Network Games

Brighten Godfrey

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Demo

Game theory basics

Games & networks: a natural fit

Game theory
Studies interaction
between selfish agents

Networking
Enables interaction
between agents

Networks make games happen!

Game theory

Components defining a game

- Two or more players
- Set of strategies for each player
- For each combination of played strategies, a payoff or utility for each player

Prisoner's Dilemma

Blue player strategies

Red player strategies

	Cooperate	Defect
Cooperate	-1, -1	-12, 0
Defect	0, -12	-5, -5

Nash equilibrium

A chosen strategy for each player such that no player can improve its utility by changing its strategy

- (In mixed Nash equilibrium: players randomize their strategies according to some distribution and no player can improve its *expected* utility)

Can you find a Nash equilibrium?

Blue player strategies

Red player strategies

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Prisoner's dilemma Nash eq.

Blue player strategies

Red player strategies

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Nash equilibrium

Price of Anarchy

[C. Papadimitriou, "Algorithms, games and the Internet", STOC 2001]

Assumes some global "cost" objective, e.g., social utility (sum of players' payoffs).

$$\text{Price of anarchy} = \frac{\text{worst Nash equilibria's cost}}{\text{optimal cost}}$$

Blue prisoner

Red prisoner	Cooperate	Defect
Cooperate	-1, -1	-10, 0
Defect	0, -10	-5, -5

Here, PoA = 10/2 = 5.

Rock Paper Scissors

Can you find a Nash equilibrium in R-P-S?

Blue player strategies

Red player strategies

	Rock	Paper	Scissors
Rock	\$0, \$0	\$0, \$1	\$1, \$0
Paper	\$1, \$0	\$0, \$0	\$0, \$1
Scissors	\$0, \$1	\$1, \$0	\$0, \$0

No pure Nash equilibrium!

Today's papers

Stable paths problem

- [Tim Griffin, Bruce Shepherd, Gordon Wilfong, ToN'02]
- A game model of BGP

How bad is selfish routing?

- [Tim Roughgarden, Eva Tardos, JACM 2002]
- Analysis of price of anarchy of latency-optimized routing

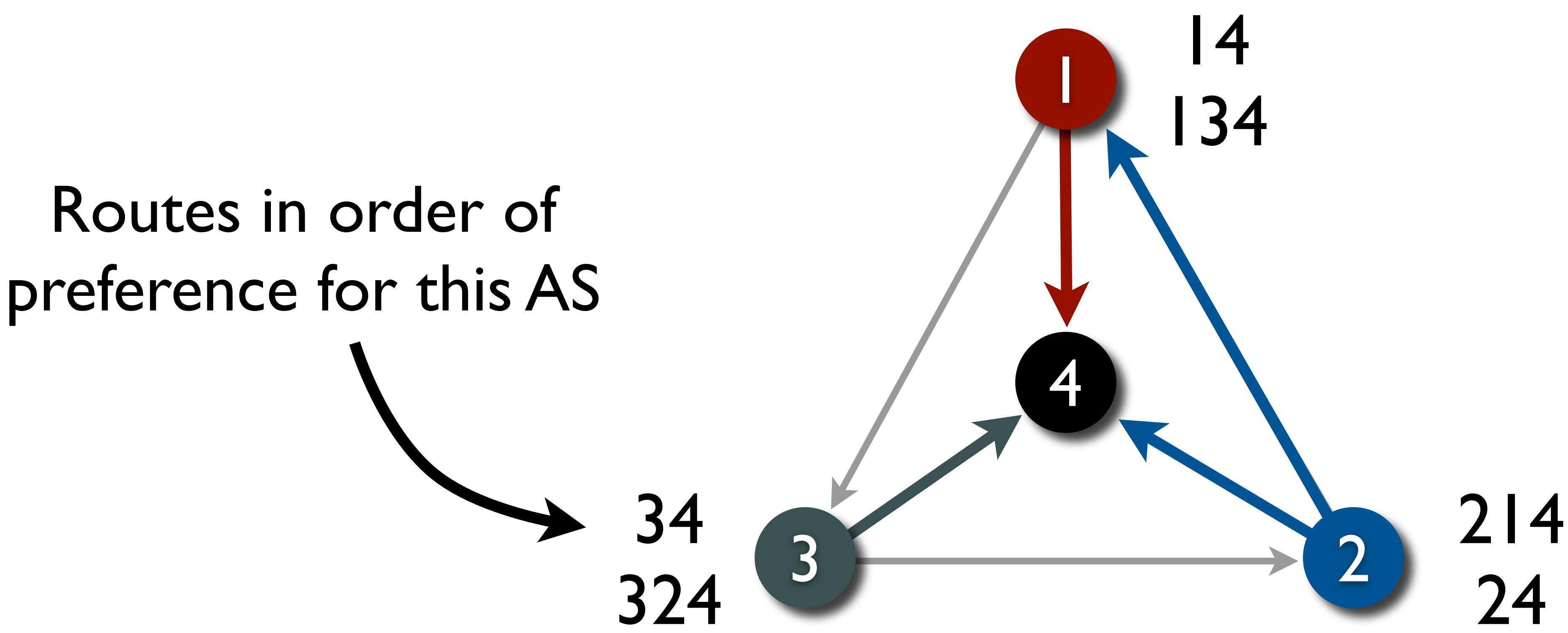
Selfish routing in Internet-like environments

- [Lili Qiu, Richard Yang, Yin Zhang, Scott Shenker, SIGCOMM'03]
- What is the price of anarchy like in practice for latency-optimized routing?

Internet routing as a game

BGP routing as a game

players	autonomous systems
strategies	pick a route, any route... (to fixed dest.)
player’s utility	arbitrary function of route (but $-\infty$ for ‘illegal’ route not offered by neighbor)



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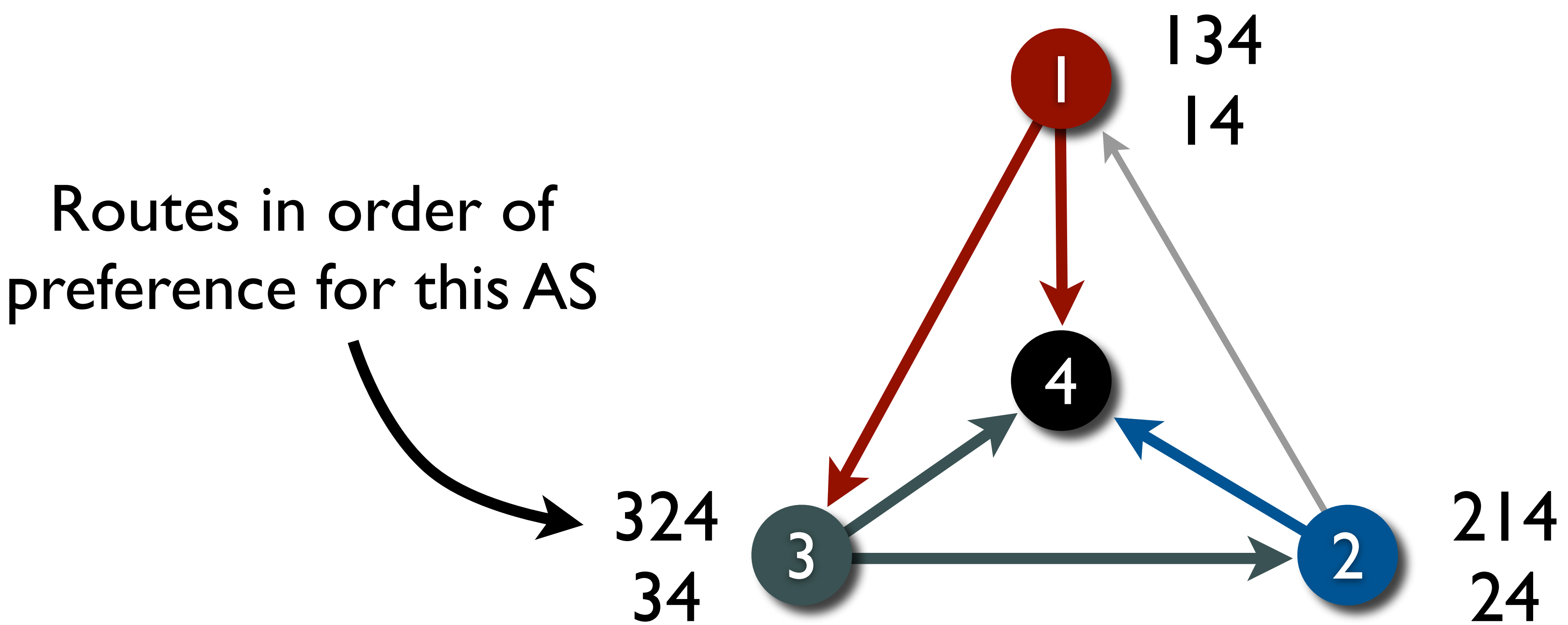


No Nash equilibrium!

BGP routing as a game

In general, NP-complete to decide whether an equilibrium exists [Griffin, Shepherd, Wilfong, ToN'02]

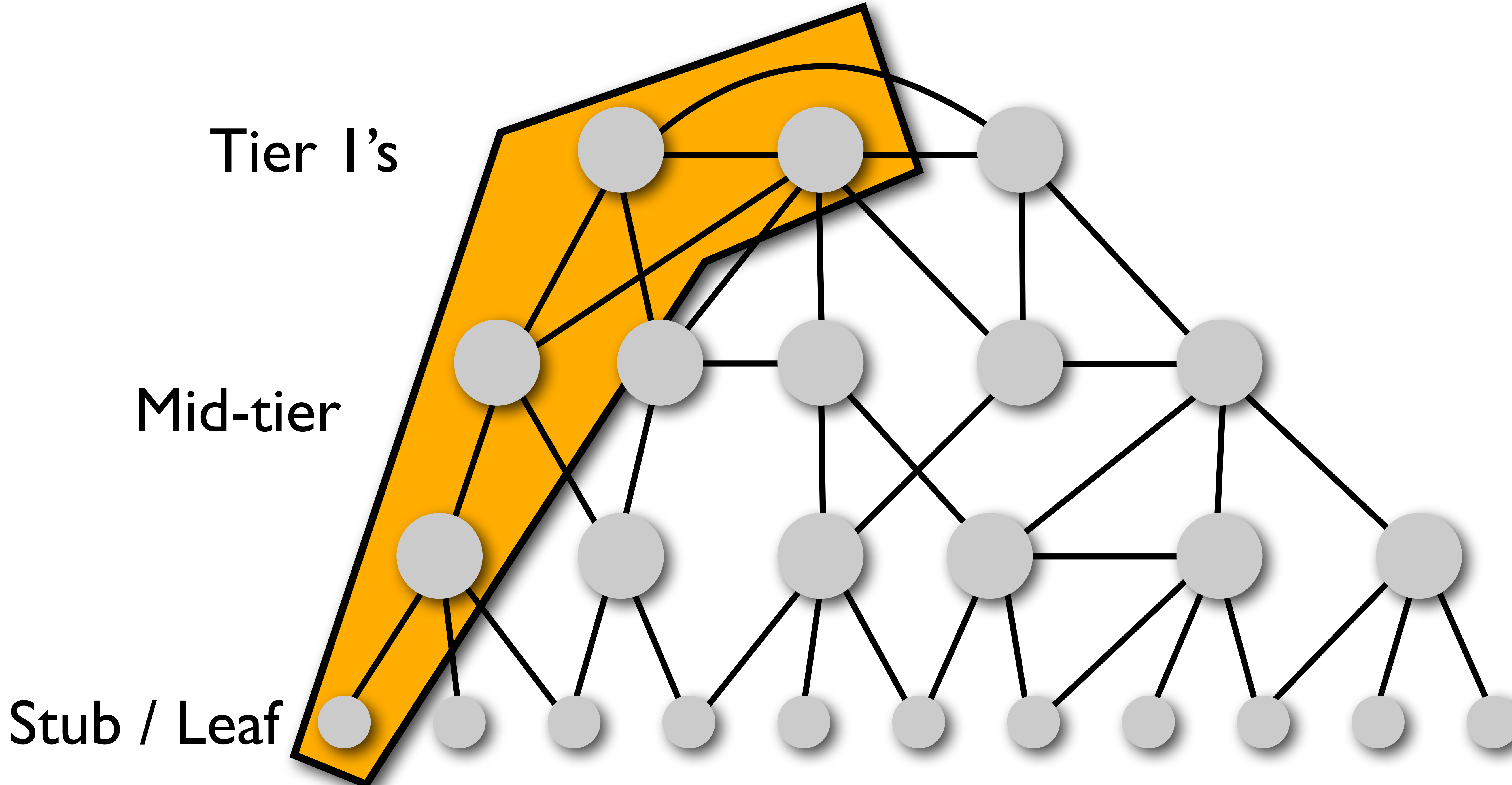
Might have 0, 1, 2, 3, ... equilibria

Even if it has an equilibrium, might not converge to it

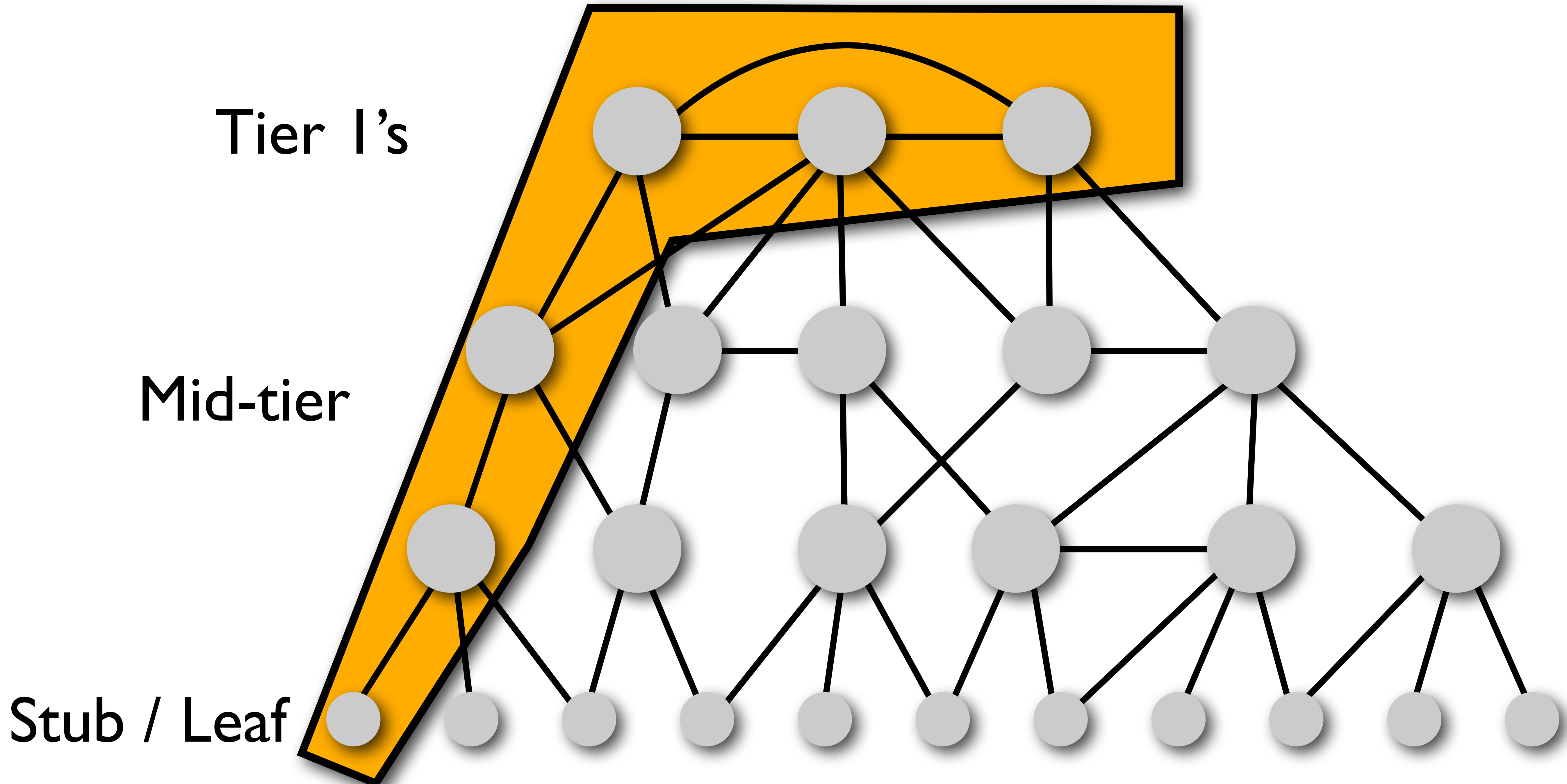
- Depends on starting state, message timing, ...
- PSPACE-complete to decide whether a given set of BGP preferences can oscillate [Fabrikant, Papadimitriou, SODA'08]

If we assume customer-provider-peer and valley-free routing, guaranteed to converge [Gao, Rexford]

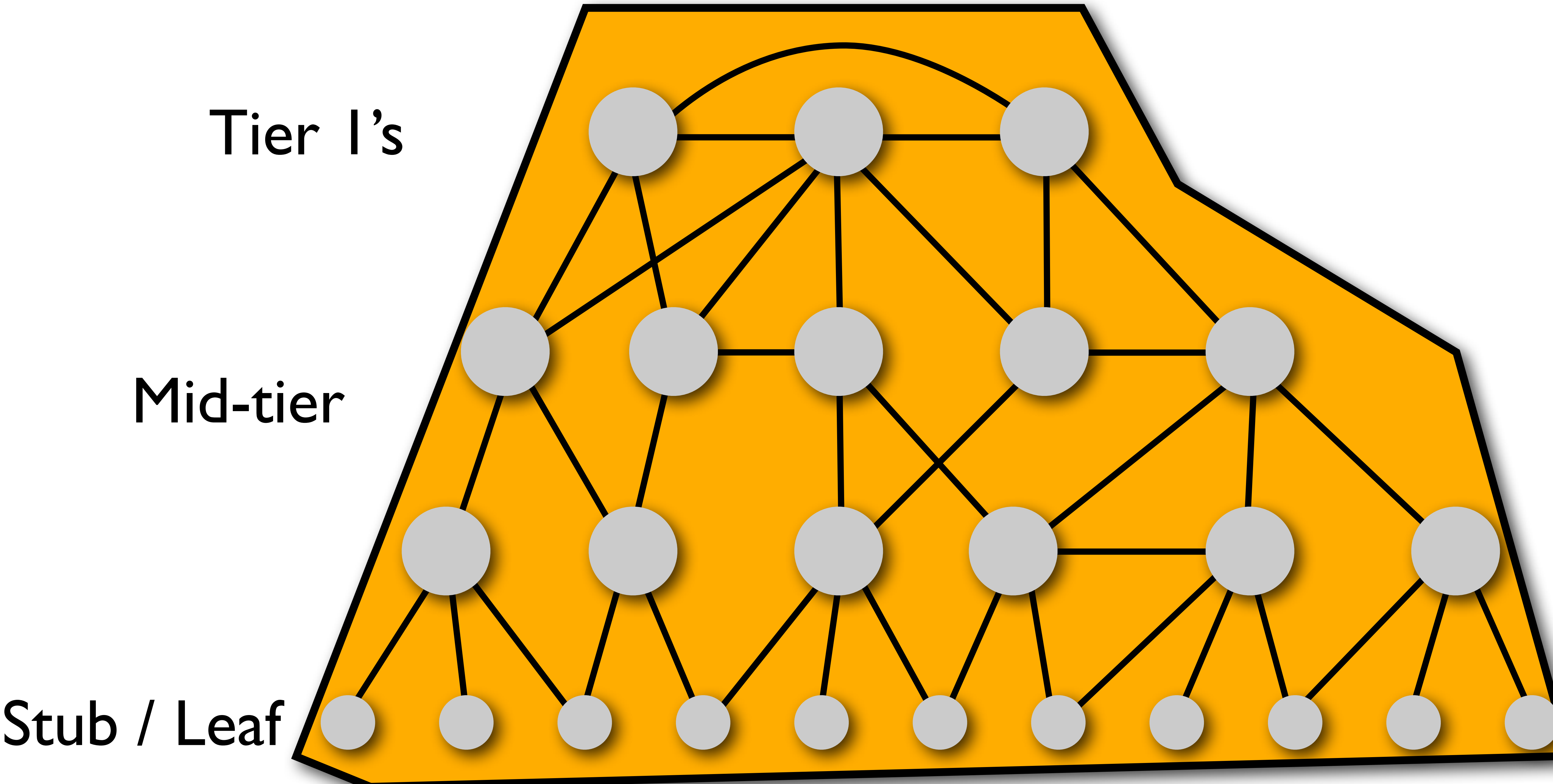
Gao-Rexford convergence



Gao-Rexford convergence



Gao-Rexford convergence

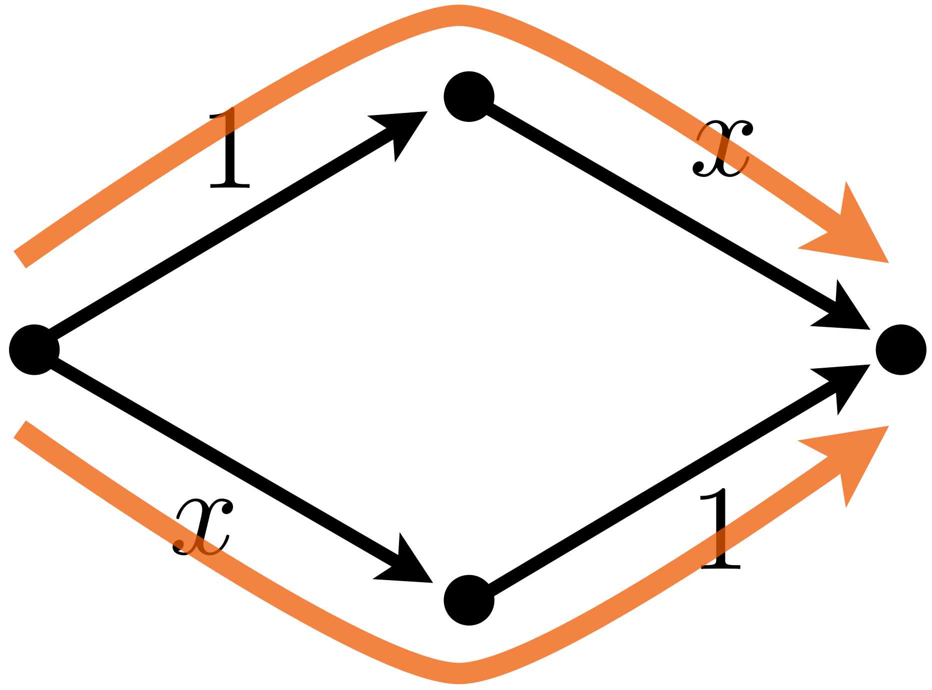


How bad is
selfish routing?

The selfish routing game

The game context:

- Directed graph
- Latency function on each edge specifying latency as function of total flow x on edge
- Path latency = sum of edge latencies



Flow x = 0.5 on each path;
Total latency = 1.5

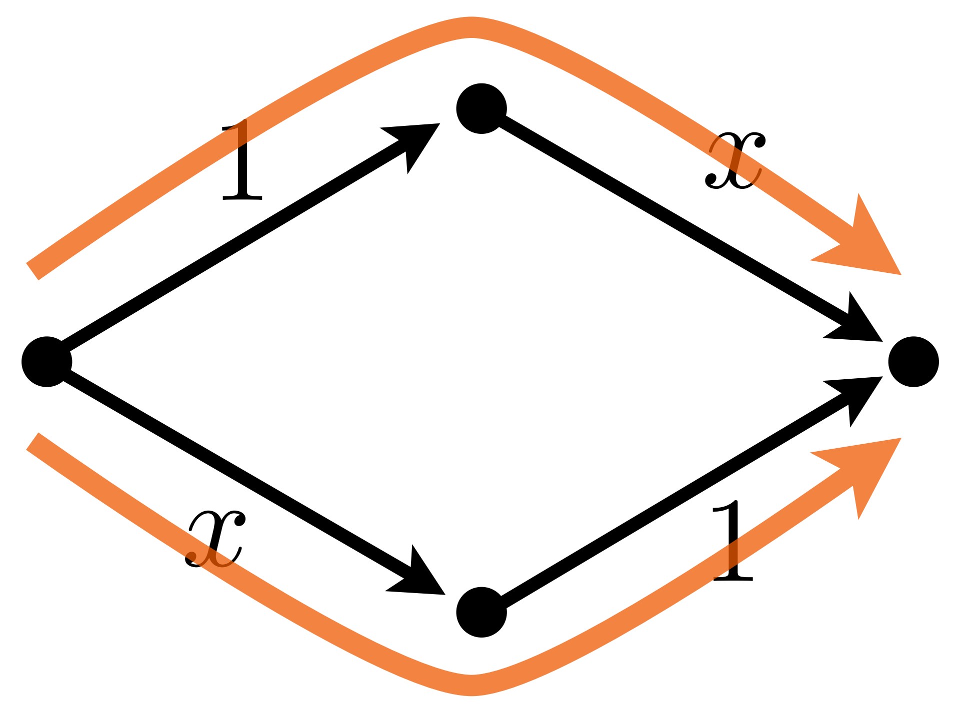
The selfish routing game

Player strategy:

- Pick a path on which to route
- Players selfishly pick paths with lowest latency (source-controlled routing)

For now assume:

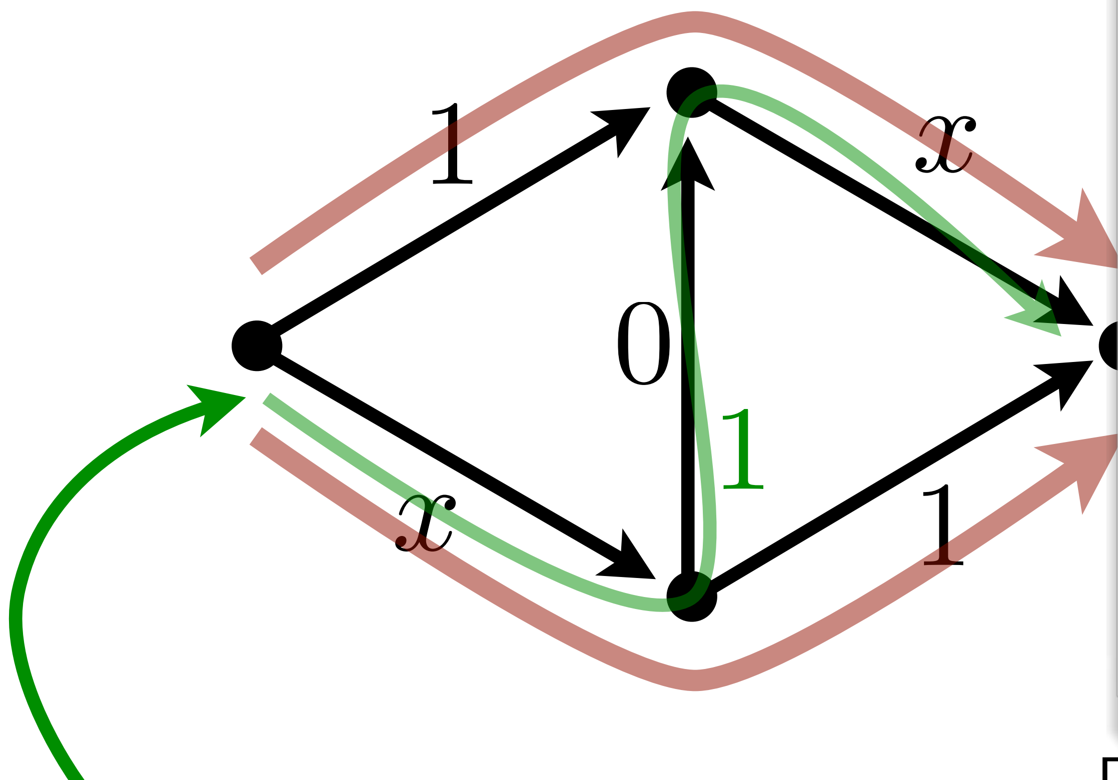
- many users
- each has negligible load
- total load = 1



Flow x = 0.5 on each path;
Total latency = 1.5

Example: Braess's paradox

[Dietrich Braess, 1968]



Green path is better.
Everyone switches to it!

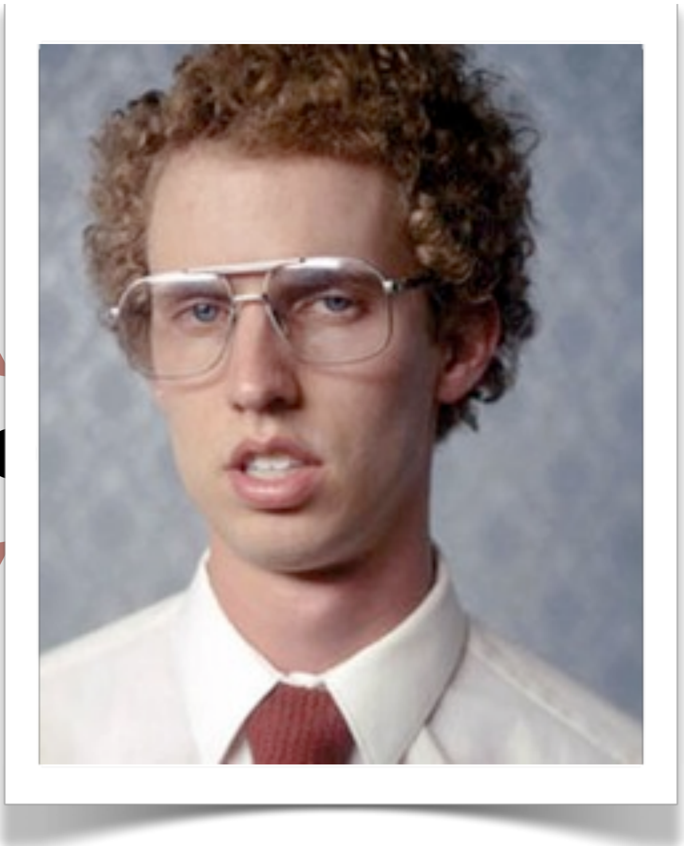


Fig 1b: N. Dynamite.

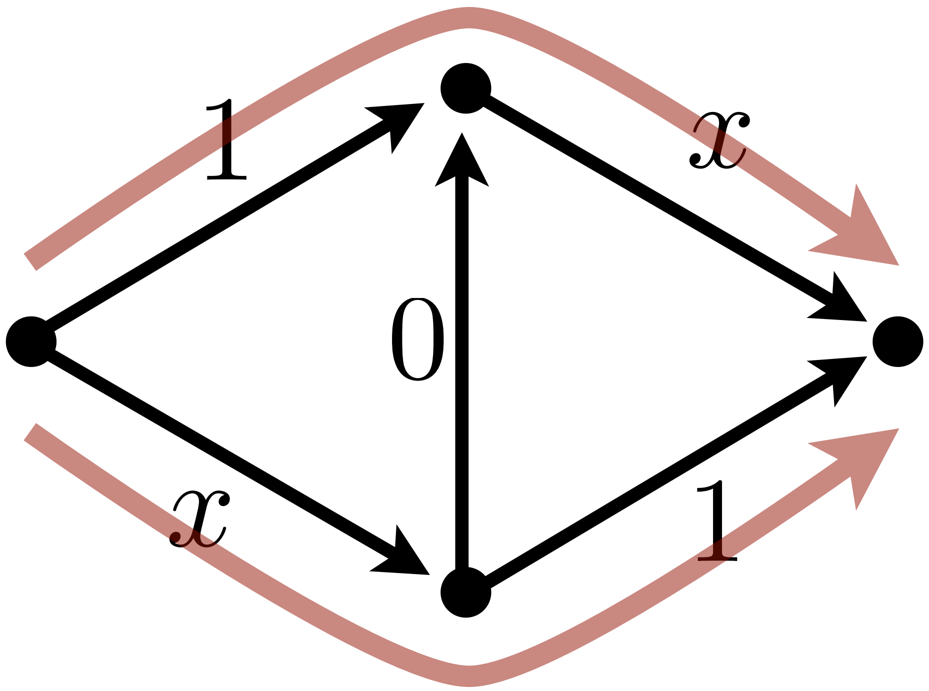
≈

Fig 1a: D. Braess.

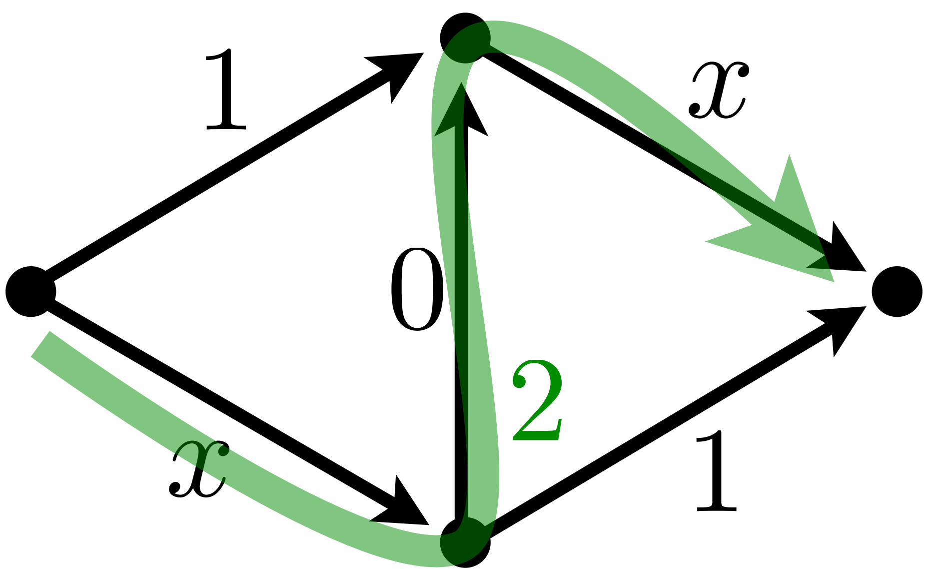
Initially: 0.5 flow along each path; latency 1+0.5 = 1.5

With new edge: all flow along greed path; latency = 2

Example: Braess's paradox



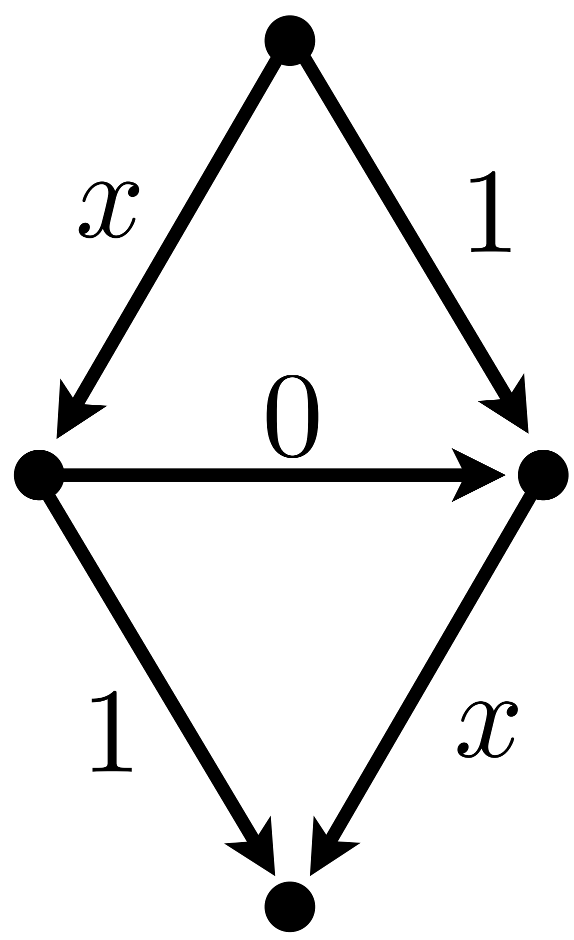
Optimal latency = 1.5

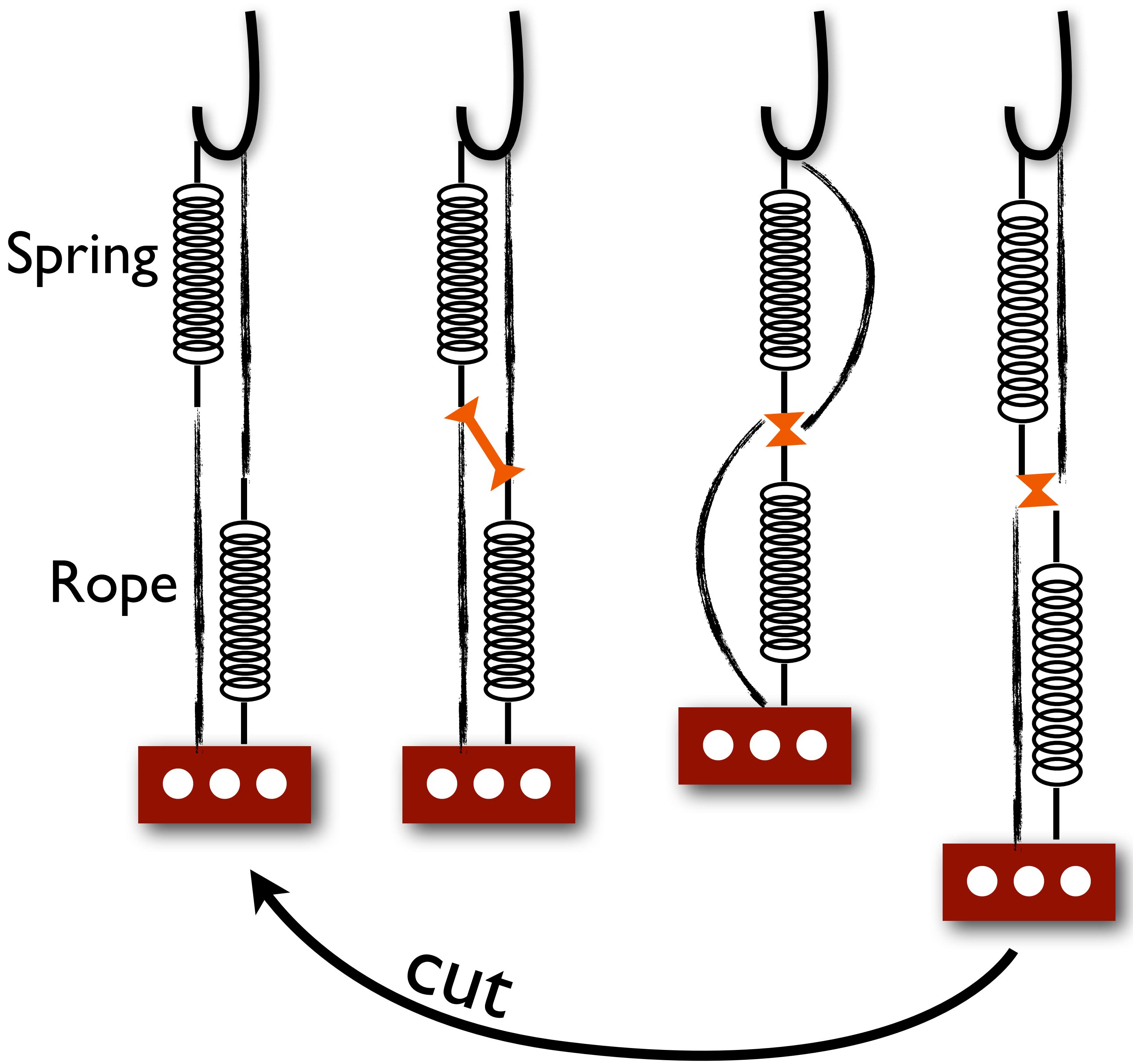


Nash equilibrium latency = 2

Thus, price of anarchy = 4/3

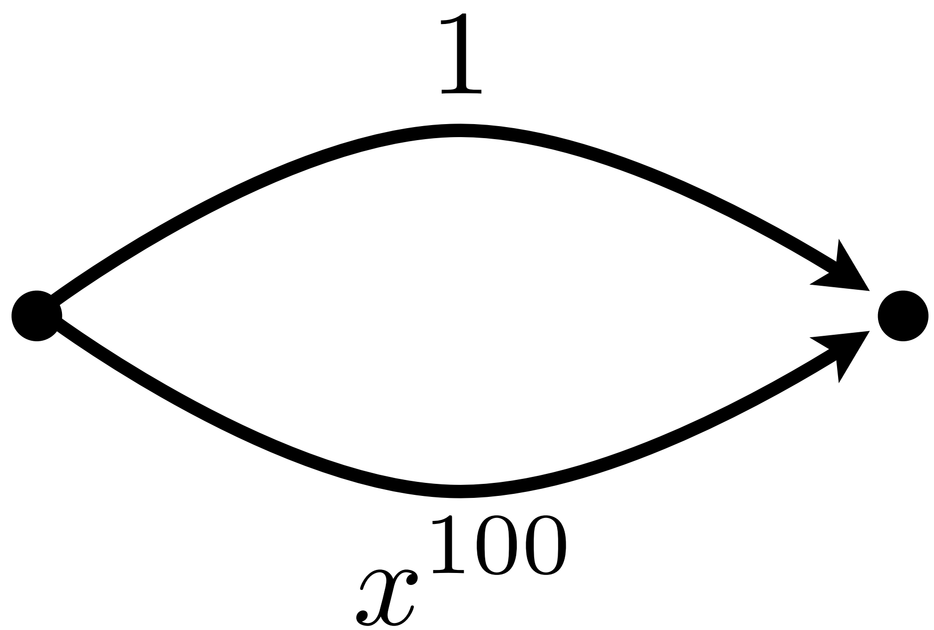
From links to springs



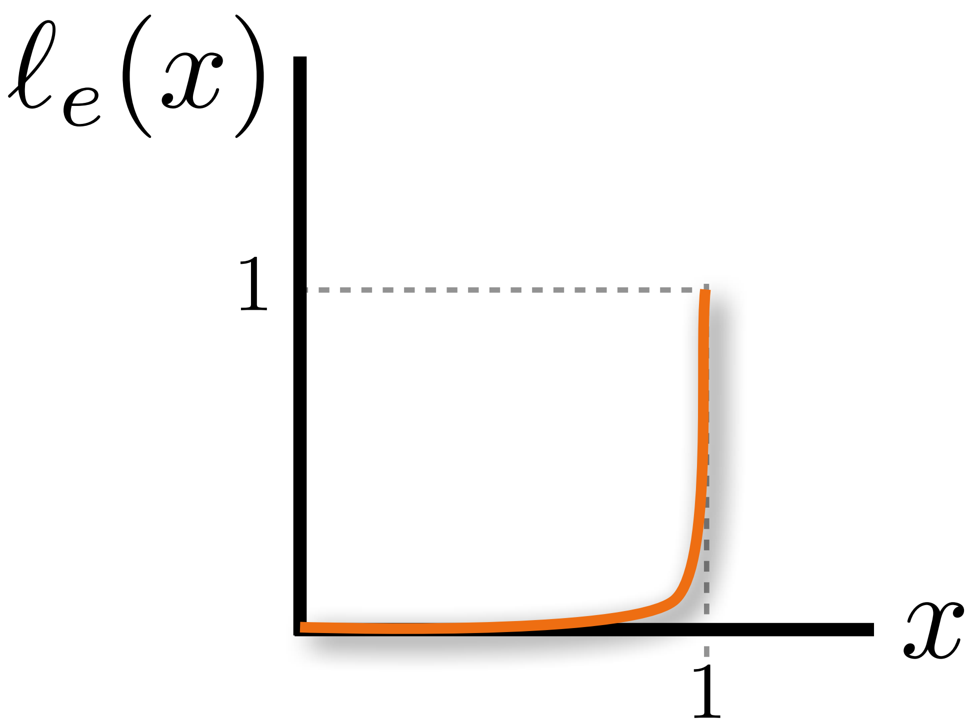


[Cohen and Horowitz, Nature 352, 699 - 701 (22 August 1991)]

Example: arbitrarily bad



Optimal: almost all flow on bottom; total latency near zero



Nash: all flow on bottom; total latency = 1

Roughgarden et al. results

As we just saw, price of anarchy can be arbitrarily high

But for linear latency functions: PoA $\leq$ 4/3

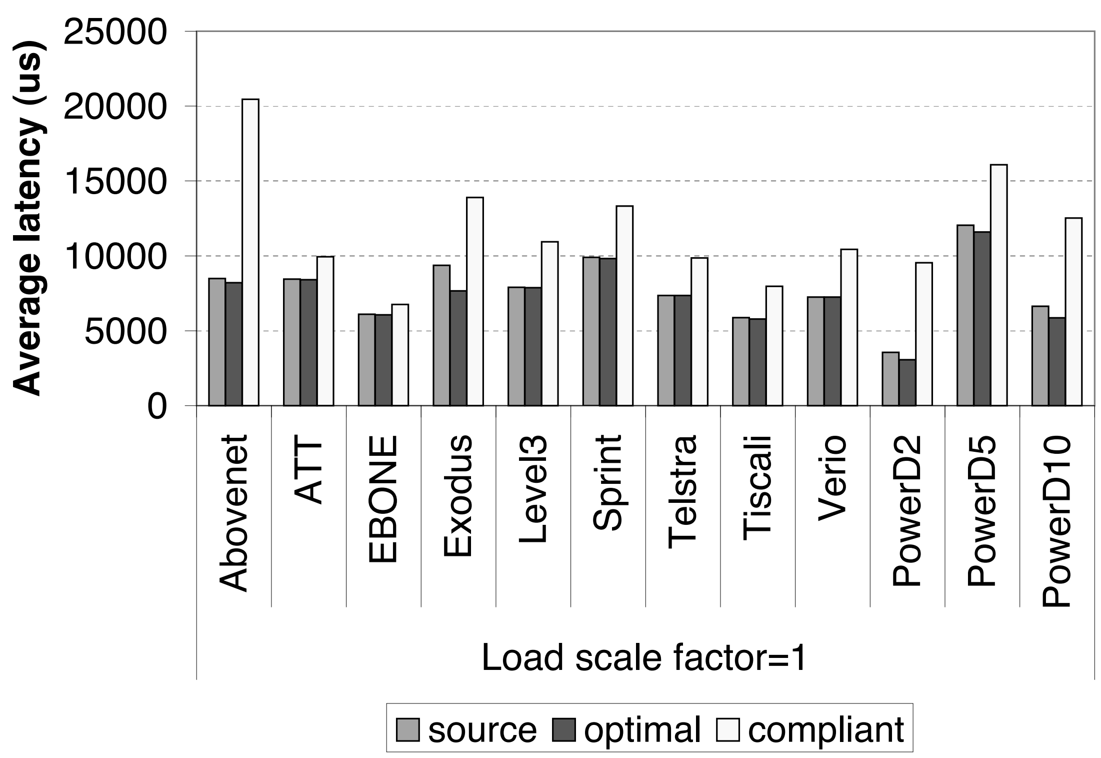
For any latency function: Nash cost is at most optimal cost of 2x as much flow

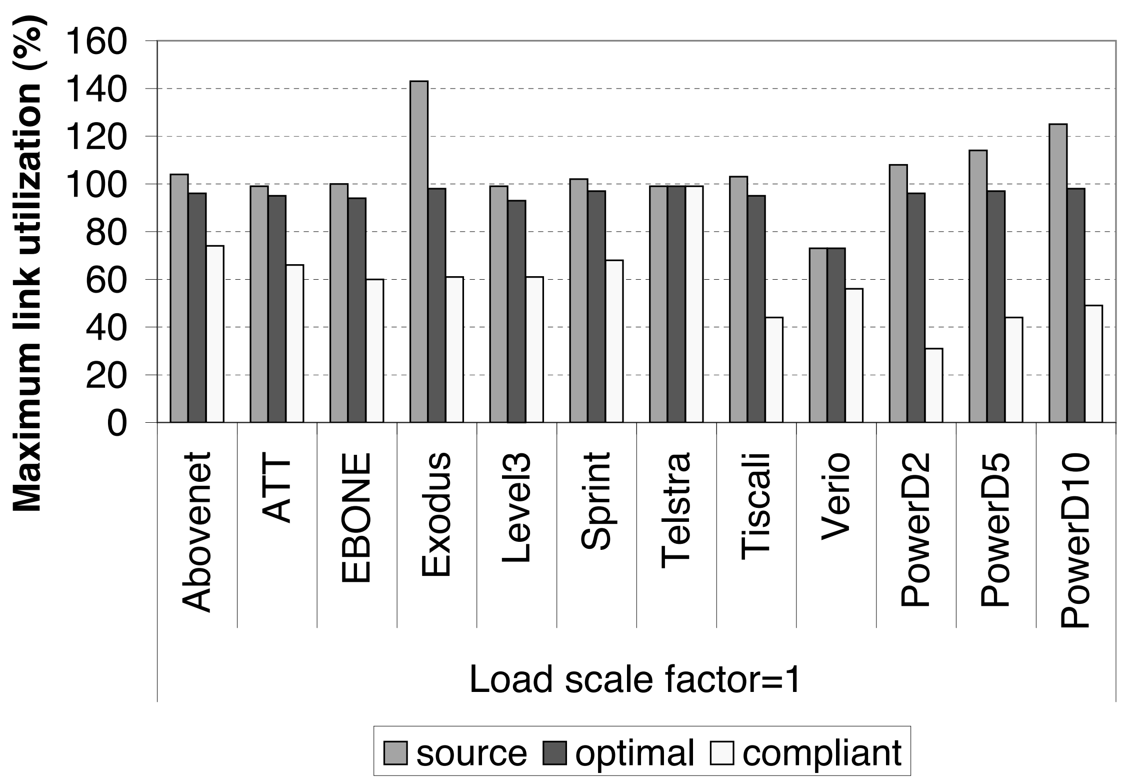
Extension to finitely many agents

- i.e., a single agent might have a nontrivial fraction of the total bandwidth
- Splittable flow: similar "2x" result
- Unsplittable flow: can be very bad

Selfish routing in realistic networks

[Qiu et al., SIGCOMM 2003]





Close to optimal latency

...but higher maximum link utilization

Discussion

Max utilization is higher in selfish. Does it matter? [Zhuotao]

Is average latency the right objective for the user? [Sangeetha, Siyu]

Discussion

How would the traffic engineering systems we learned about earlier interact with this framework? [Anthony]

- Suppose the network is running a near-optimal TE underneath selfish overlay routing. Would the overlay end up doing anything nontrivial?

Discussion

What would results be as a function of percentage of nodes being selfish? [Nirupam]

Beyond routing...

Game theory used in networking to model

- Equilibria of distributed algorithms
- ISPs competing with each other
- Spread of new technology in social networks
- ...

Many more applications of game theory to CS

- ...and applications of CS to game theory!
- See Nisan, Roughgarden, Tardos, Vazirani's book Algorithmic Game Theory, available free online

Announcements

Next Tuesday

- Survey of BGP security Issues (Butler, 2010)

Projects

- By now you should be moving along and have some initial progress
- Midterm presentations in less than 3 weeks
- Benchmark: demonstrate concrete progress