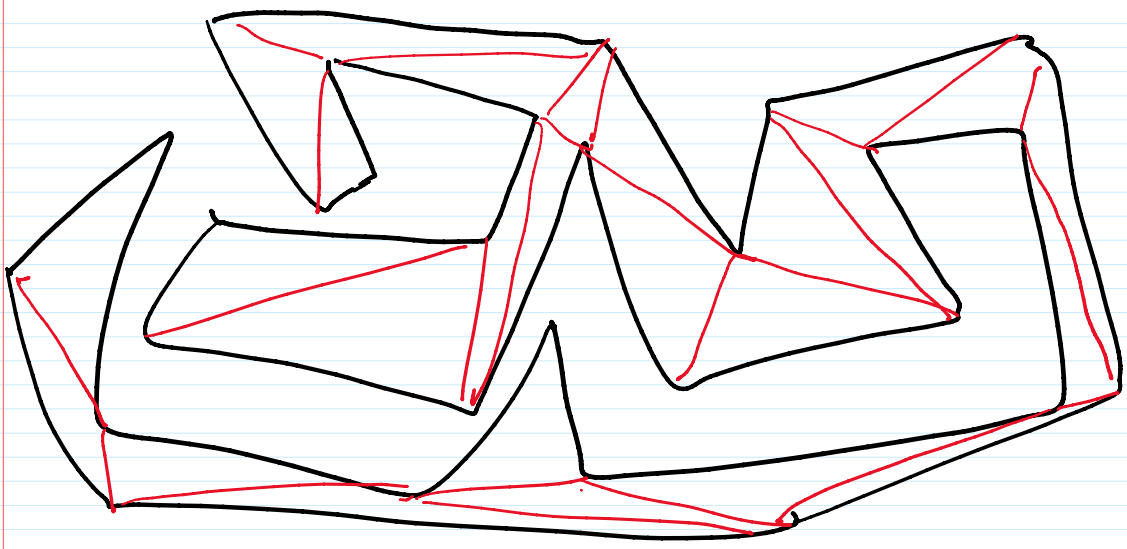
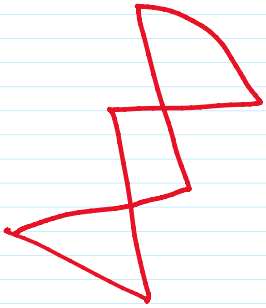
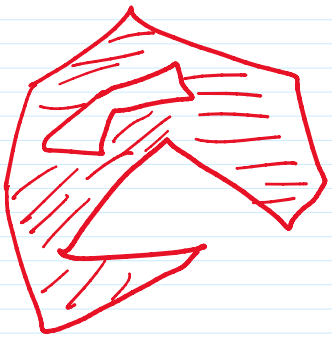
Polygon Triangulation

#triangles = n-2

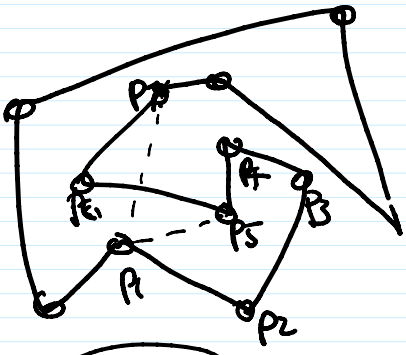


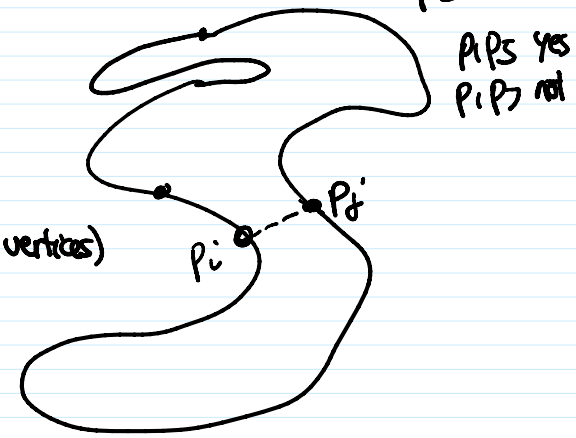
Problem Given a simple polygon P with n vertices, partition P into triangles, without using new vertices.





holes



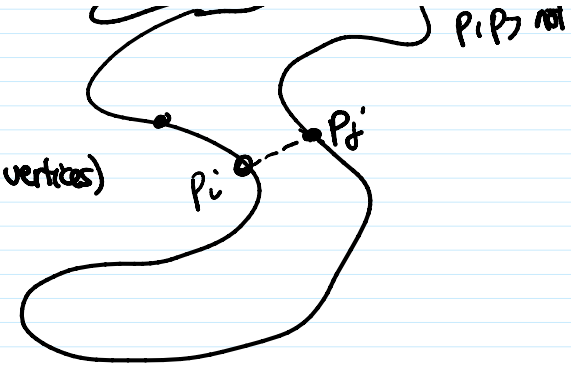


Earliest? Proof (Lennes 1911)

1. find a line seg P_iP_j ← diagonal (P_i, P_j vertices) not intersecting P
2. recurse on two sides

Earliest? Proof (Lemes 1911)

1. find a line seg $p_i p_j$ (diagonal) (p_i, p_j vertices) not intersecting P
2. recurse on two sides



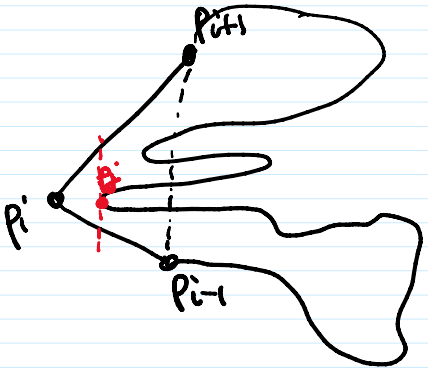
Proof that a diagonal exists:

$O(1)$ → 1.1. let p_i be leftmost vertex

$O(n)$ → 1.2. if $p_{i-1} p_{i+1}$ is a diagonal, return $p_{i-1} p_{i+1}$

$O(n)$ → 1.3. let p_j be leftmost vertex inside $\triangle p_{i-1} p_i p_{i+1}$

1.4. return $p_i p_j$.



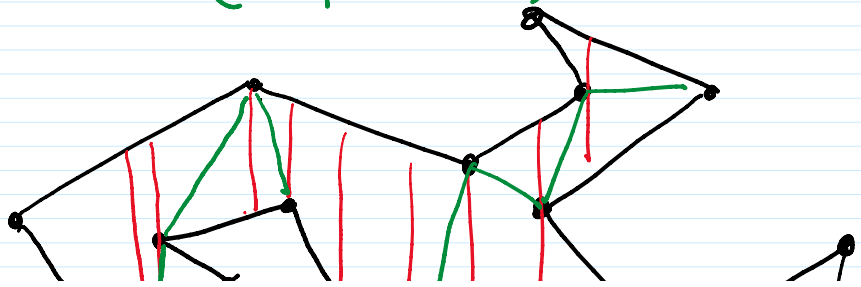
$$T(n) \leq T(n_1) + T(n_2) + O(n)$$
for some $n_1 + n_2 = n + 2$

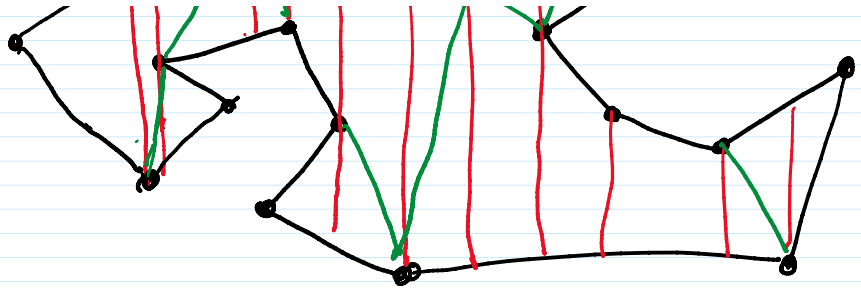
$$\Rightarrow \boxed{O(n^2)} \text{ time}$$

Chazelle '82: find a diagonal (in $O(n)$ time) with $n_1, n_2 \leq \frac{2}{3}n$
$\Rightarrow O(n \log n)$

Method 2: Reduce to Trapezoidal Decomposition

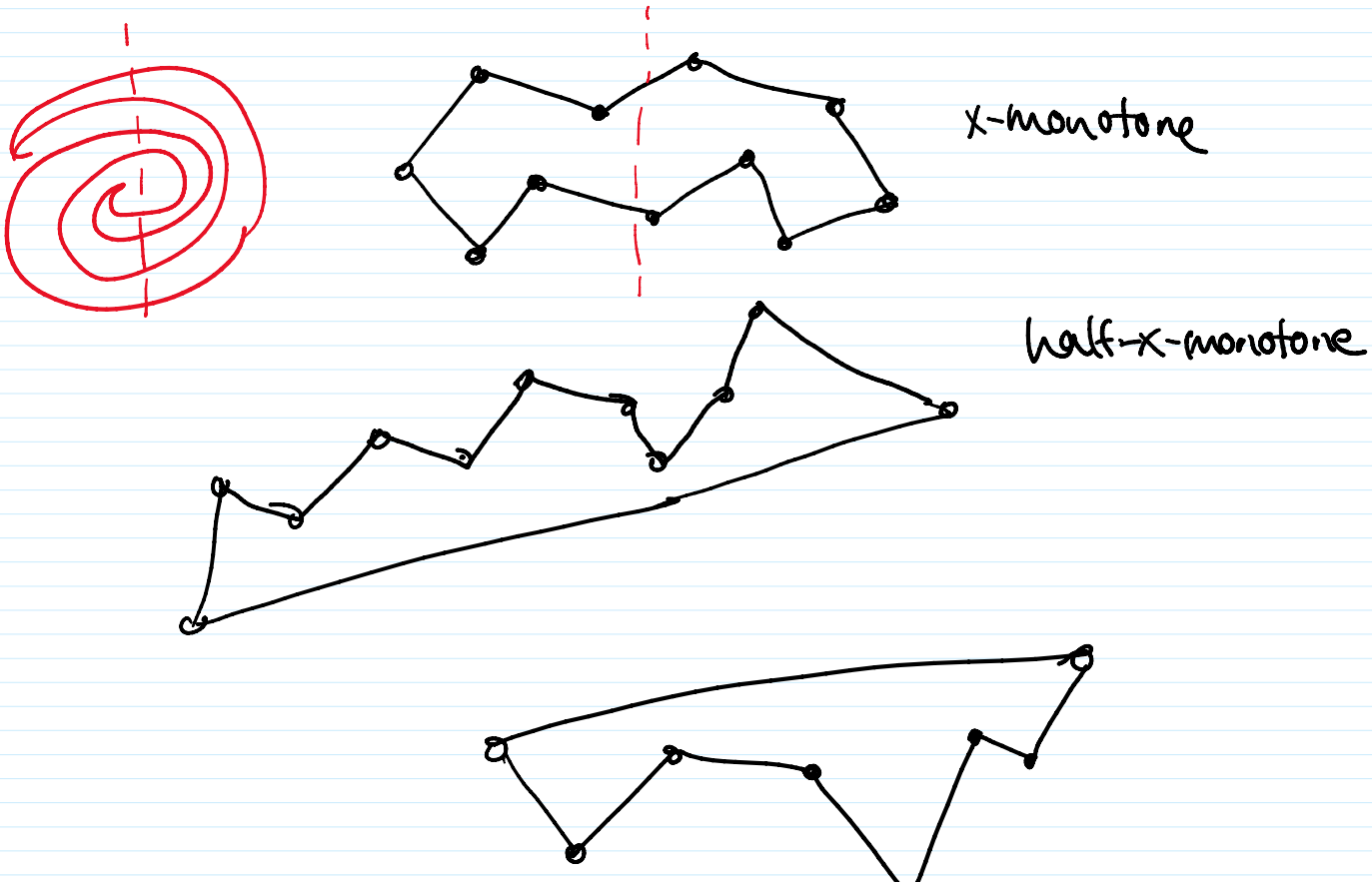
(Garey et al. 1978)

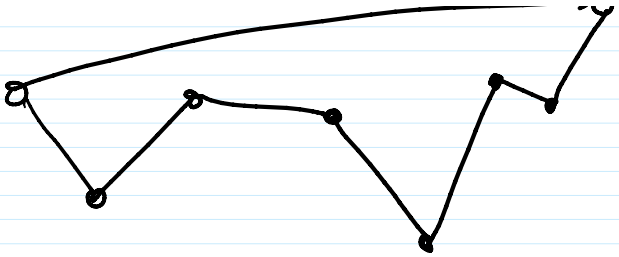


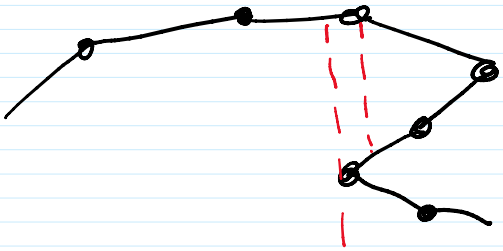


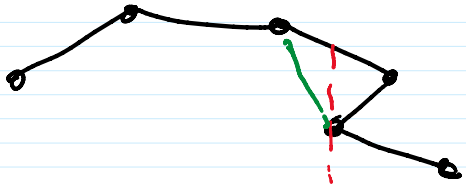
1. compute TD within P in $O(n \log n)$ time
   (Bentley-Ottmann sweep or rand incremental)
2. for each trapezoid τ
   if τ contains 2 nonadj vertices
   draw diagonal
3. triangulate each subpolygon

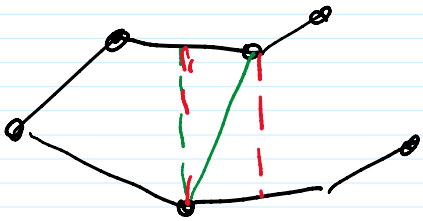
Obs A Each subpolygon is a half-x-monotone polygon



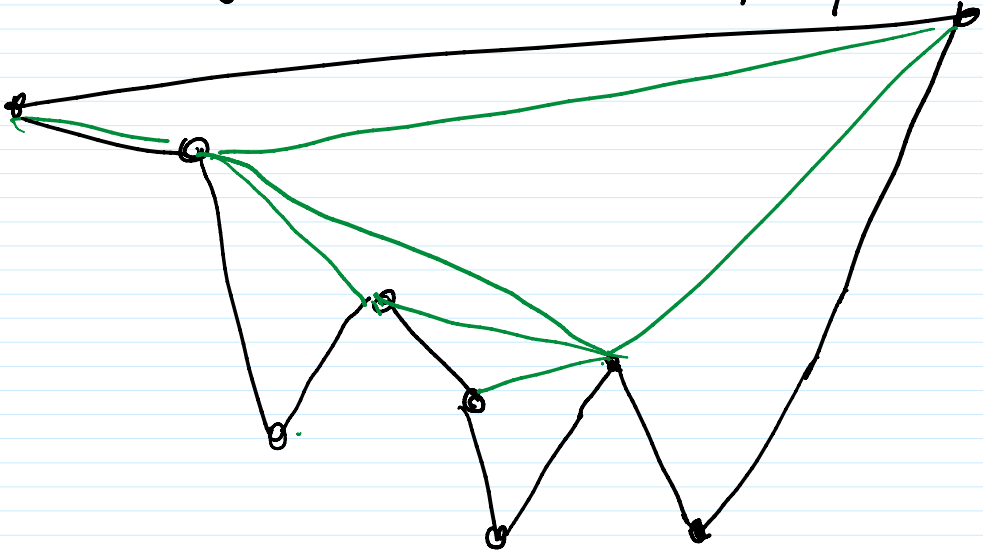








Obs B Each half-x-monotone polygon can be triangulated in linear time, by Graham's scan!!



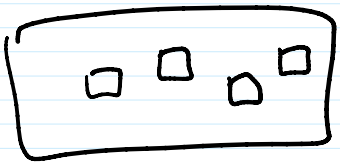
Analysis:

line 1	$O(n \log n)$ ←
line 2	$O(n)$
line 3	$O(n)$

$\Rightarrow$ $O(n \log n)$ time

Question: better??

- no for TD of arbitrary disj line segs.
- no for trians of polygons with holes

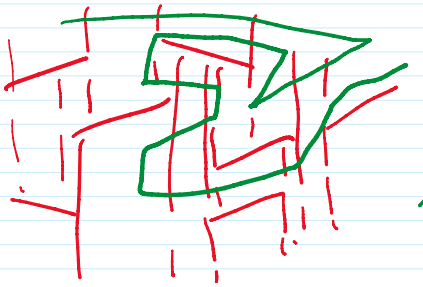


for simple polygon??

Analysis: yes!

Tarjan, van Wyk '88	$O(n \log\log n)$
Clarkson et al. '89, Seidel '91	$O(n \log^* n)$ rand.
Chazelle '91	$O(n)$ det.

($n \log\log n$; $\frac{n}{\log n}$



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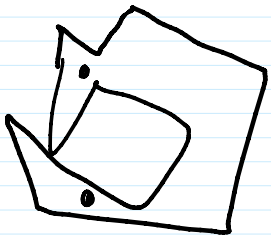
Appl'ns – check whether a polygon is simple (no self-intersect)

– do 2 simple polygons intersect? $O(n)$ time

$n+h\log n$ — pt loc in simple polygon
w. $O(n)$ preproc time

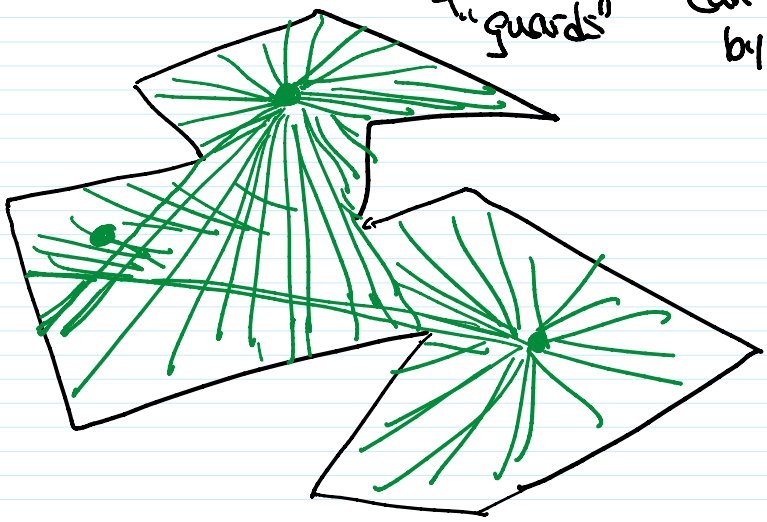


ray shooting
shortest path
⋮

$n+k$

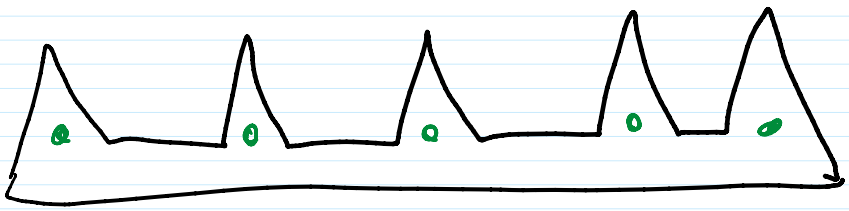
A Visibility Appl'n

The Art Gallery Problem Given simple polygon P,
find smallest # of pts s.t. every pt inside P can be seen by some "guard".



note- NP-hard!
- combinatorial worst-case bound?

bad ex:



Chvatal's comb

guards = $\lfloor n/3 \rfloor$ for this

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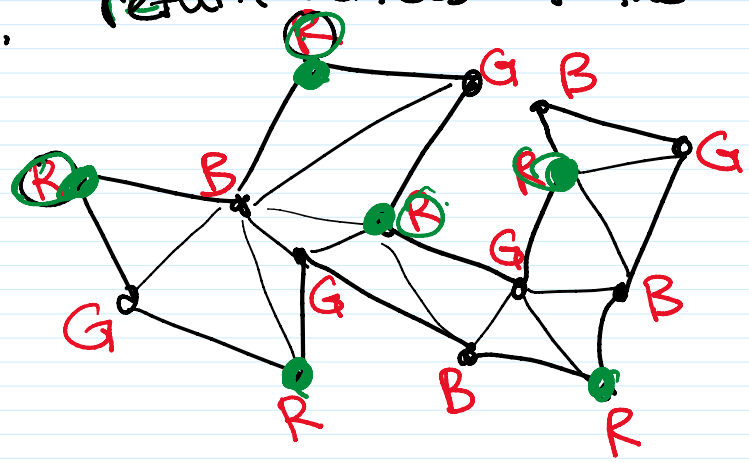
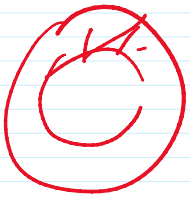
Chvatal's Thm ('75) for every simple polygon,
guards $\leq \lfloor n/3 \rfloor$.

Fisk's Pf ('78):

1. compute a triangulation T
2. compute a 3-coloring of T
3. return vertices of the least popular color
$\leq \left(\frac{n}{3}\right)$.





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