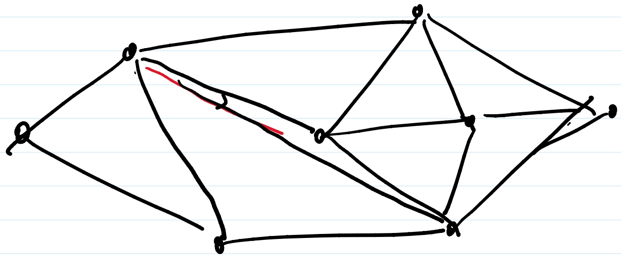
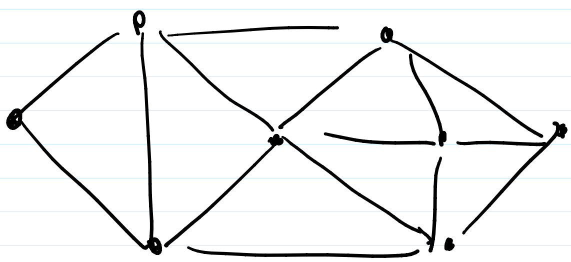
Prob 4 "Best" triangulation of a point set

one way to define "best"

- maximize smallest angle
- in case of tie, maximize next smallest angle, etc.

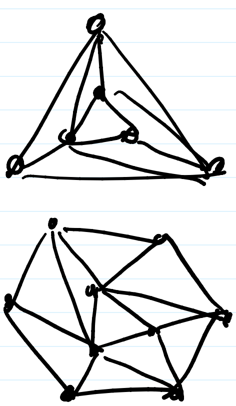
"lexicographic"

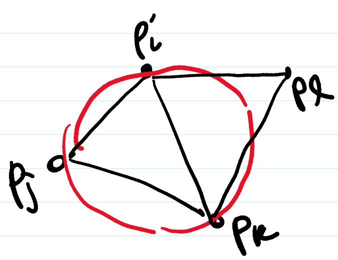


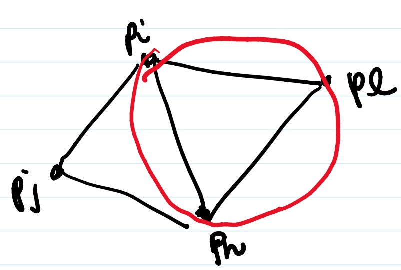


motivation - piecewise-linear interpolation
finite-element methods
⋮

Def A triangulation T is <u>locally Delaunay</u>
iff $\forall$ pair of adj triangles $\Delta p_i p_j p_k$, $\Delta p_i p_j p_l$, in T
p_l is outside circumcircle thru p_i, p_j, p_k



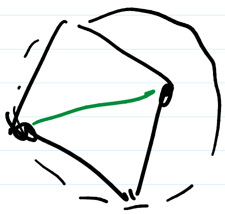


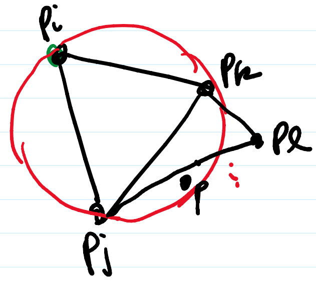


Fact T is locally Delaunay $\Leftrightarrow$ T is the DT.

Pf: ($\Leftarrow$) obvious

($\Rightarrow$)





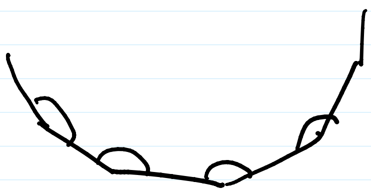
by induction...

another way:

lift to 3D   lower halls

locally convex $\to$ convex □

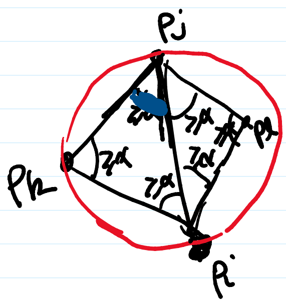
intuitively



Thm The "best" triangulation T is the DT.

Pf: Suffice to show best triang T is locally Delaunay,

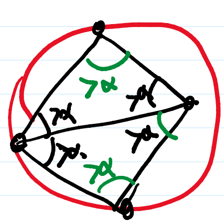
Assume T is not.

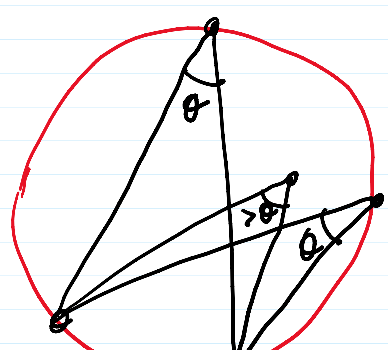


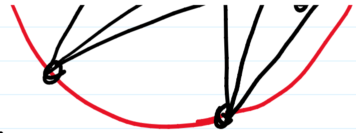
let α be smallest of these 6 angles

$\Downarrow$

flip edge $p_ip_j \to p_kp_l$ to get T'.







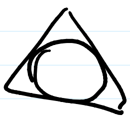
T' is better : contra! $\square$

<u>Cor of Pf</u>: any triangulation can be transformed into another using only edge flips

...

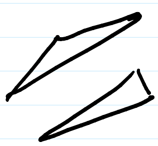
Rmk other criterion:

min largest circumradius $\Rightarrow$ DT



max total inradii $\Rightarrow$ DT

⋮

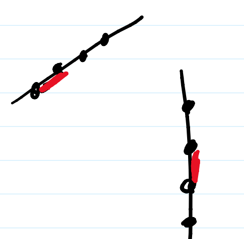


min largest angle $\Rightarrow$ not DT but poly time

min total length $\Rightarrow$ not DT, NP-hard (Mulzer, Rote '06)

Extensions of VD/DT

- 3D VD/DT
  lifts to 4D CH
  worst-case size $\Omega(n^2)$

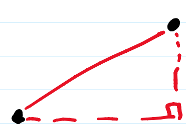


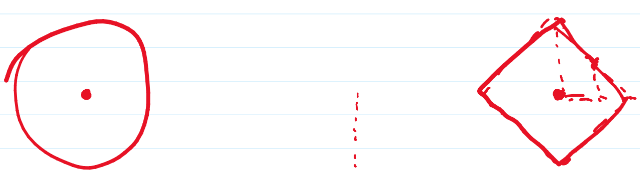
- different distance fns e.g.

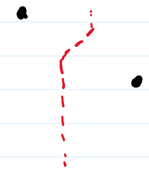
$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

rectilinear metric (Manhattan) (L_1)

$|x_1-x_2|+|y_1-y_2|$

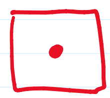






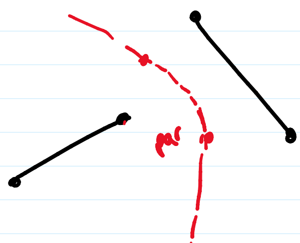
L_p $\quad \left(|x_1-x_2|^p + |y_1-y_2|^p\right)^{1/p}$ $\quad p \geq 1$

L_∞ $\quad \max\{|x_1-x_2|, |y_1-y_2|\}$



weighted.

- different kinds of sites e.g.
  line segments



- "constrained Delaunay triang".

⋮

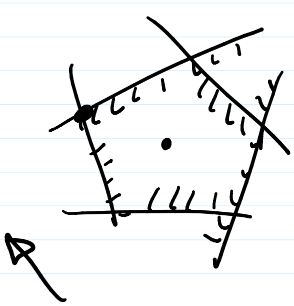
Linear Programming (LP)

optimization problem involving linear constraints

$$\begin{aligned} \text{min/max} \quad & c_1x_1 + c_2x_2 + \dots + c_dx_d \\ \text{s.t.} \quad & a_{11}x_1 + a_{12}x_2 + \dots + a_{1d}x_d \leq b_1 \\ & \vdots \\ & a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nd}x_d \leq b_n \end{aligned}$$

over vars $x_1, \dots, x_d \in \mathbb{R}$.

geometric interpretation:



intersection of halfspaces