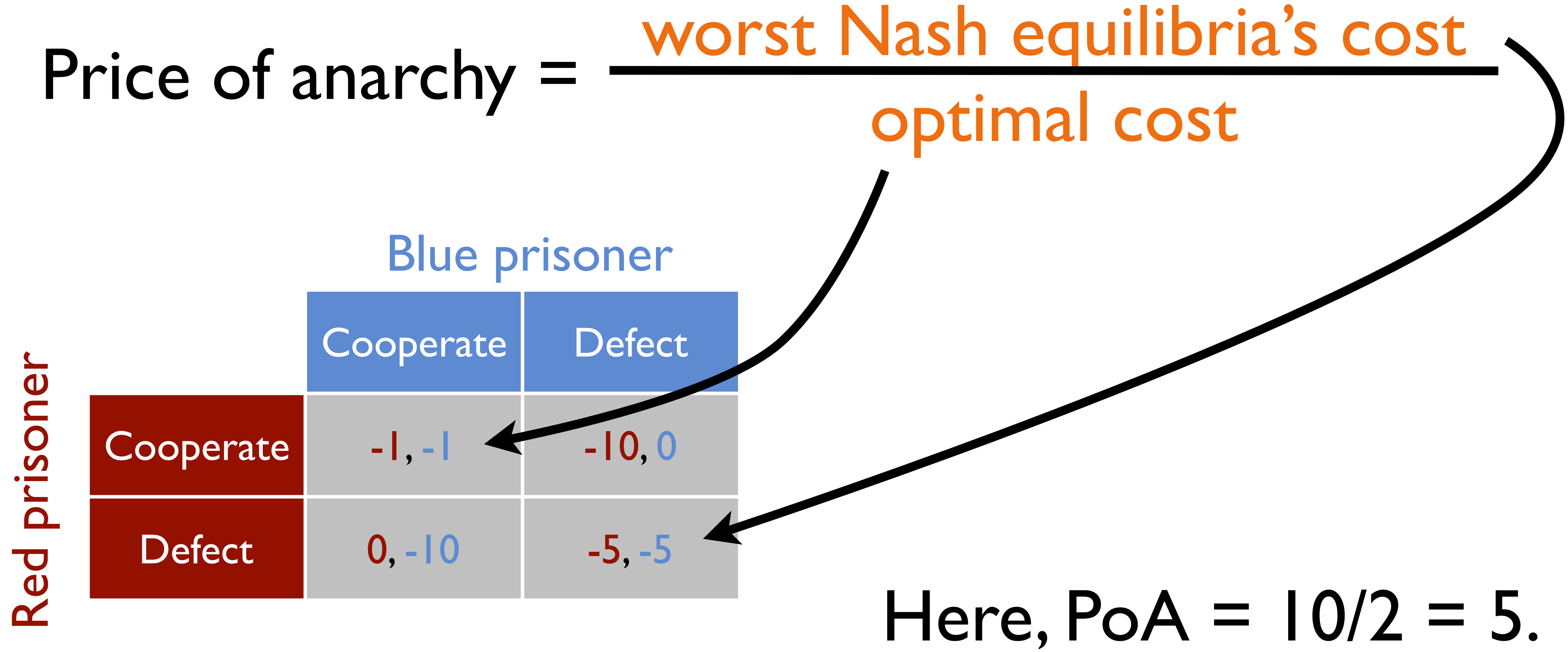
Red player strategies		Cooperate	Defect
	Cooperate	-1, -1	-12, 0
	Defect	0, -12	-5, -5

Nash equilibrium

Price of Anarchy

[C. Papadimitriou, "Algorithms, games and the Internet", STOC 2001]

Assumes some global "cost" objective, e.g., social utility (sum of players' payoffs).



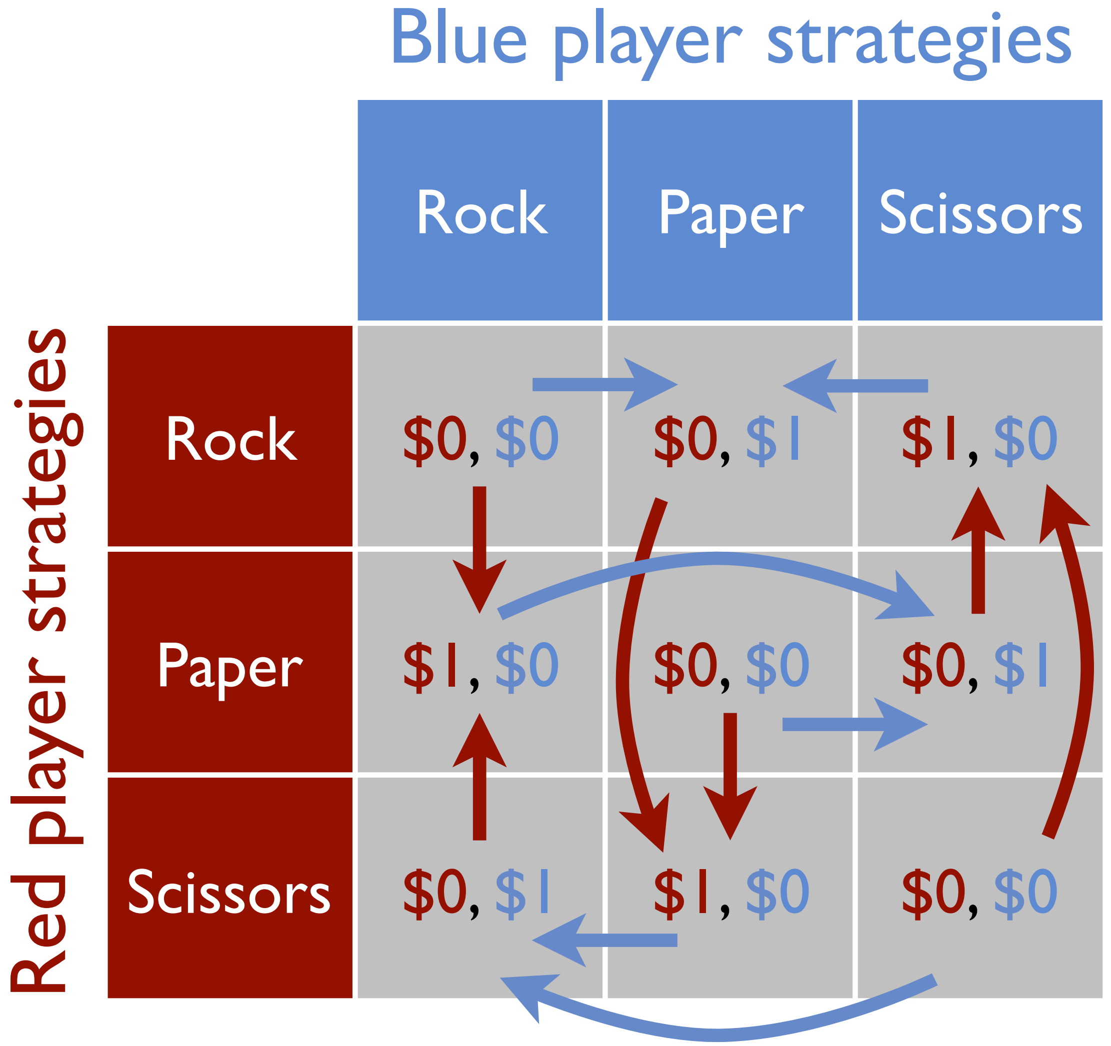
$$\text{Price of anarchy} = \frac{\text{worst Nash equilibria's cost}}{\text{optimal cost}}$$

		Blue prisoner	
		Cooperate	Defect
Red prisoner	Cooperate	-1, -1	-10, 0
	Defect	0, -10	-5, -5

Here, PoA = 10/2 = 5.

Rock Paper Scissors

Can you find a Nash equilibrium in R-P-S?



Blue player strategies

Red player strategies

	Rock	Paper	Scissors
Rock	$0, $0	$0, $1	$1, $0
Paper	$1, $0	$0, $0	$0, $1
Scissors	$0, $1	$1, $0	$0, $0

No pure Nash equilibrium!

Games in Networks: 3 Examples

The Stable Paths Problem

Tim Griffin, Bruce Shepherd, Gordon Wilfong

ACM Transactions on Networking, 2002

A game model of BGP

How Bad is Selfish Routing?

Tim Roughgarden, Eva Tardos

JACM 2002

Analysis of price of anarchy of latency-optimized routing

Selfish routing in Internet-like environments

Lili Qiu, Richard Yang, Yin Zhang, Scott Shenker

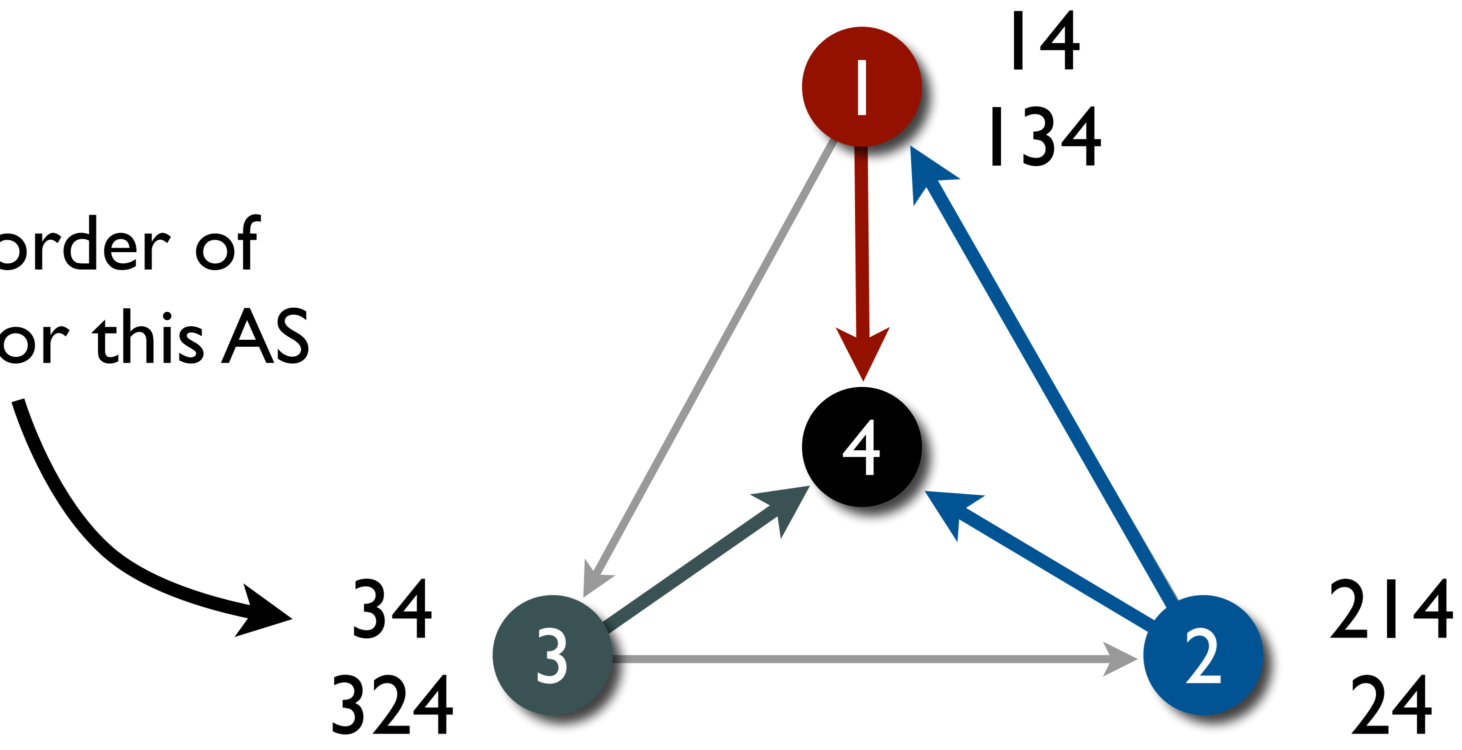
SIGCOMM 2003

What is the price of anarchy in practice for latency-optimized routing?

Internet routing as a game

BGP routing as a game

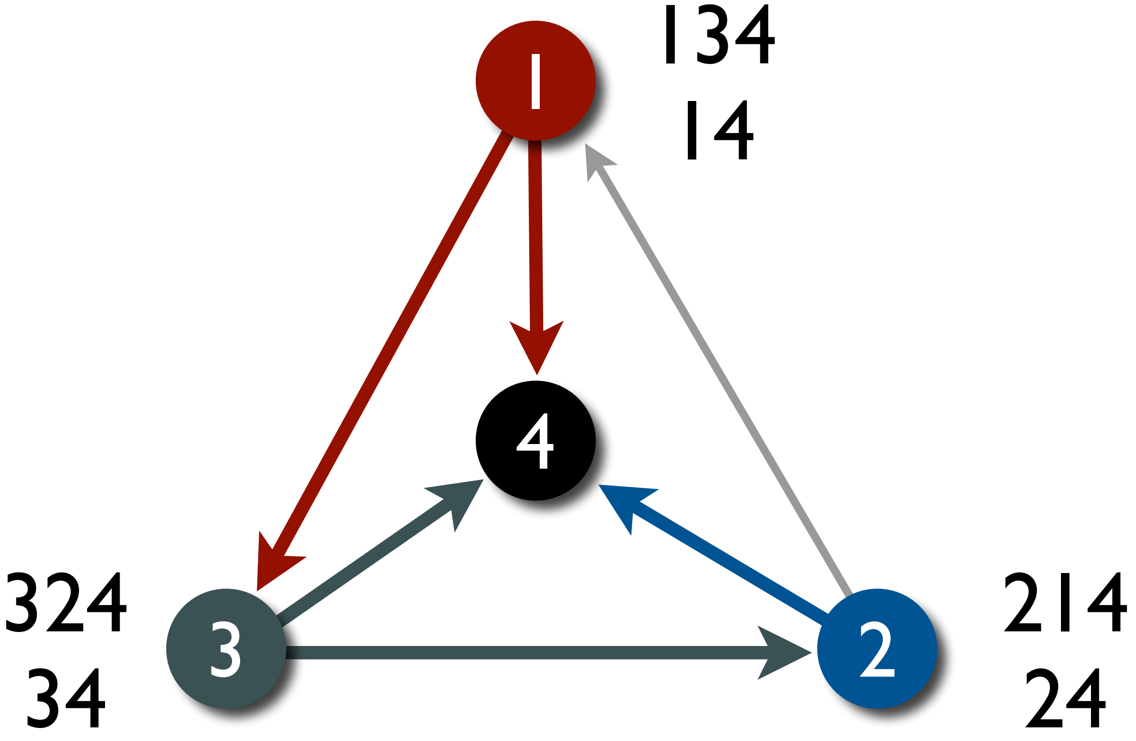
players	autonomous systems
strategies	pick a route, any route... (to fixed dest.)
player's utility	arbitrary function of route (but $-\infty$ for 'illegal' route not offered by neighbor)



Routes in order of preference for this AS

BGP routing as a game

players	autonomous systems
strategies	pick a route, any route... (to fixed dest.)
player’s utility	arbitrary function of route (but –∞ for ‘illegal’ route not offered by neighbor)



Routes in order of preference for this AS

No Nash equilibrium!

BGP routing as a game

In general, NP-complete to decide whether an equilibrium exists [Griffin, Shepherd, Wilfong, ToN'02]

Might have 0, 1, 2, 3, ... equilibria

Even if it has an equilibrium, might not converge to it

- Depends on starting state, message timing, ...
- PSPACE-complete to decide whether a given set of BGP preferences can oscillate [Fabrikant, Papadimitriou, SODA'08]

If we assume customer-provider-peer and valley-free routing, guaranteed to converge [Gao, Rexford]

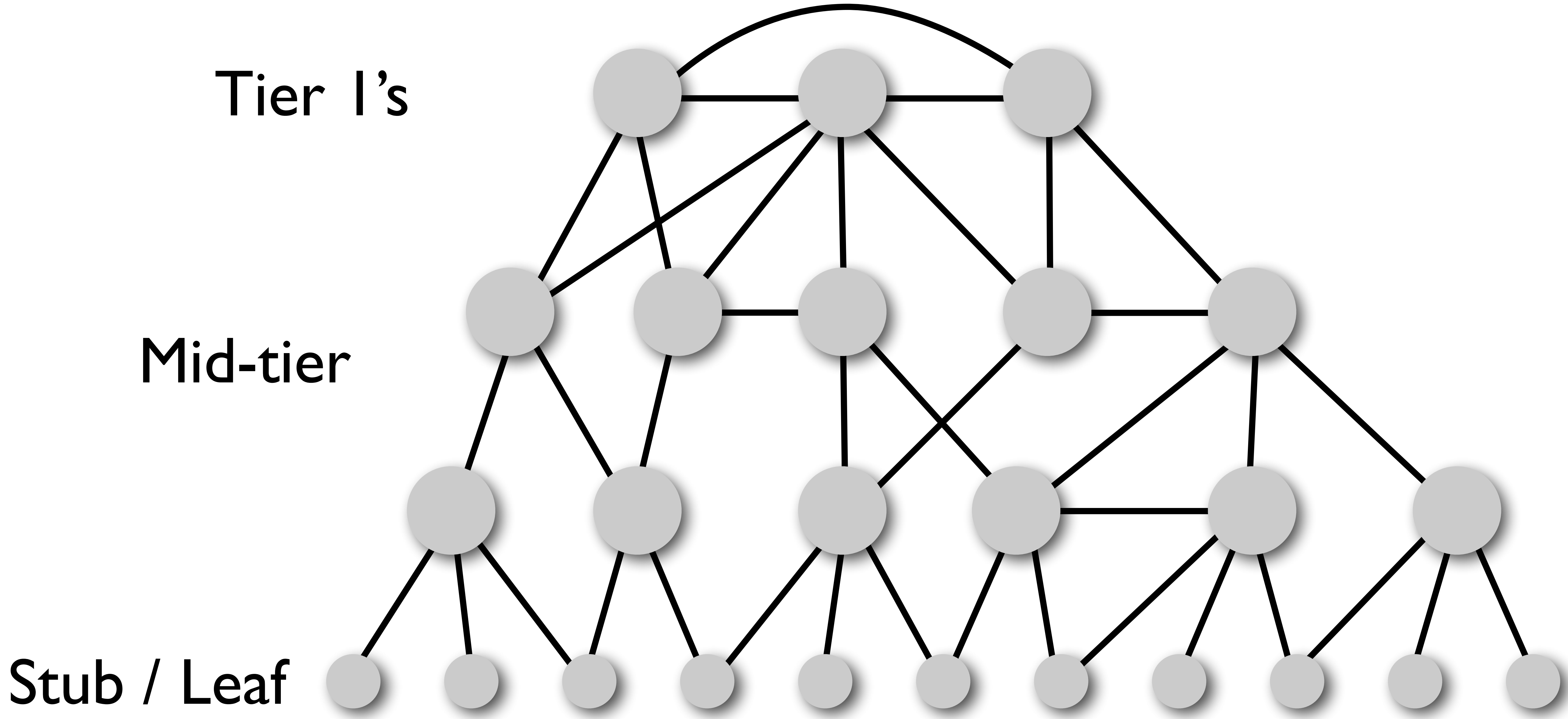
Gao-Rexford convergence

Recall "Gao-Rexford" policies:

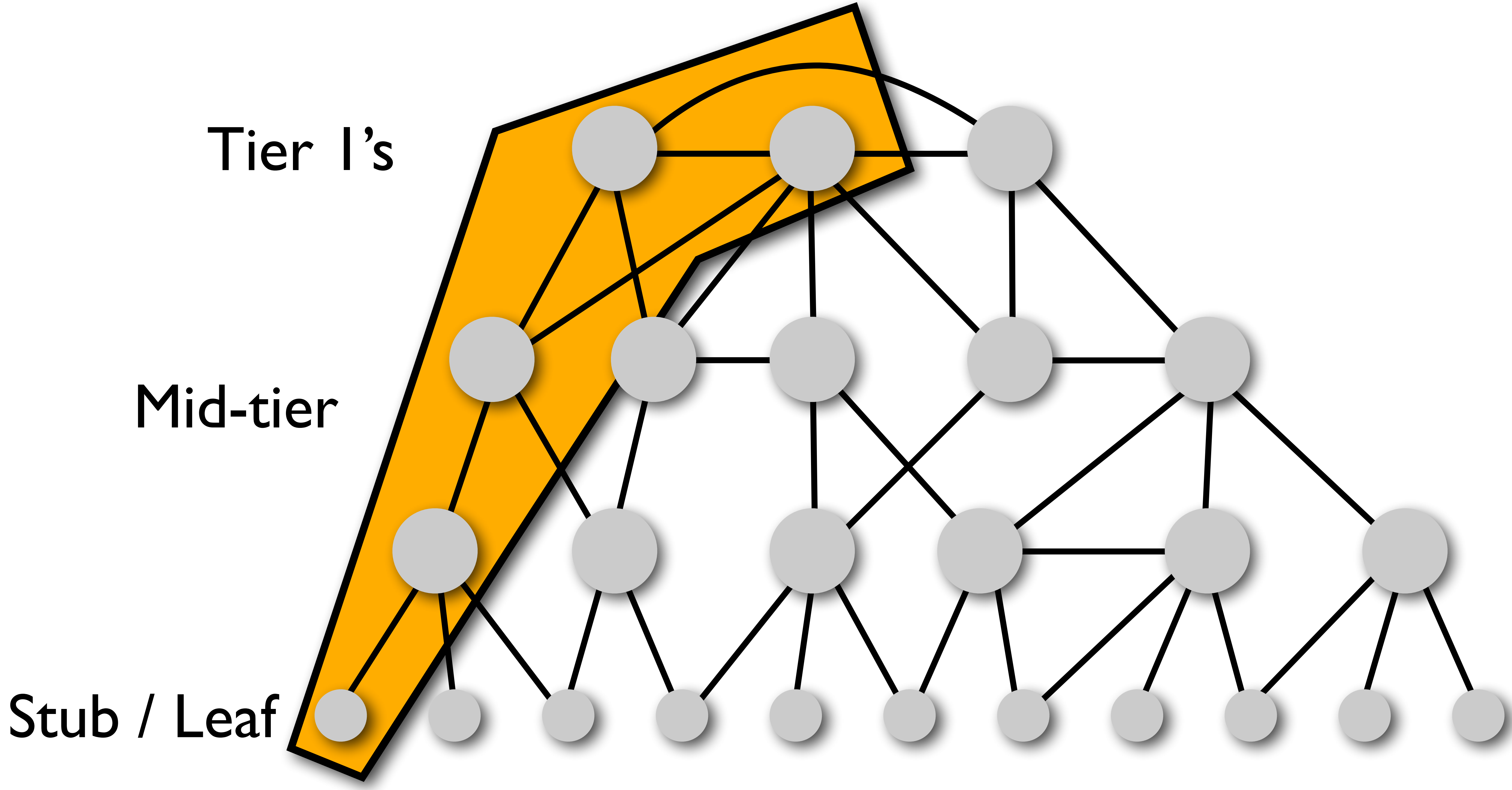
- Prefer customer > peer > provider
- Export all routes to customers
- Export customer routes to everyone
- (…and export nothing else: "valley-free" routes only)
- Further assume no provider-customer cycles
  - Not allowed: A is customer of B which is customer of … which is customer of A

Subject to these constraints, BGP will converge

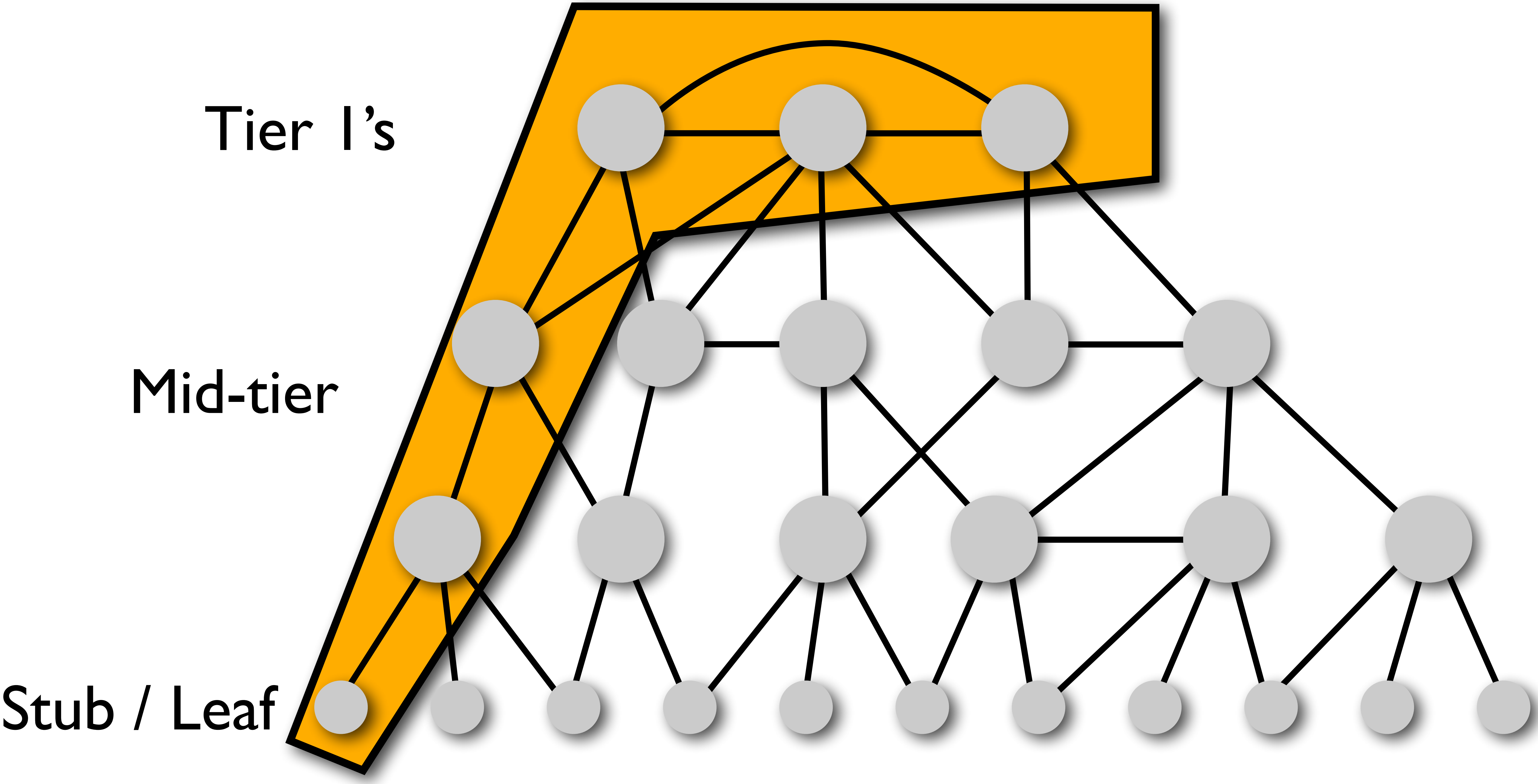
Gao-Rexford convergence



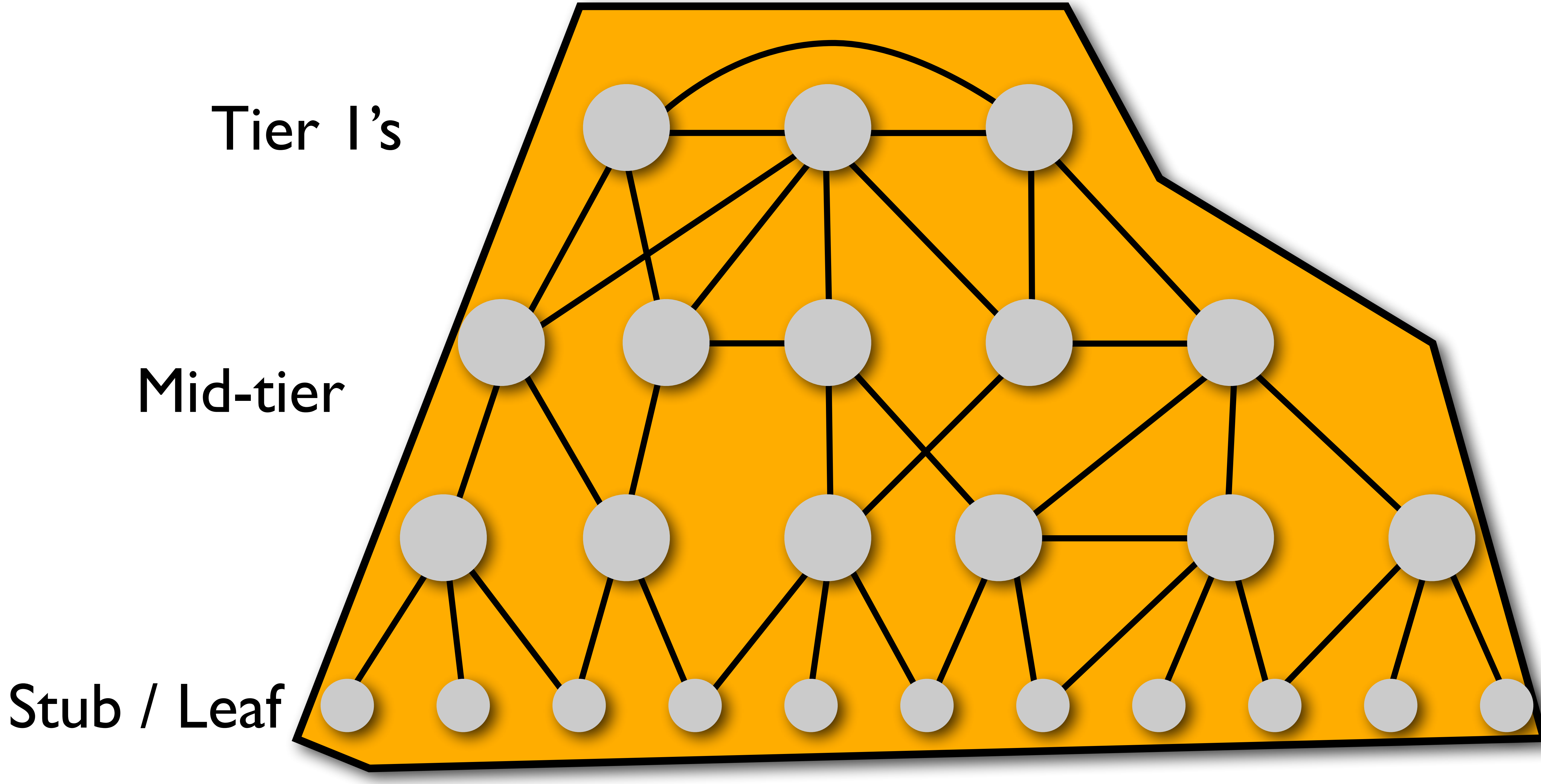
Gao-Rexford convergence



Gao-Rexford convergence



Gao-Rexford convergence

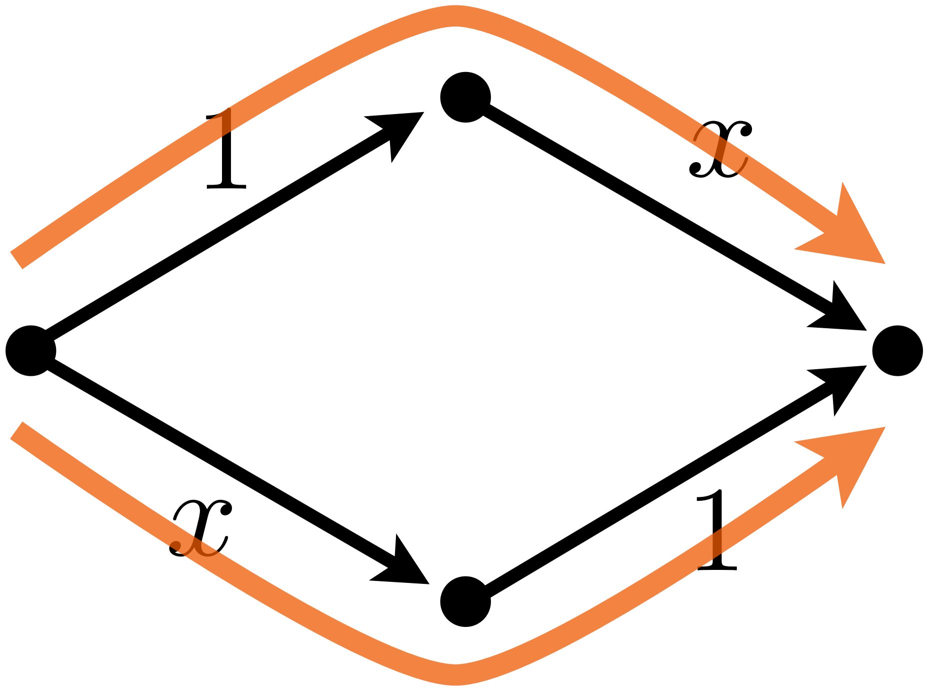


How bad is selfish routing?

The selfish routing game

The game context:

- Directed graph
- Latency function on each edge specifying latency as function of total flow *x* on edge
- Path latency = sum of edge latencies



Flow *x* = 0.5 on each path;
Total latency = 1.5

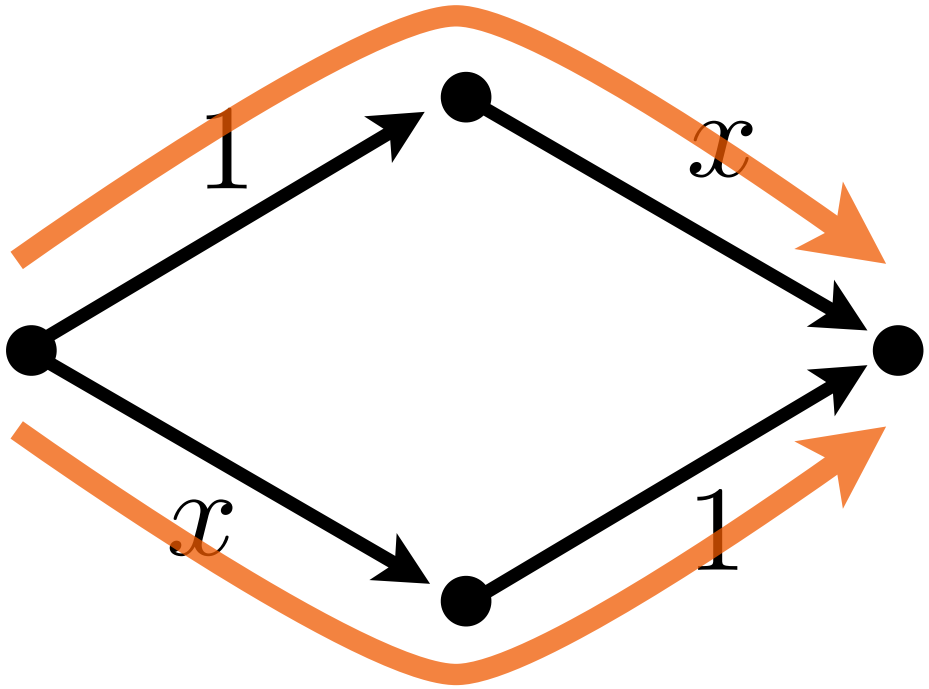
The selfish routing game

Player strategy:

- Pick a path on which to route
- Players selfishly pick paths with lowest latency (source-controlled routing)

For now assume:

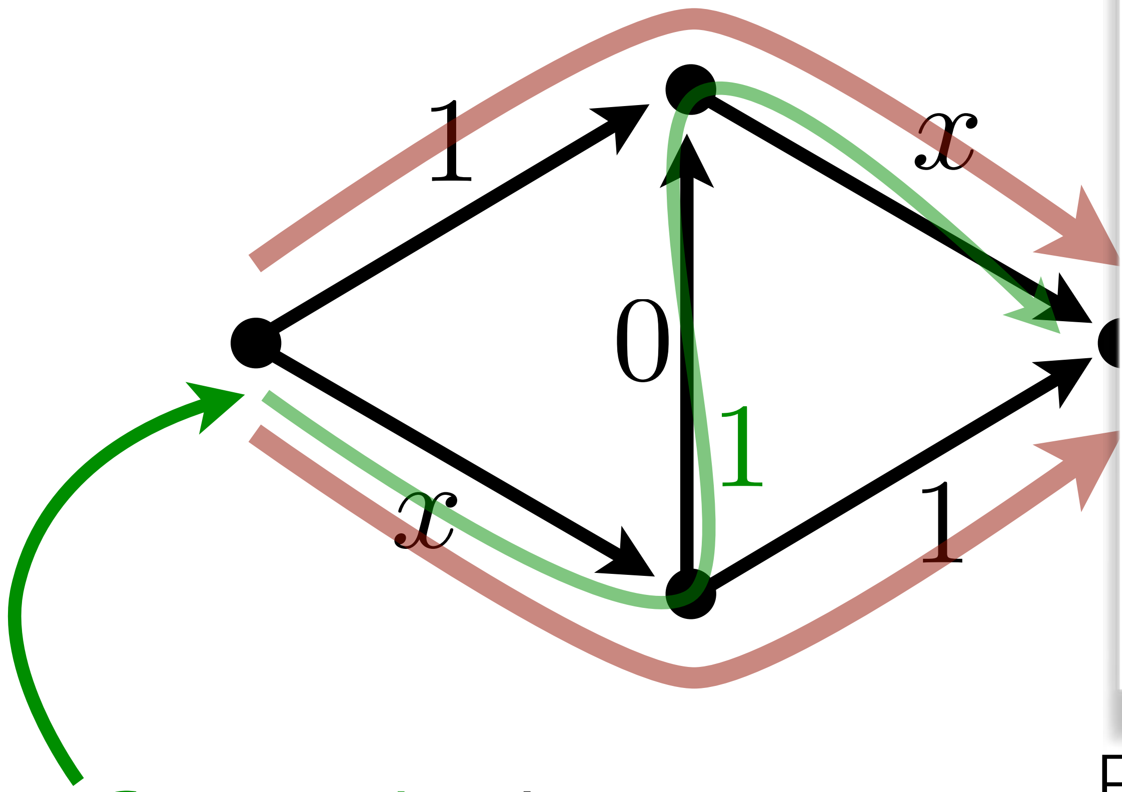
- many users
- each has negligible load
- total load = 1



Flow x = 0.5 on each path;
Total latency = 1.5

Example: Braess's paradox

[Dietrich Braess, 1968]



Green path is better.
Everyone switches to it!

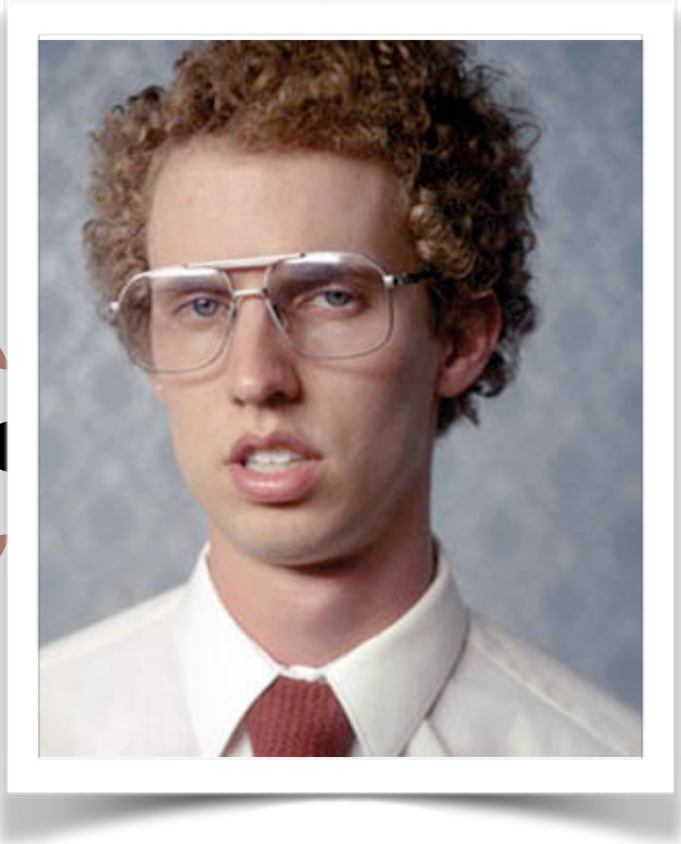


Fig 1b: N. Dynamite.

≈

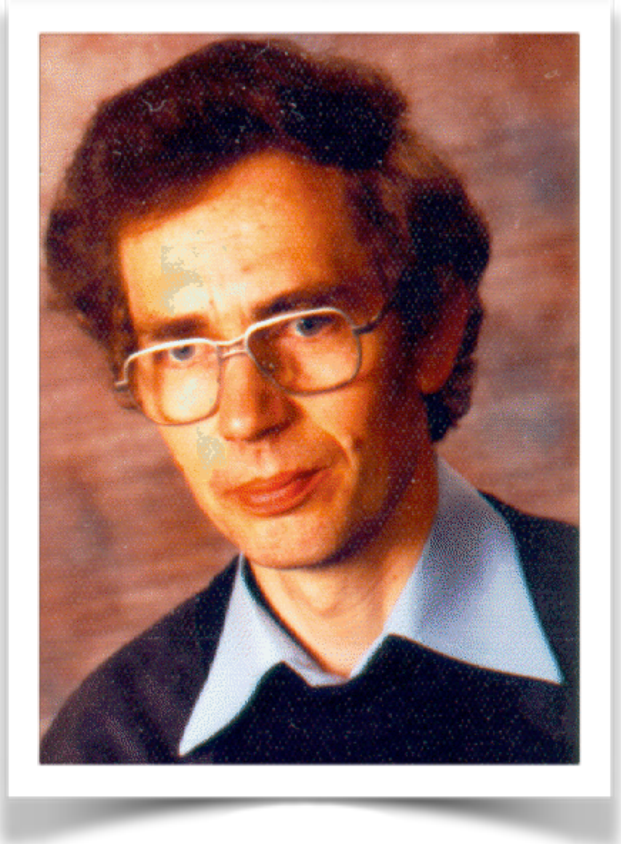
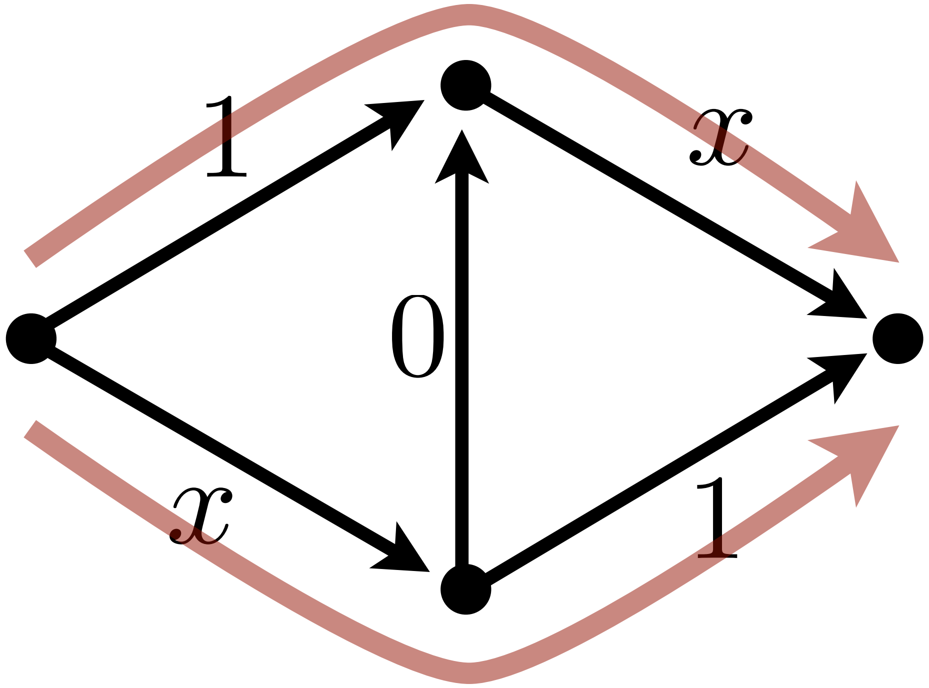


Fig 1a: D. Braess.

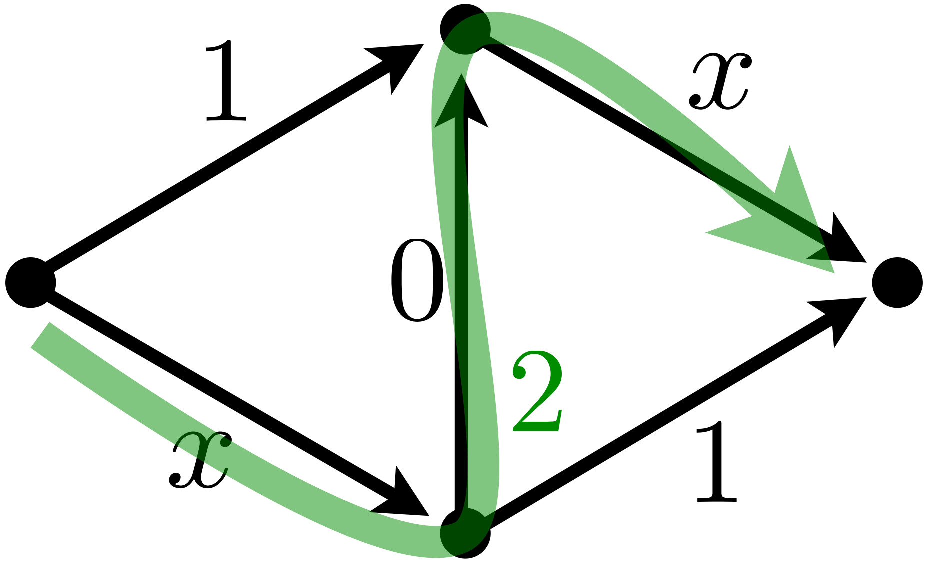
Initially: 0.5 flow along each path; latency 1+0.5 = 1.5

With new edge: all flow along greed path; latency = 2

Example: Braess's paradox



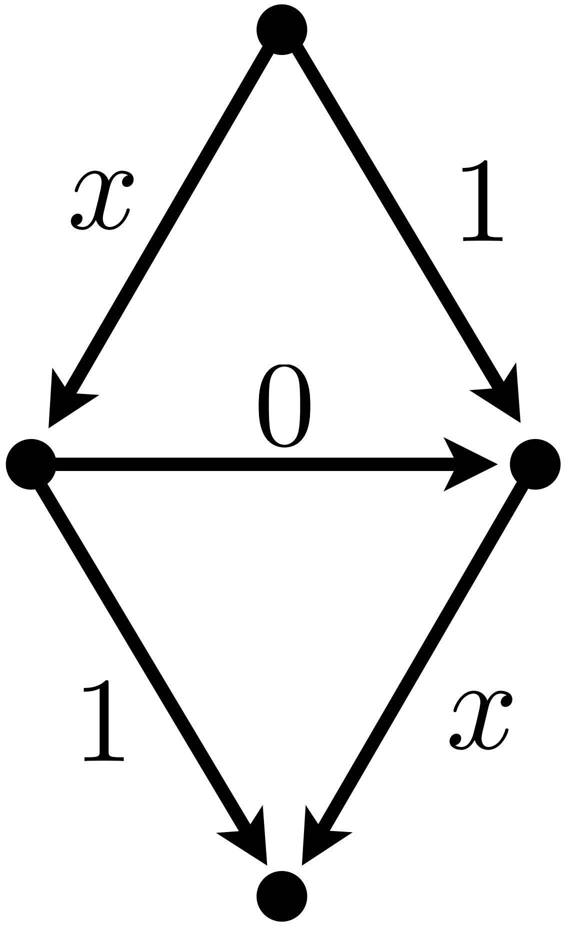
Optimal latency = 1.5

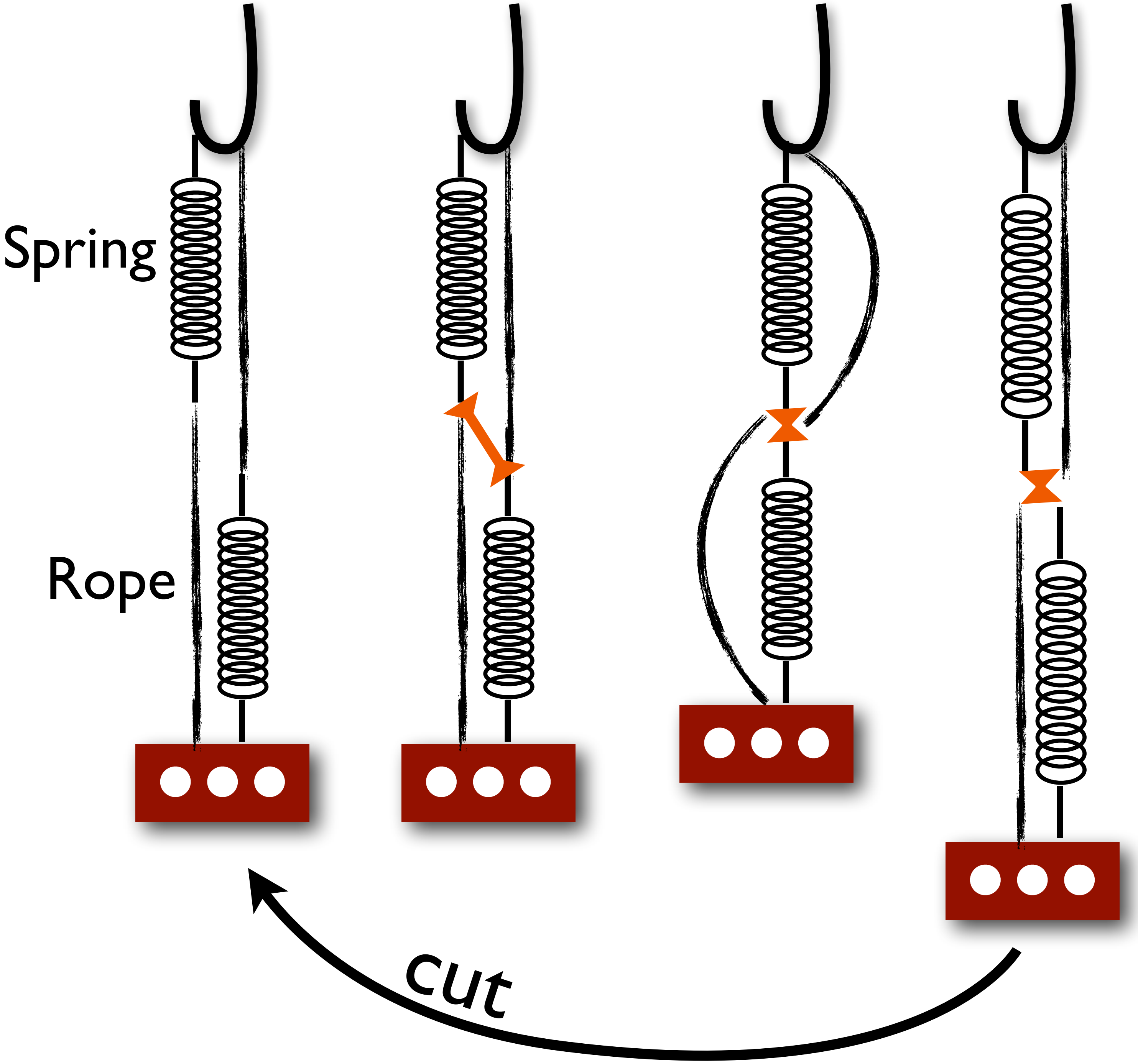


Nash equilibrium latency = 2

Thus, price of anarchy = 4/3

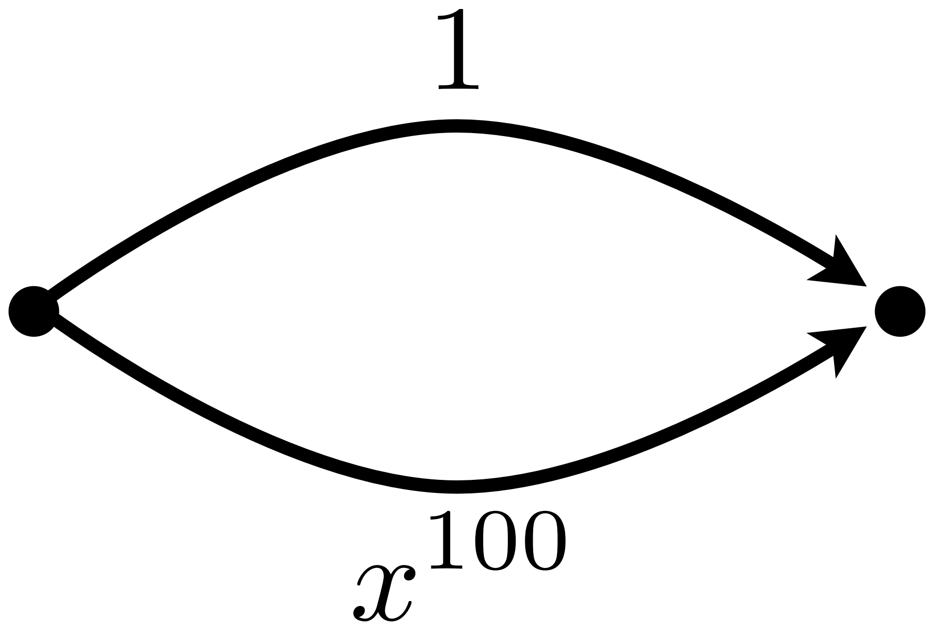
From links to springs



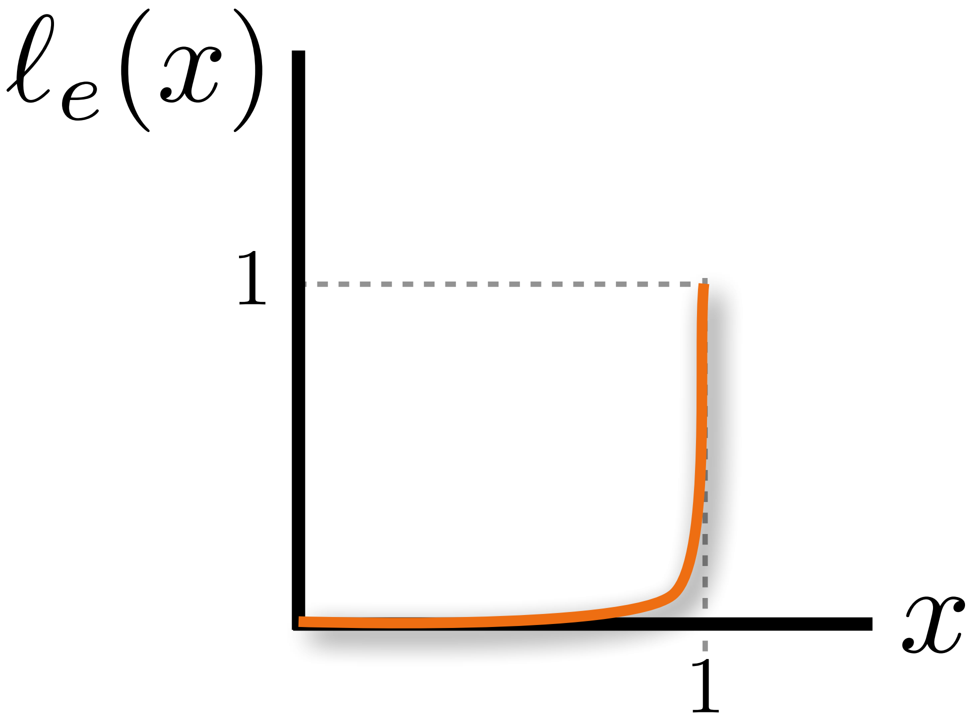


[Cohen and Horowitz, Nature 352, 699 - 701 (22 August 1991)]

Example: arbitrarily bad



Optimal: almost all flow on bottom; total latency near zero



Nash: all flow on bottom; total latency = 1

Roughgarden et al. results

As we just saw, price of anarchy can be arbitrarily high

But for linear latency functions: PoA $\leq$ 4/3

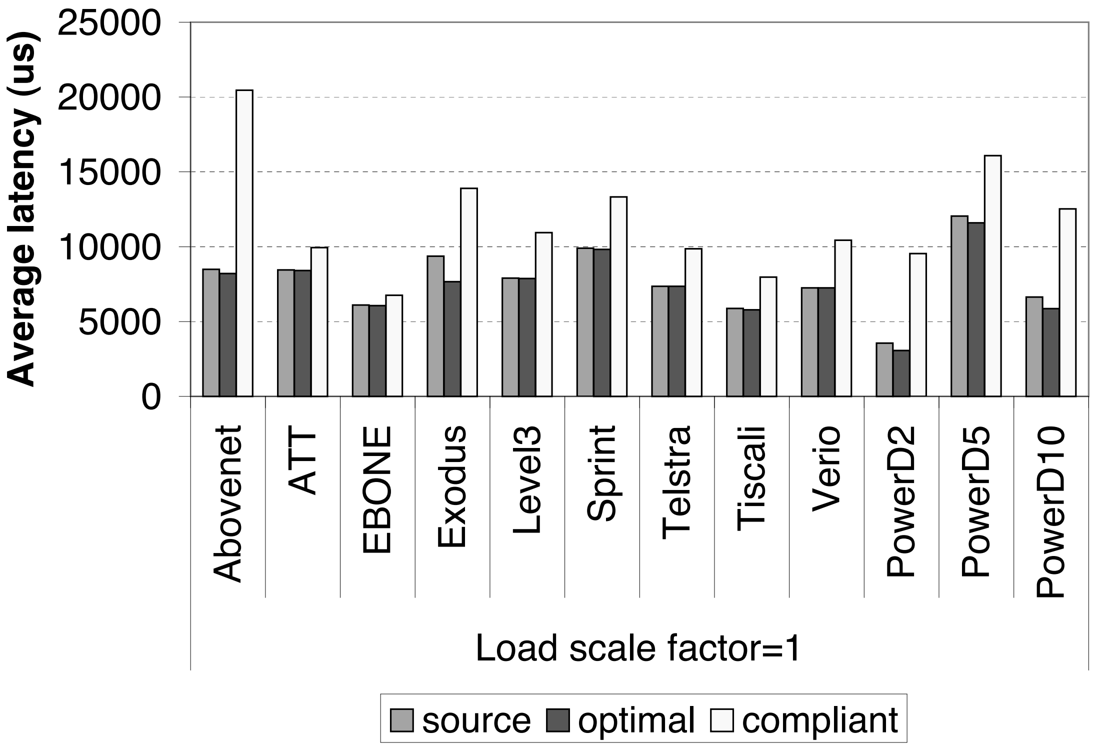
For any latency function: Nash cost is at most optimal cost of 2x as much flow

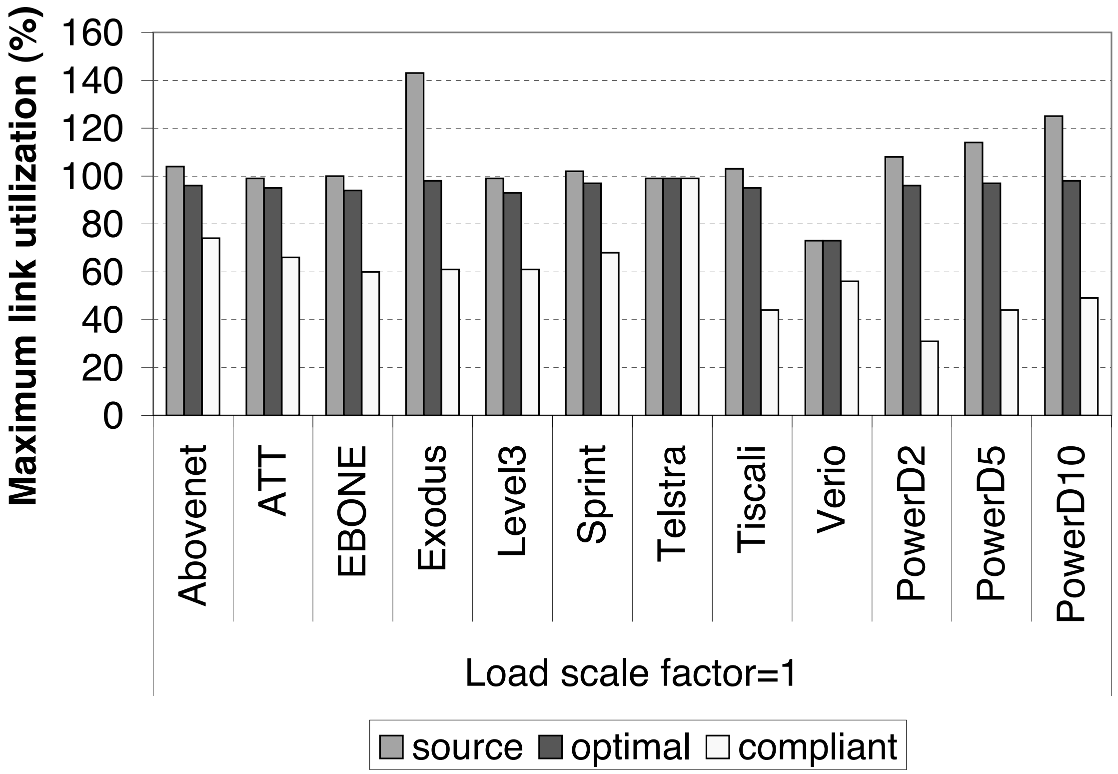
Extension to finitely many agents

- i.e., a single agent might have a nontrivial fraction of the total bandwidth
- Splittable flow: similar "2x" result
- Unsplittable flow: can be very bad

Selfish routing in realistic networks

[Qiu et al., SIGCOMM 2003]





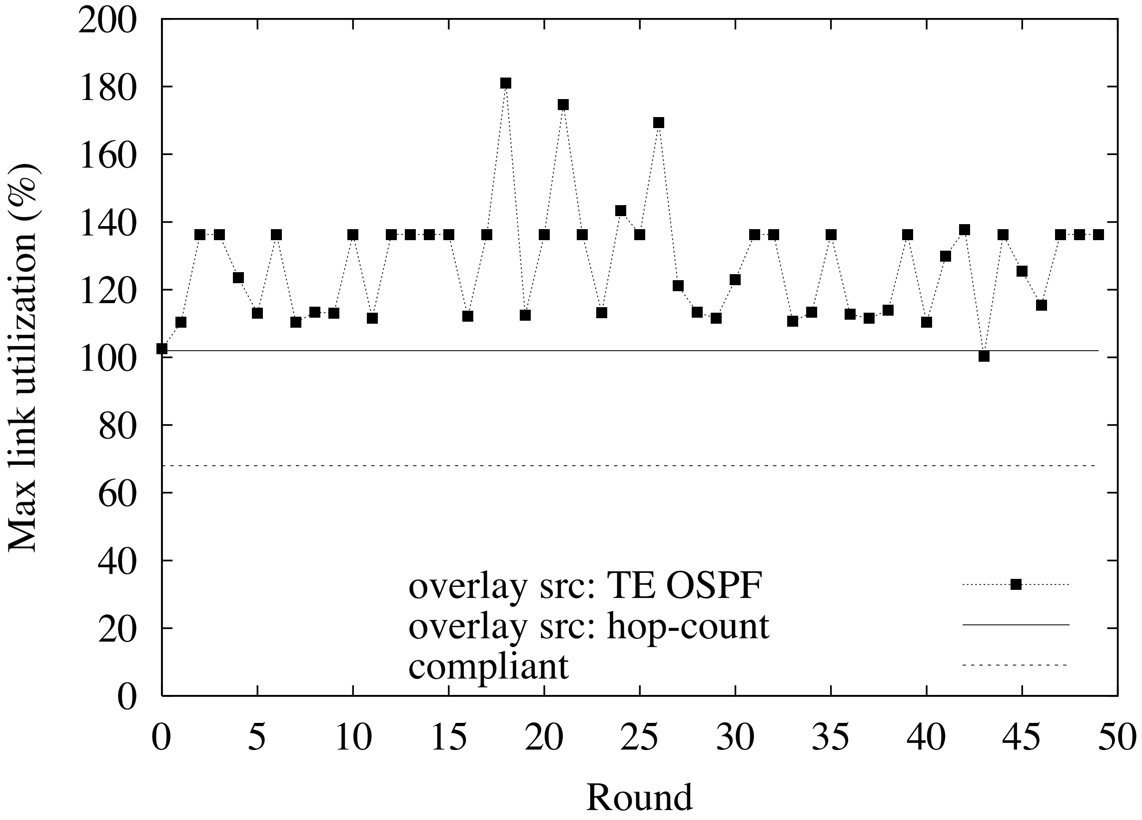
Close (but not equal) to optimal latency

...but higher maximum link utilization

Qiu et al: Selfish competing w/TE

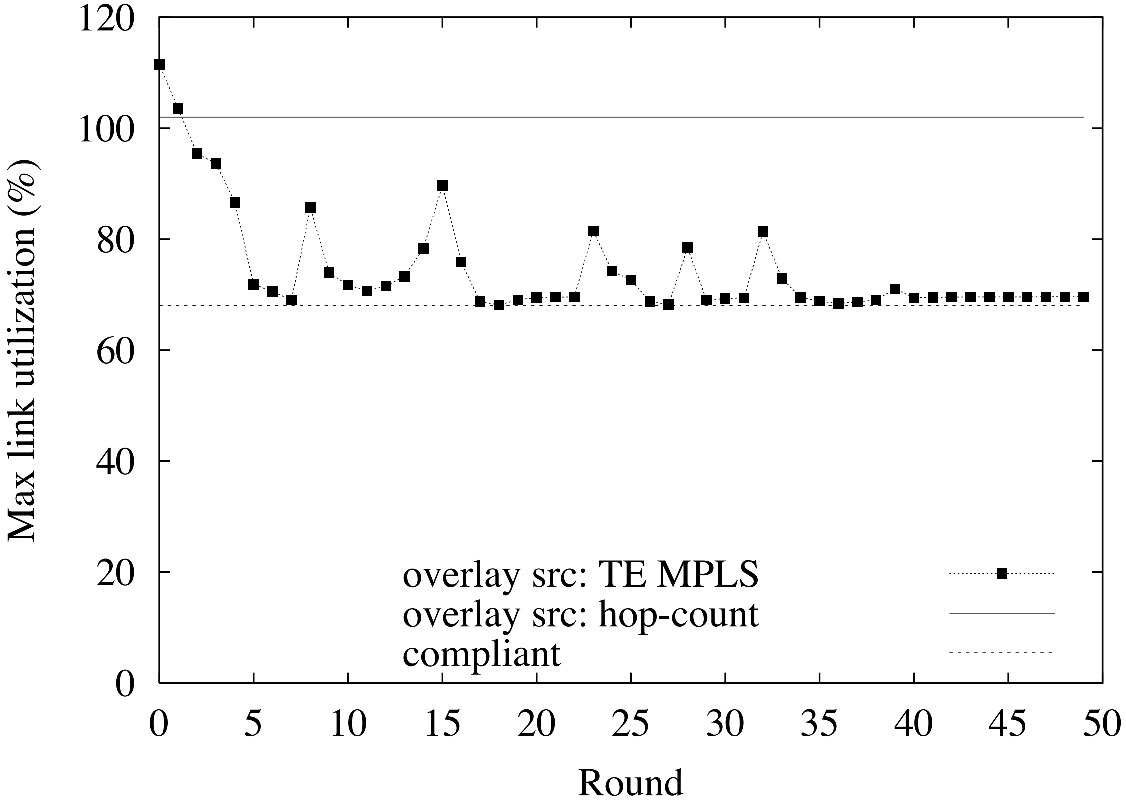
Competing systems

- Senders pick lowest latency paths
- TE computes its paths
- But now lowest latency paths have changed… iterate!



Discussion

- Are these results positive or negative?
- Examples of similar competing overlays?



Discussion

How would the traffic engineering systems we learned about earlier interact with this framework?

- Suppose the network is running a near-optimal TE underneath selfish overlay routing. Would the overlay end up doing anything nontrivial?

Discussion

Max utilization is higher in selfish. Does it matter?

Is average latency the right objective for the user?

Beyond routing...

Game theory used in networking to model

- Equilibria of distributed algorithms
- ISPs competing with each other
- Spread of new technology in social networks
- ...

Many more applications of game theory to CS

- ...and applications of CS to game theory!
- See Nisan, Roughgarden, Tardos, Vazirani's book Algorithmic Game Theory, available free online