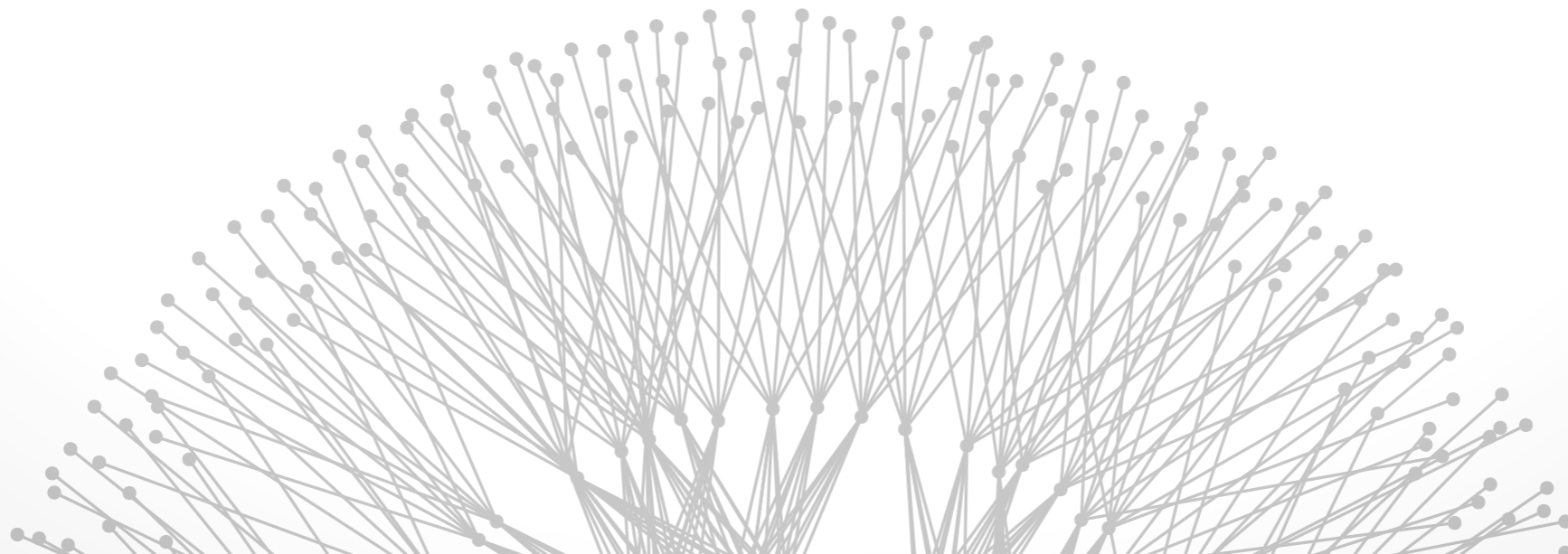


Network Games

Brighten Godfrey
CS 538 March 15 2017



Demo

Game theory basics

Games & networks: a natural fit



Game theory

Studies interaction
between selfish agents

Networking

Enables interaction
between agents

Networks make games happen!



Components defining a game

- Two or more **players**
- Set of **strategies** for each player
- For each combination of played strategies, a **payoff** or utility for each player

Red player strategies

Prisoner's Dilemma

Blue player strategies

	Cooperate	Defect
Cooperate	-1, -1	-12, 0
Defect	0, -12	-5, -5

Nash equilibrium



A chosen strategy for each player such that no player can improve its utility by changing its strategy

- (In **mixed** Nash equilibrium: players randomize their strategies according to some distribution and no player can improve its *expected* utility)

Can you find a Nash equilibrium?

Blue player strategies

Red player strategies

	Cooperate	Defect
Cooperate	-1, -1	-12, 0
Defect	0, -12	-5, -5

Prisoner's dilemma Nash eq.



Blue player strategies

Red player strategies

	Cooperate	Defect
Cooperate	-1, -1	-12, 0
Defect	0, -12	-5, -5

Nash equilibrium

Price of Anarchy



[C. Papadimitriou, "Algorithms, games and the Internet", STOC 2001]

Assumes some global "cost" objective, e.g., social utility (sum of players' payoffs).

Price of anarchy = $\frac{\text{worst Nash equilibria's cost}}{\text{optimal cost}}$

		Blue prisoner	
		Cooperate	Defect
Red prisoner	Cooperate	-1, -1	-10, 0
	Defect	0, -10	-5, -5

Here, $PoA = 10/2 = 5$.

Rock Paper Scissors



Can you find a Nash equilibrium in R-P-S?

Blue player strategies

		Rock	Paper	Scissors
Red player strategies	Rock	\$0, \$0	\$0, \$1	\$1, \$0
	Paper	\$1, \$0	\$0, \$0	\$0, \$1
	Scissors	\$0, \$1	\$1, \$0	\$0, \$0

No pure Nash equilibrium!



Stable paths problem

- [Tim Griffin, Bruce Shepherd, Gordon Wilfong, ToN'02]
- A game model of BGP

How bad is selfish routing?

- [Tim Roughgarden, Eva Tardos, JACM 2002]
- Analysis of price of anarchy of latency-optimized routing

Selfish routing in Internet-like environments

- [Lili Qiu, Richard Yang, Yin Zhang, Scott Shenker, SIGCOMM'03]
- What is the price of anarchy like in practice for latency-optimized routing?

Internet routing as a game

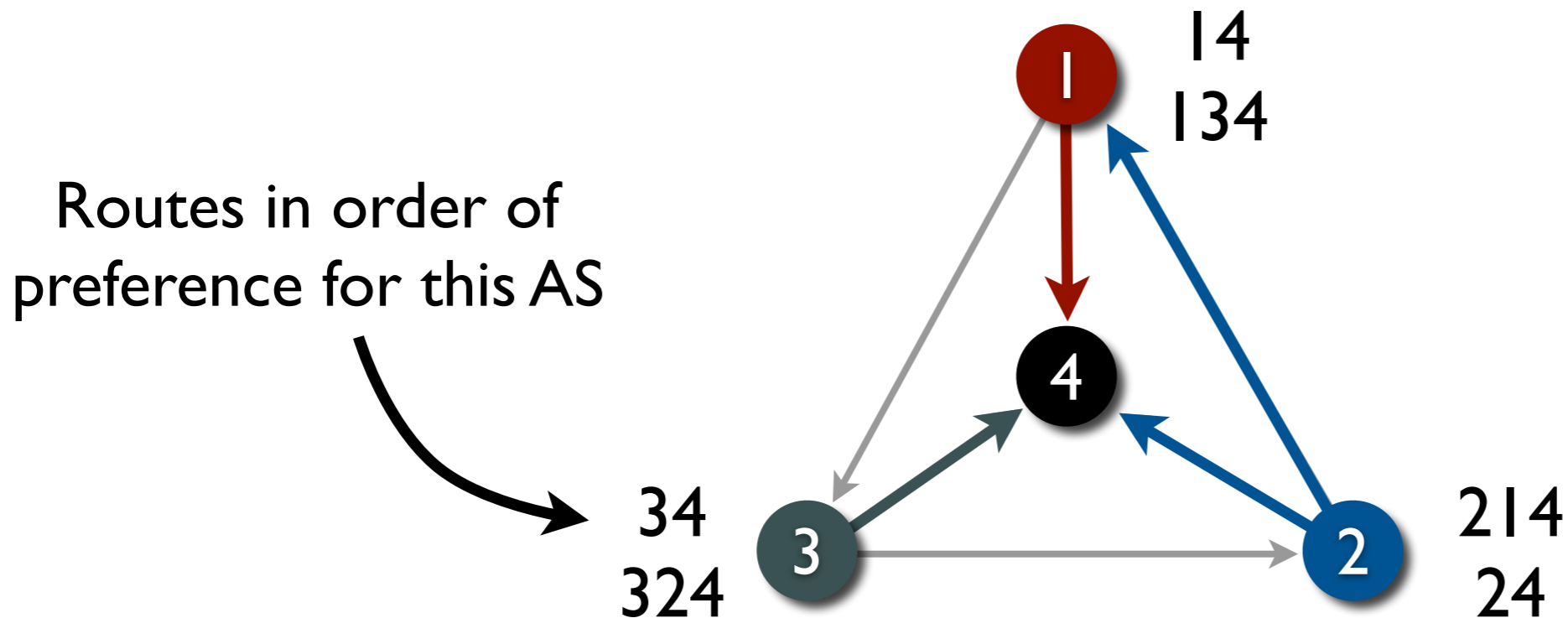
BGP routing as a game



players autonomous systems

strategies pick a route, any route... (to fixed dest.)

player's utility arbitrary function of route (but $-\infty$ for 'illegal' route not offered by neighbor)



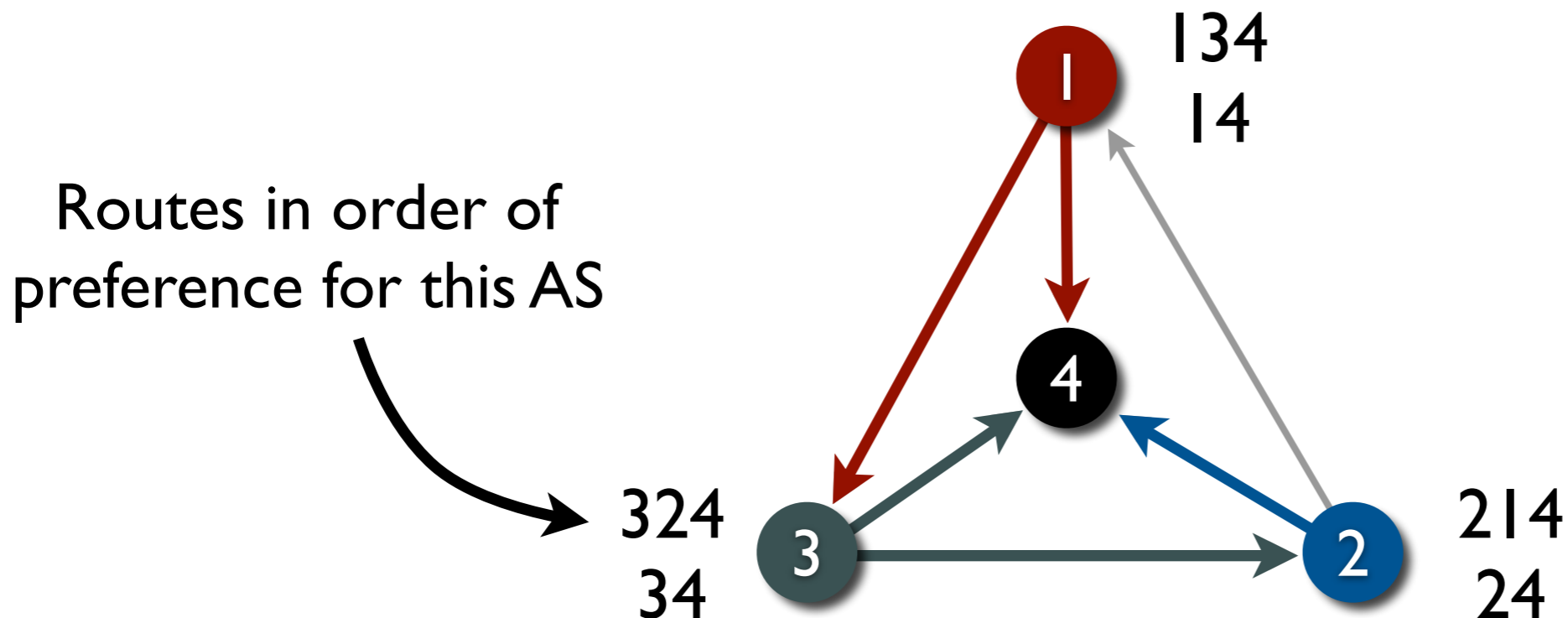
BGP routing as a game



players autonomous systems

strategies pick a route, any route... (to fixed dest.)

player's utility arbitrary function of route (but $-\infty$ for 'illegal' route not offered by neighbor)



No Nash equilibrium!

BGP routing as a game



In general, NP-complete to decide whether an equilibrium exists [Griffin, Shepherd, Wilfong, ToN'02]

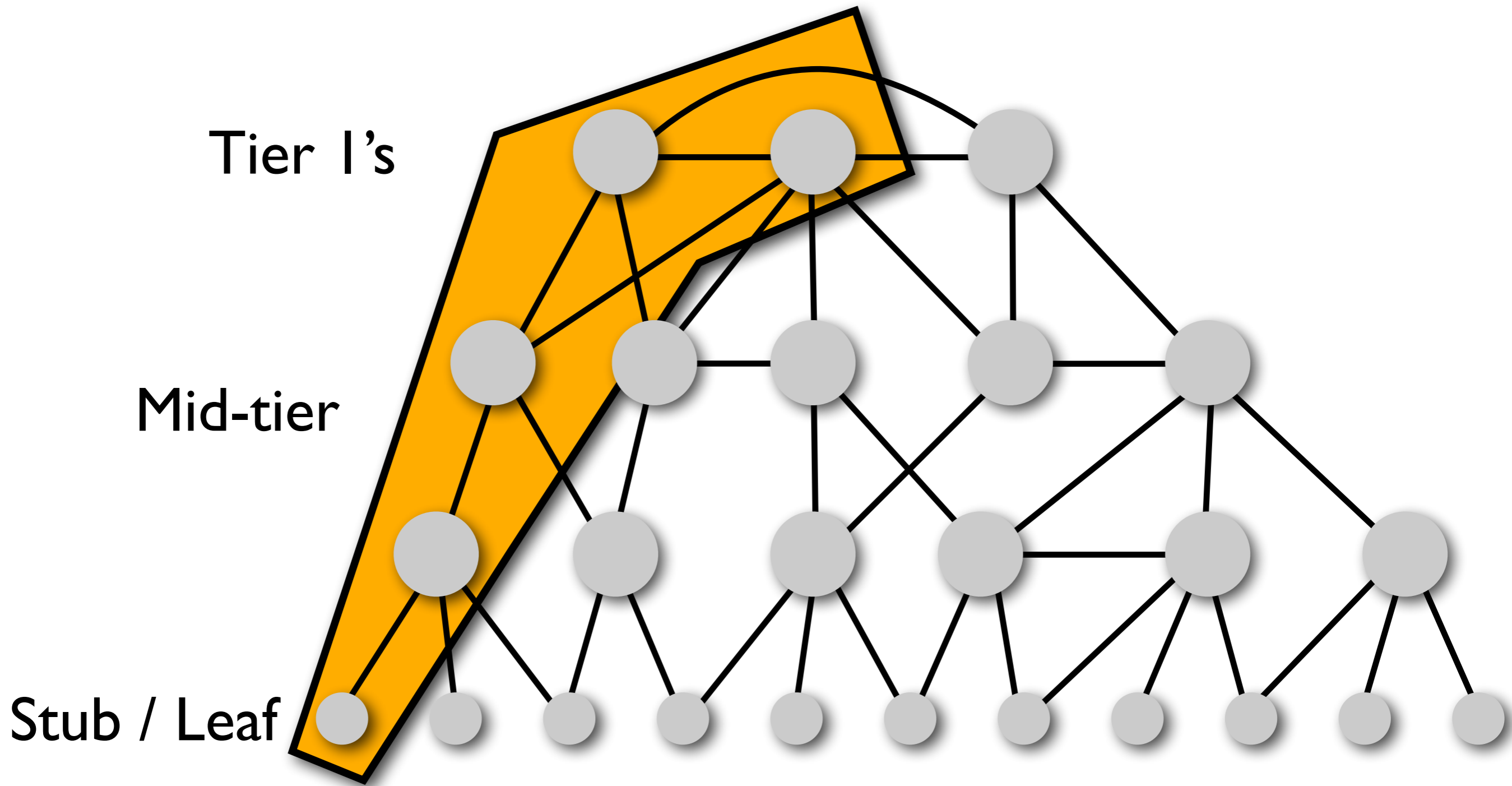
Might have 0, 1, 2, 3, ... equilibria

Even if it has an equilibrium, might not converge to it

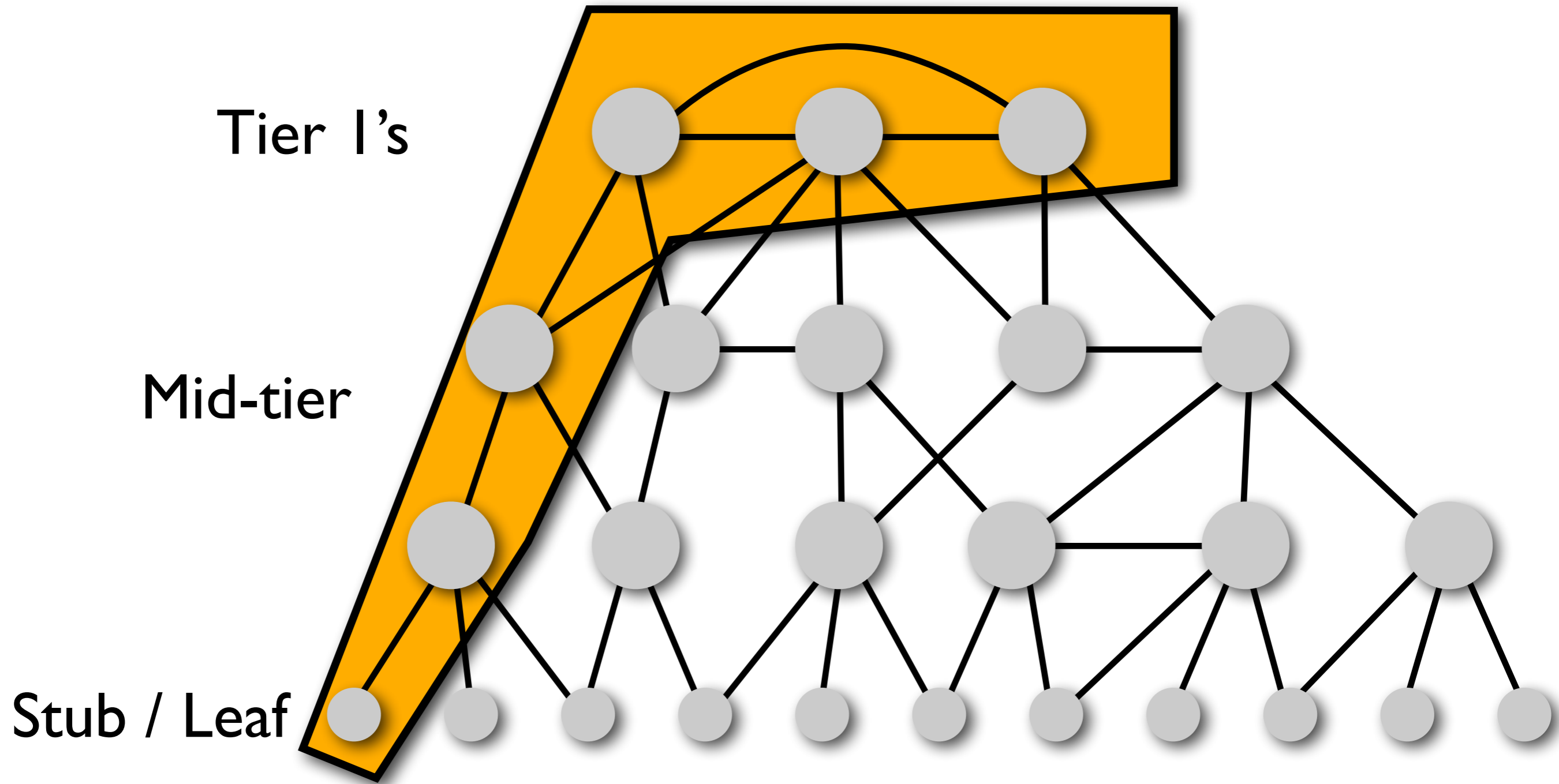
- Depends on starting state, message timing, ...
- PSPACE-complete to decide whether a given set of BGP preferences can oscillate [Fabrikant, Papadimitriou, SODA'08]

If we assume customer-provider-peer and valley-free routing, guaranteed to converge [Gao, Rexford]

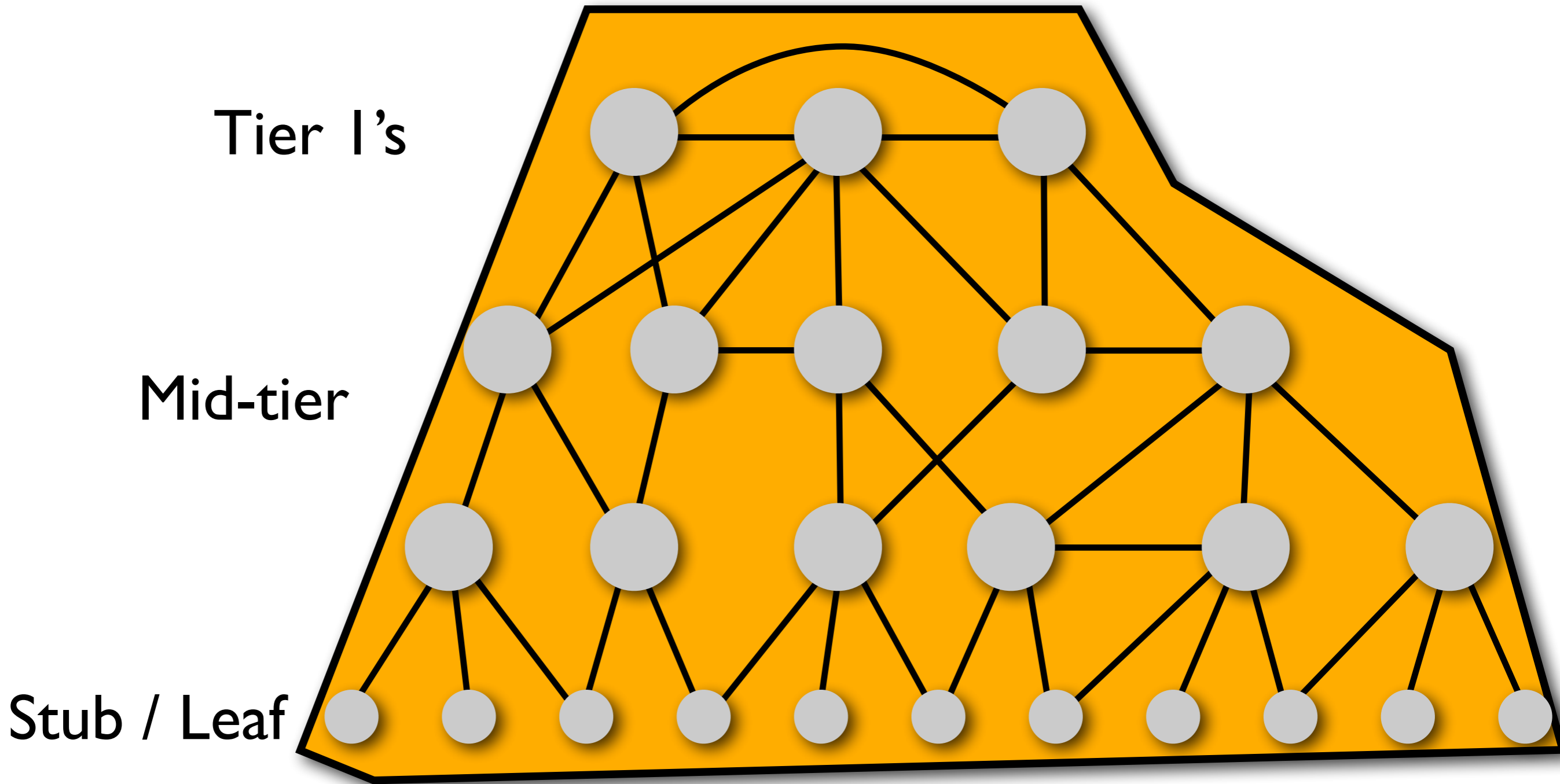
Gao-Rexford convergence



Gao-Rexford convergence



Gao-Rexford convergence



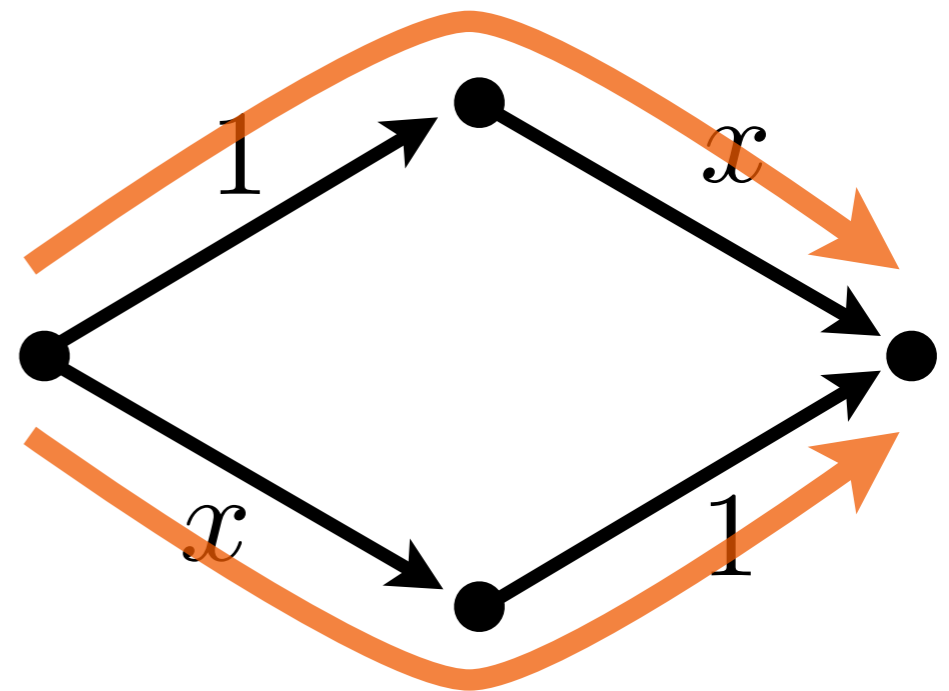
How bad is
selfish routing?

The selfish routing game



The game context:

- Directed graph
- **Latency function** on each edge specifying latency as function of total flow x on edge on edge
- **Path latency** = sum of edge latencies



Flow $x = 0.5$ on each path;
Total latency = 1.5

The selfish routing game

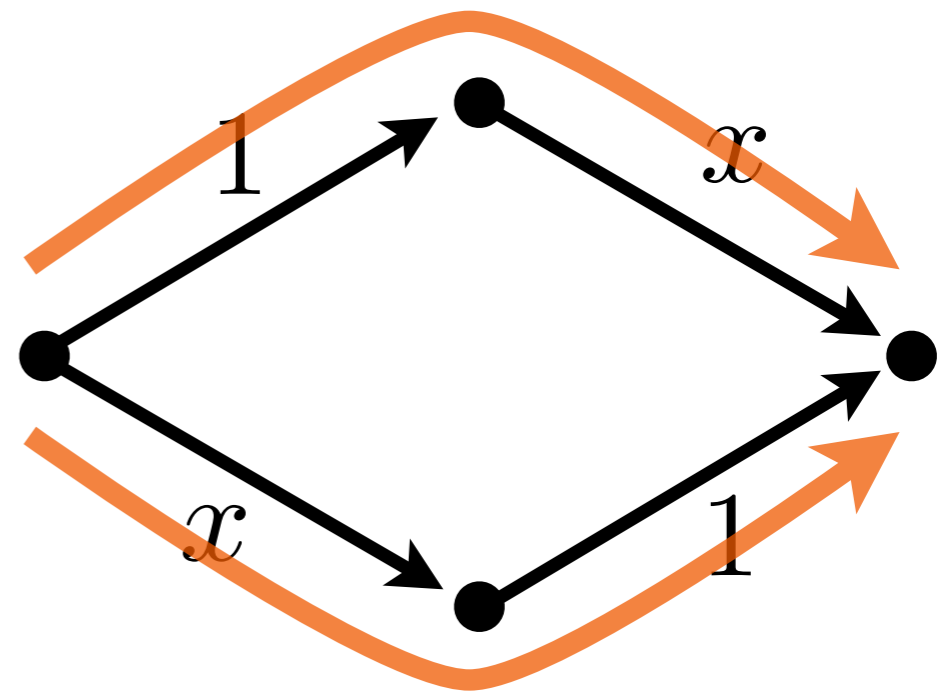


Player strategy:

- Pick a path on which to route
- Players selfishly pick paths with lowest latency (source-controlled routing)

For now assume:

- many users
- each has negligible load
- total load = 1



Flow $x = 0.5$ on each path;
Total latency = 1.5

Example: Braess's paradox



[Dietrich Braess, 1968]

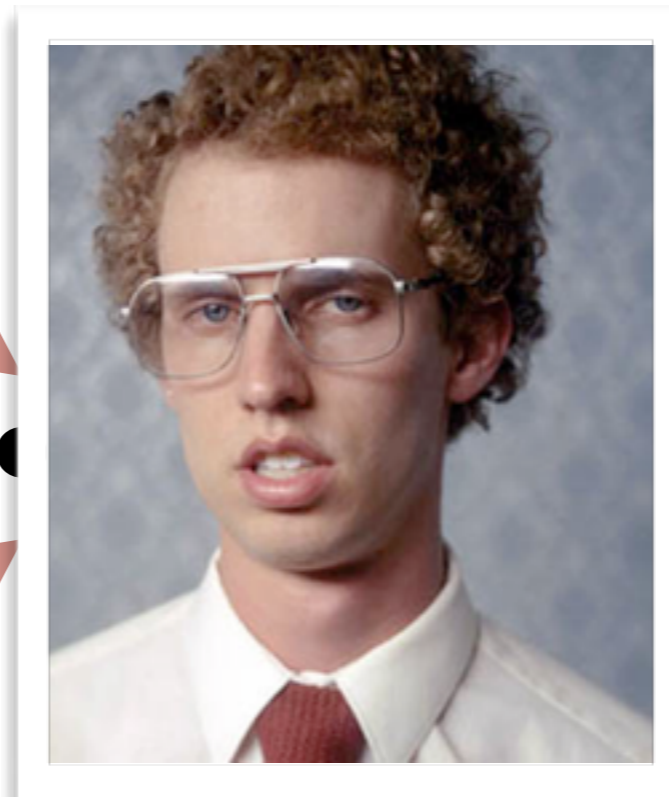
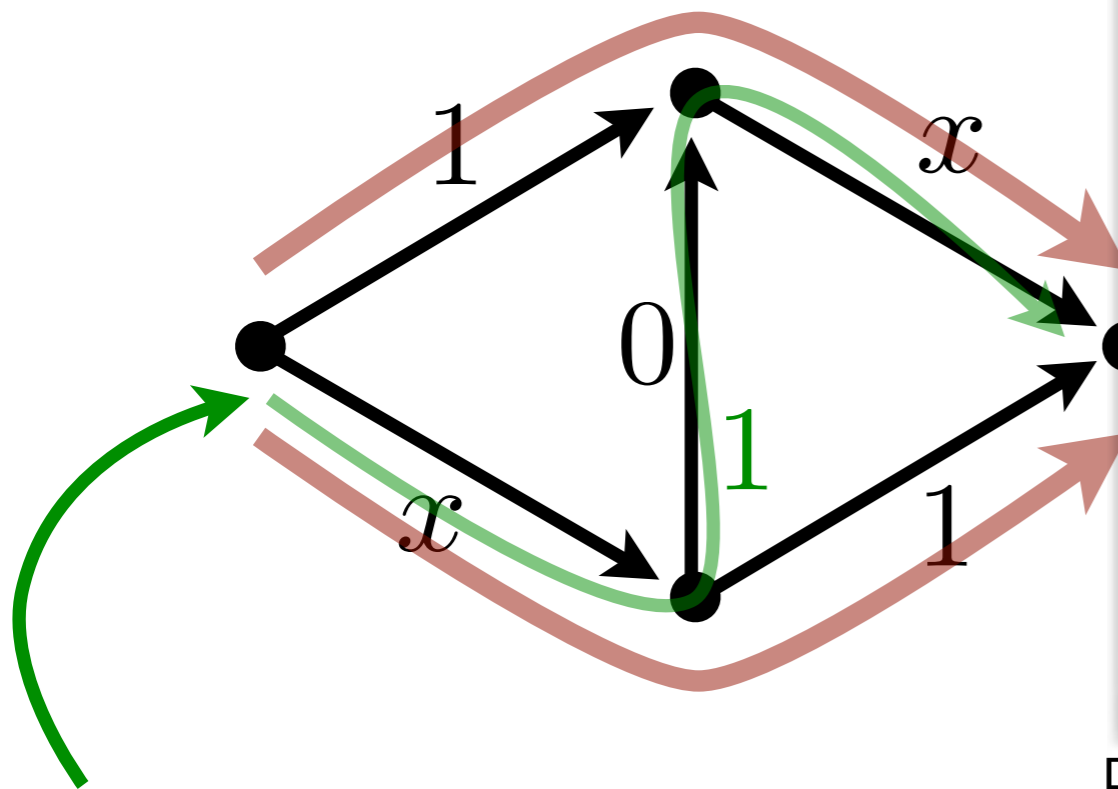


Fig 1b: N. Dynamite.

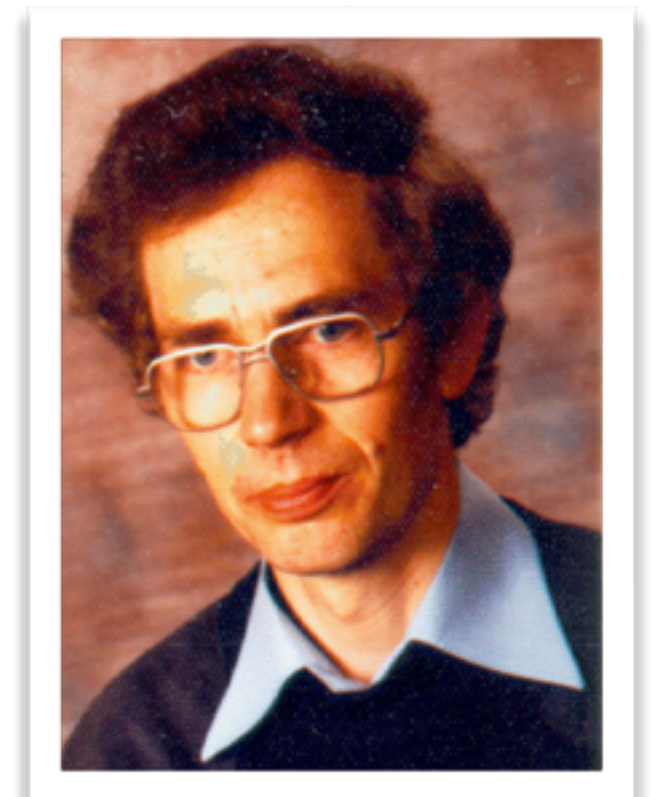


Fig 1a: D. Braess.

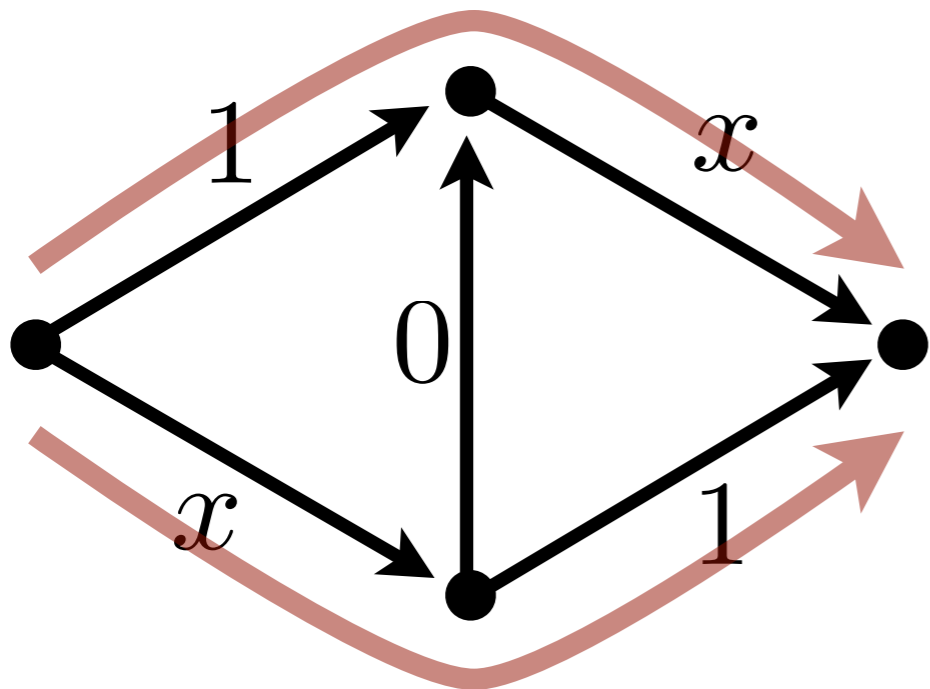
≈

Green path is better.

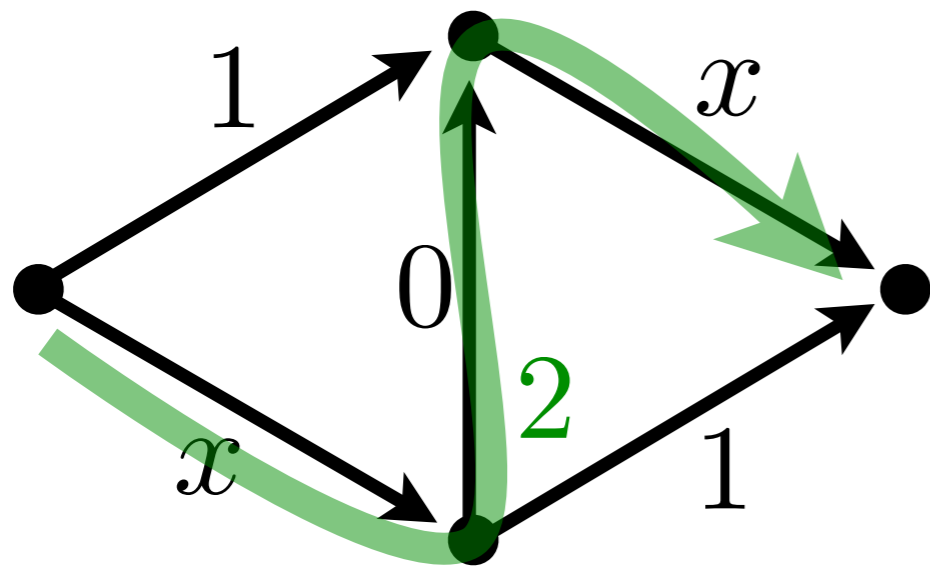
Everyone switches to it!

Initially: 0.5 flow along each path; latency $1 + 0.5 = 1.5$

Example: Braess's paradox



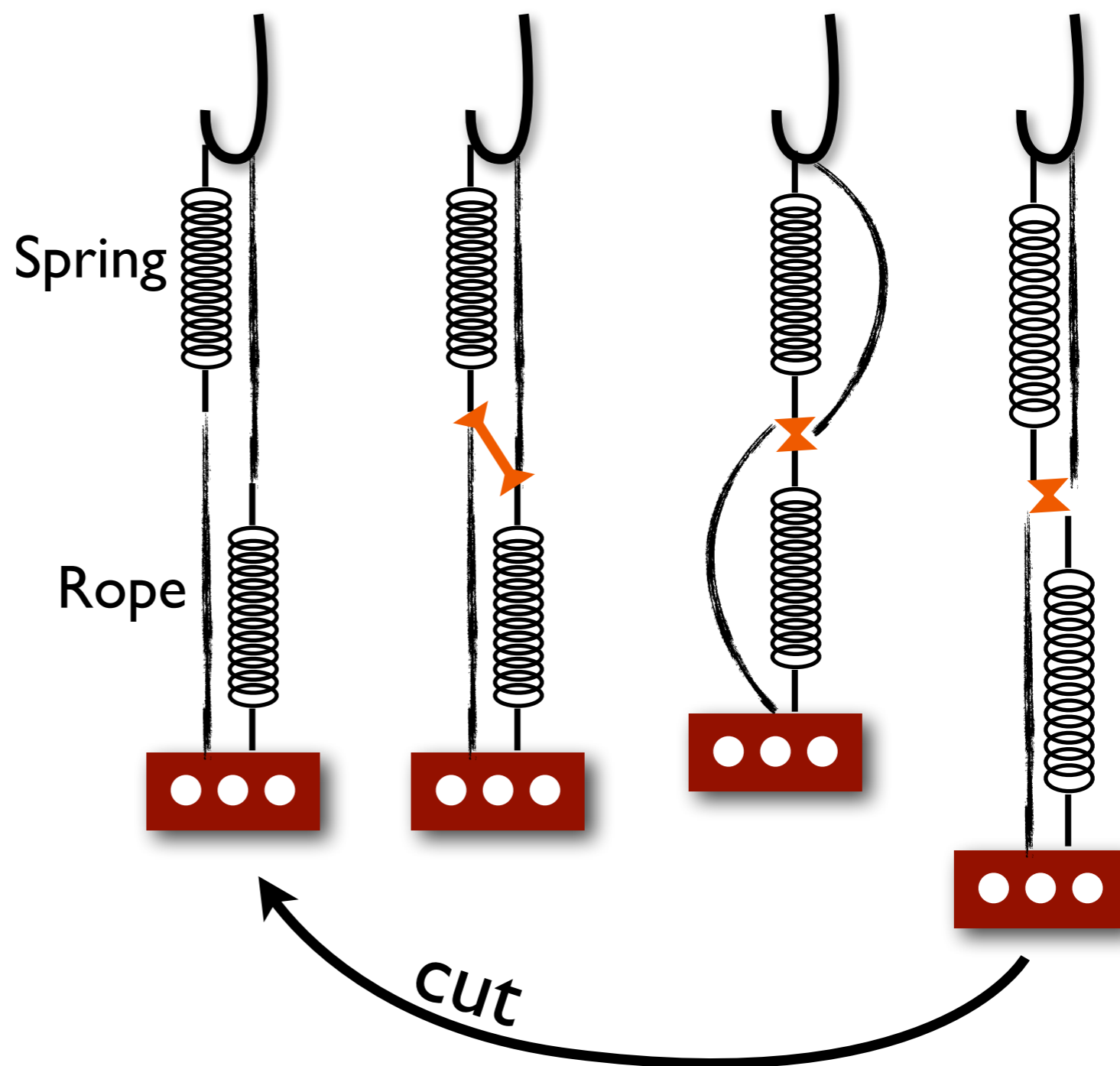
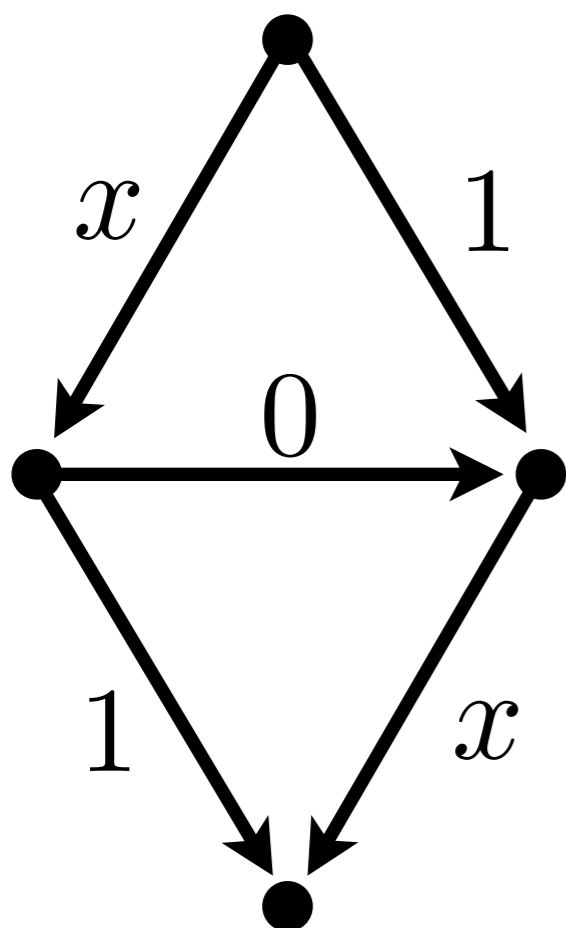
Optimal latency = 1.5



Nash equilibrium latency = 2

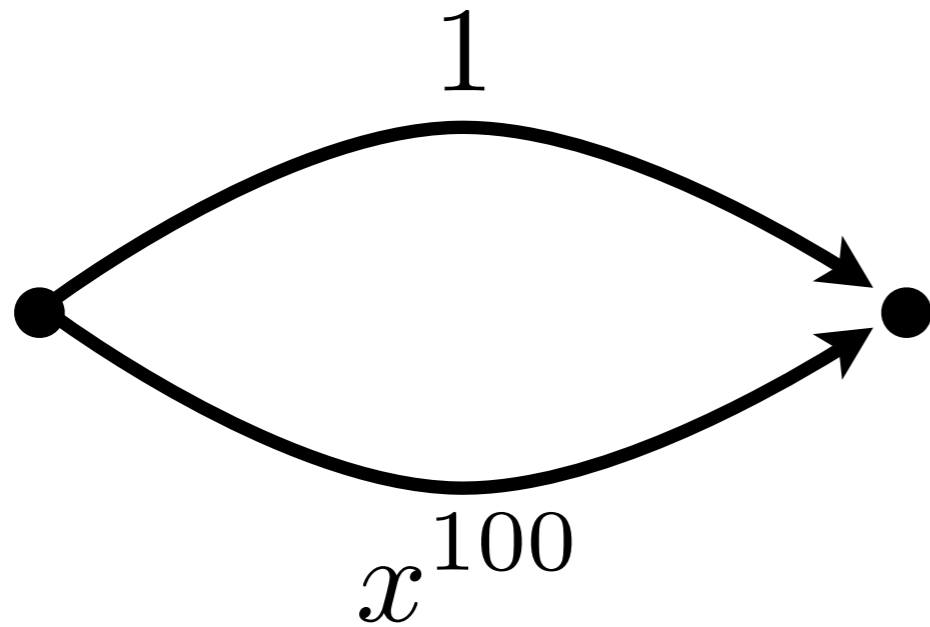
Thus, price of anarchy = $4/3$

From links to springs

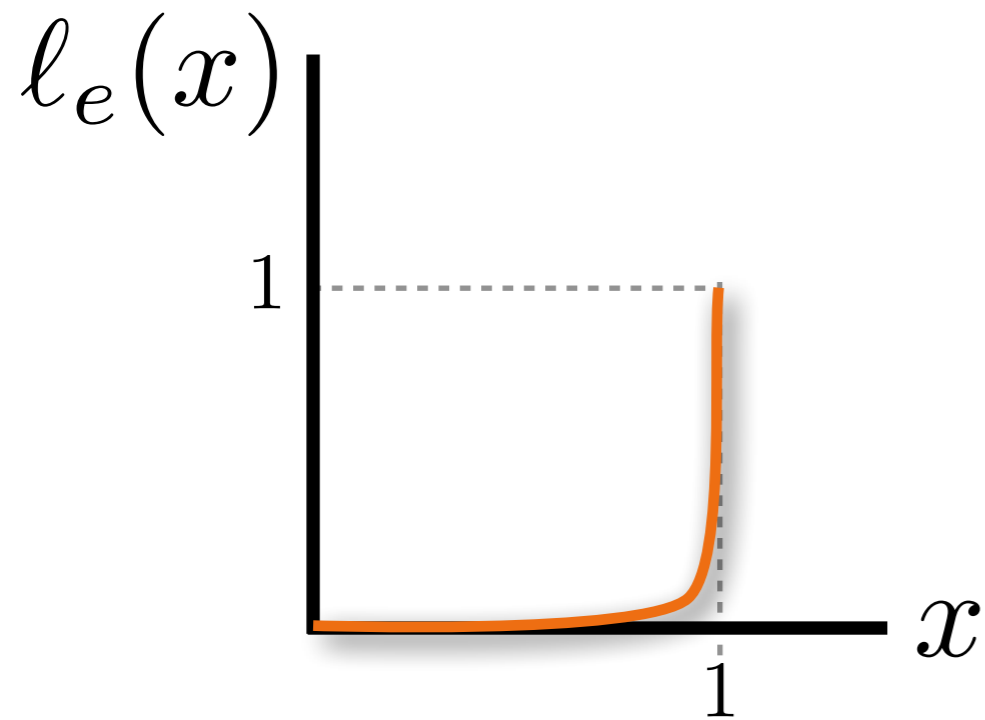


[Cohen and Horowitz, Nature 352, 699 - 701 (22 August 1991)]

Example: arbitrarily bad



Optimal: **almost** all flow on bottom; total latency near zero



Nash: all flow on bottom;
total latency = 1



As we just saw, price of anarchy can be arbitrarily high

But for linear latency functions: $\text{PoA} \leq 4/3$

For any latency function: Nash cost is at most optimal cost of $2x$ as much flow

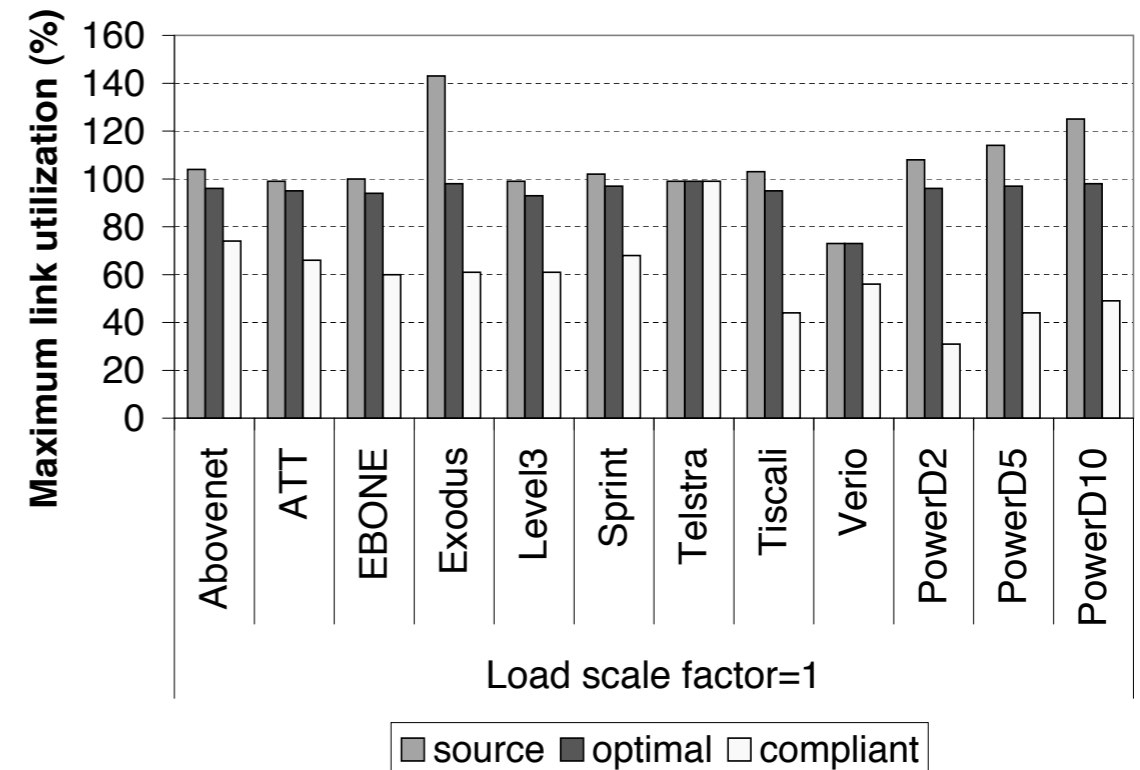
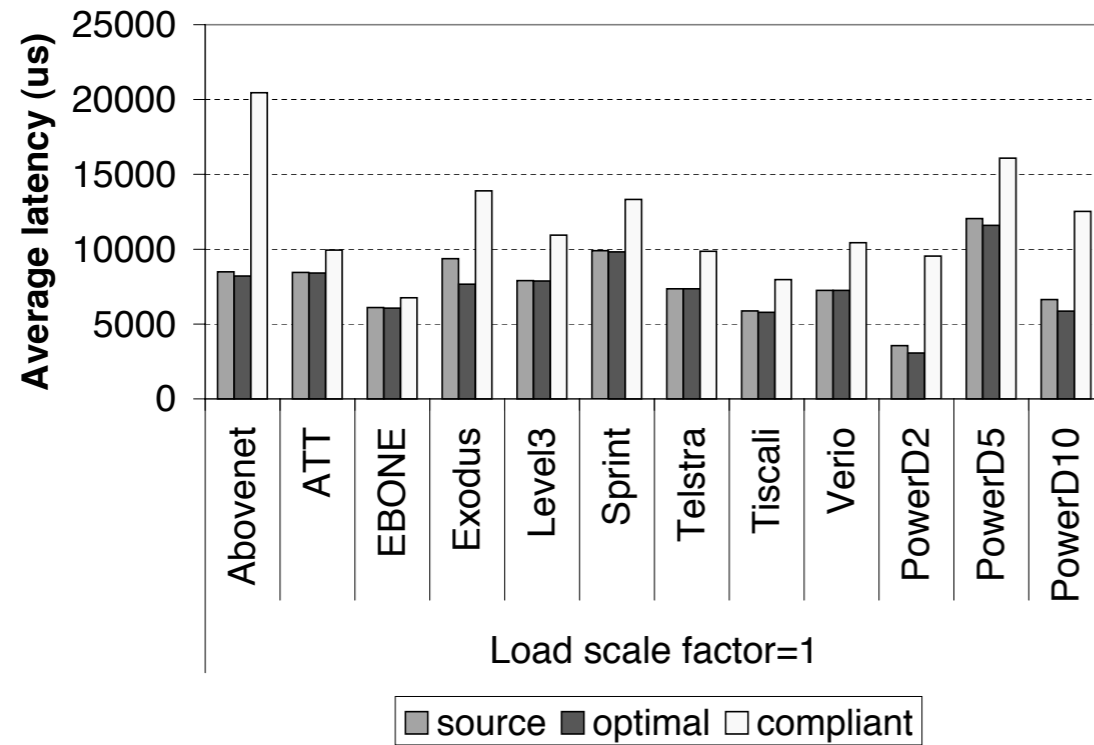
Extension to finitely many agents

- i.e., a single agent might have a nontrivial fraction of the total bandwidth
- Splittable flow: similar “ $2x$ ” result
- Unsplittable flow: can be very bad

Selfish routing in realistic networks



[Qiu et al., SIGCOMM 2003]



Close to optimal latency

...but higher maximum link utilization



How would the traffic engineering systems we learned about earlier interact with this framework?

- Suppose the network is running a near-optimal TE underneath selfish overlay routing. Would the overlay end up doing anything nontrivial?



Max utilization is higher in selfish. Does it matter?

Is average latency the right objective for the user?



Game theory used in networking to model

- Equilibria of distributed algorithms
- ISPs competing with each other
- Spread of new technology in social networks
- ...

Many more applications of game theory to CS

- ...and applications of CS to game theory!
- See Nisan, Roughgarden, Tardos, Vazirani's book **Algorithmic Game Theory**, available free online

Project Midterm Presentations



Two key goals

- Benchmark: Demonstrate concrete progress
- Feedback & discussion with your peers

Content

- What problem are you solving?
- Why has past work not addressed the problem?
- What is your approach for solving it?
- What are your preliminary results & progress?

Logistics

- 10 minutes total: 6 min presentation + 4 min discuss
- Be ready to present on Monday after spring break