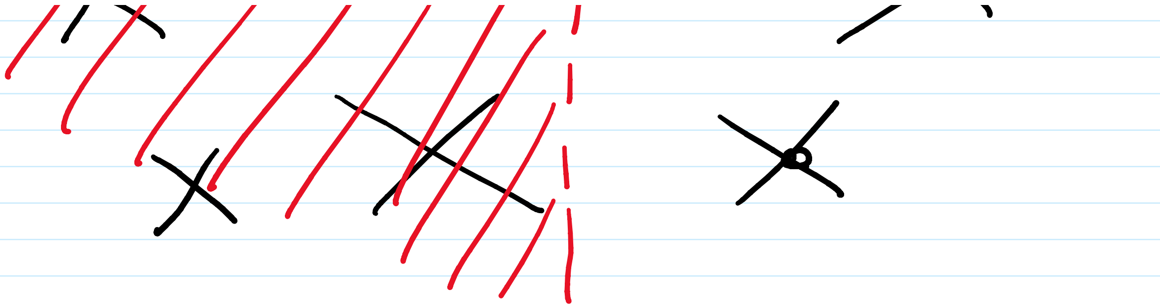
m=0



Megiddo '84 $O\left(2^{O(2^d)} n\right)$

Dyer $O\left(2^{O(d^2)} n\right)$

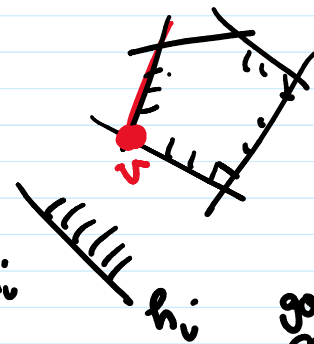
current best deterministic $O\left(2^{O(d \log d)} n\right)$

## Seidel's Randomized Incremental Alg'm in 2D ('90)

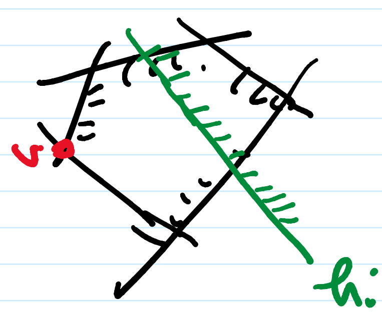
idea - add halfspaces $h_1, \ldots, h_n$ one at a time (in rand order)
& maintain current sol'n $v$

if $v \in h_i$, no change

else need to recompute sol'n!
but new $v$ must on $\partial h_i$




good case




bad case

$LP_{\cancel{2}\,d}(H)$:

0. let $h_1, \ldots, h_n$ be a random order of H
1. $v$ = a pt at $\infty$

VERY

0. let $h_1, \ldots, h_n$ be ...

VERY SIMPLE!

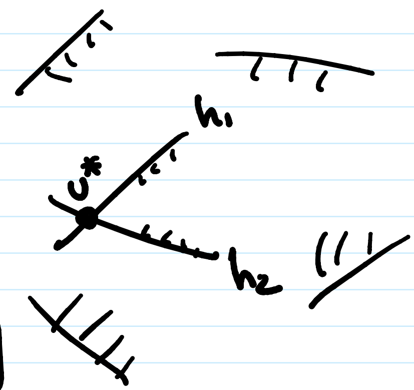
1. $v =$ a pt at $\infty$
2. for $i = 1, \ldots, n$ do
3. if $v \notin h_i$ then
4. $v = LP_{\cancel{d}\; d-1}(\{h_1 \cap \partial h_i, \ldots, h_{i-1} \cap \partial h_i\})$
5. return $v$

## Worst-case analysis:

line 4 $O(n)$ $\Rightarrow$ total time $O(n^2)$ in 2D
(or $O(n^d)$ in $\mathbb{R}^d$).

## Rand analysis: ("backwards" analysis)

look at $n$-th iteration
let $v^*$ be opt final soln
defined by $h_1$ and $h_2$




$$Pr[\text{bad case}]$$
$$= Pr[\underset{\substack{\uparrow \\ \text{last iter}}}{h_i} = h_1 \text{ or } h_i = h_2]$$
$$= \frac{2}{n}.$$

expected time

$$T_2(n) = \overbrace{\frac{2}{\cancel{n}} \cdot O(\cancel{n})}^{O(1)} + \left(1 - \frac{2}{n}\right) \cdot O(1) + T_2(n-1)$$

$$\Rightarrow T_2(n) = O(1) + T_2(n-1)$$
$$\Rightarrow \boxed{O(n)}$$

$$\Rightarrow \boxed{O(n)}$$

$$T_d(n) = \frac{d}{n} \cdot T_{d-1}(n) + O(1) + T_d(n-1)$$

$$T_3(n) = O(\cancel{\tfrac{3}{n}} \cdot \cancel{T_2(n)}) + T_3(n-1) \Rightarrow T_3(n) = O(n)$$

$$T_4(n) = O(\cancel{\tfrac{4}{n}} \cdot T_3\cancel{(n)}) + T_4(n-1) \Rightarrow T_4(n) = O(n)$$

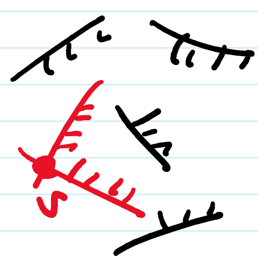
$$\vdots$$

$$T_d(n) = \boxed{O(d!\, n)} \quad \text{expected time}$$

## Clarkson's Random Sampling Alg'm ('88)

LP(H):

1. choose a random subset $R \subseteq H$ of size $r = \sqrt{n}$
2. repeat $d+1$ times {
3. $\quad v = LP(R)$
4. $\quad R = R \cup \{h \in H : v \notin h\}$



v violates h

Lemma $\Rightarrow$ $\leq \frac{cdn}{r} \log n$

VERY SIMPLE!

}

return $v$

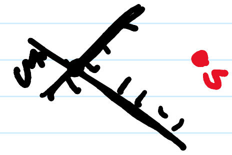
**Correctness: (of $d+1$)**

let $v^*$ be opt sol'n, defined by $d$ halfspaces $B^*$.

at each iteration,

if $v \in h \;\; \forall h \in B^*$,

then $v = v^*$



else some halfspace of $B^*$ violates $v$

& will be added to $R$.

So $d+1$ iterations $\Rightarrow$ $v = v^*$.

Expected time analysis:

"ε-Net" Lemma

Let $R$ be a rand subset of $H$ of size $r$.

If $v$ violates $\geq \underbrace{cd\frac{n}{r}\log n}$ halfspaces in $H$,

then $v$ violates $\geq 1$ halfspace in $R$.

w.h.p.

Pf: Fix $v$ that violates $\geq cd\frac{n}{r}\log n$ halfspaces in $H$.

$$\Pr[\,R \text{ does not include any of violating halfspaces}\,]$$

$$\lesssim \left(1 - \frac{cd\frac{n}{r}\log n}{n}\right)^{r}$$

$1 - x \leq e^{-x}$

$$\lesssim \left(e^{-\frac{cd\frac{\cancel{n}}{r}\log n}{\cancel{n}}}\right)^{\cancel{r}}$$

$$\approx \frac{1}{n^{cd}}$$

$\Pr(A \cup B) \leq \Pr(A) + \Pr(B)$

$$\Pr(\text{err}) \leq n^{d} \cdot \frac{1}{n^{cd}} = \frac{1}{n^{(c-1)d}}.$$

□

$$\Rightarrow \quad T(n) = \underline{(d+1)}\, T\left( O(\sqrt{n}\, \cancel{\log} n)\right) + O(\underline{d^2} n)$$

$$\Rightarrow \quad T(n) = \boxed{O(n)}$$ for any const d

$$O\left( d^2 n + \underline{(d \log n)}^{O(d)} \right)$$