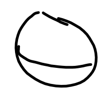
CS 519: Scientific Visualization　　　　　　　　　　　　Fall 2015
In-class Worksheet

Meshes

1. The Euler Formula

> The Euler Formula states the following relationship for the elements of a closed and connected surface mesh:
>
> $$V - E + F = 2(1 - G)$$
>
> Assume $G = 0$
>
> ***V*** is the number of vertices
> ***E*** is the number of edges
> ***F*** is the number of faces
> ***G*** is the genus of the surface (how holes/handles it has)



Show that for a triangle mesh with no holes we have $\boxed{F \approx 2V}$. Hint: each face has 3 edges and each edge is shared by 2 faces.

$$V - E + F = 2$$
$$V - \frac{3F}{2} + F = 2$$
$$V - F/2 = 2$$
$$V = F/2 + 2$$
$$2V = F + 4$$

$$2V \approx F$$

IFM { V 1.3 2.9 -5. ; v ; ~ ; f 1 5 6 }

2. Memory Requirements

Using the fact that F≈2V, compare the storage requirements for an indexed face mesh and a triangle soup. Assume the mesh has V vertices and a number requires 4 bytes of space. Derive functions for the number of bytes the mesh will require as a function of V.

△ soup → each F has 3 vertices w/3 coords
IFM → vertices w/3 coords + faces w/3 indices

Laplacian Smoothing

Can be viewed as an iterative averaging process using the following formulation:

$$\mathbf{p}_i \leftarrow \mathbf{p}_i + \lambda L(\mathbf{p}_i)$$

$$L(p_i) = \sum_{n_j} \frac{1}{w_j}(n_j - p_i) \text{ and } \lambda \text{ is in } [0,1]$$

with n_j being the neighboring vertices of p_i and w_j a weight

3. **Laplacian Smoothing**

   Consider a linear curve of three vertices: (4,2) to (12, 2) to (16, 2)

   a. Assume the endpoints always stay fixed. What is the position of the middle vertex after 2 iterations of Laplacian smoothing using uniform weights and $\lambda = 1/2$?

   b. If you iterate until convergence, what final position will the middle vertex be in?

   c. What weights in the smoothing formula would result in the middle vertex never moving?

4. **Mesh Simplification**

   Simplify the triangle mesh below using the grid to perform vertex clustering. Use cell centers for the vertex placement