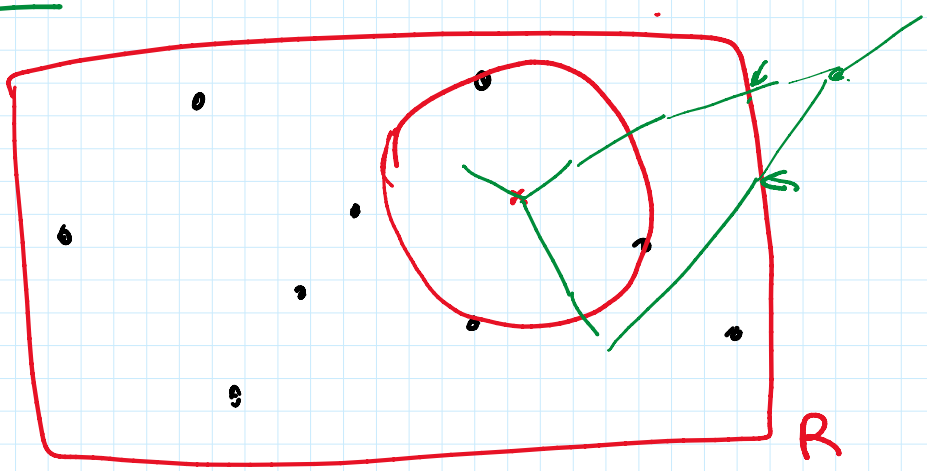
October 3, 2024 3:15 PM

Applications of Voronoi Diagrams / Delaunay Triangulations

Prob 2 Largest empty circle



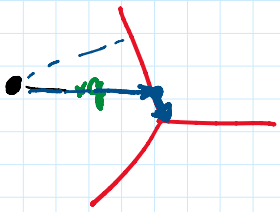
Obs optimal center is either

(i) a Vor vertex or

(ii) intersection of a Vor edge with an edge of R or

(iii) vertex of R.

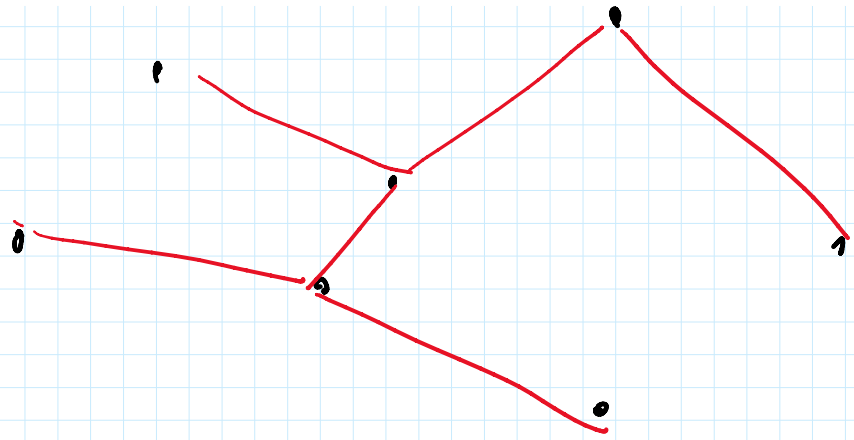
Pf: sketch to maximize $f(q)$ = dist from q to nearest site over $q \in R$

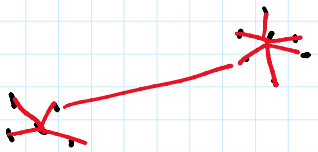


$\Rightarrow$ $O(n \log n)$ + $O(n)$ = $\boxed{O(n \log n)}$ time

(for VD)

Prob 3 Euclidean Min Spanning Tree (EMST)





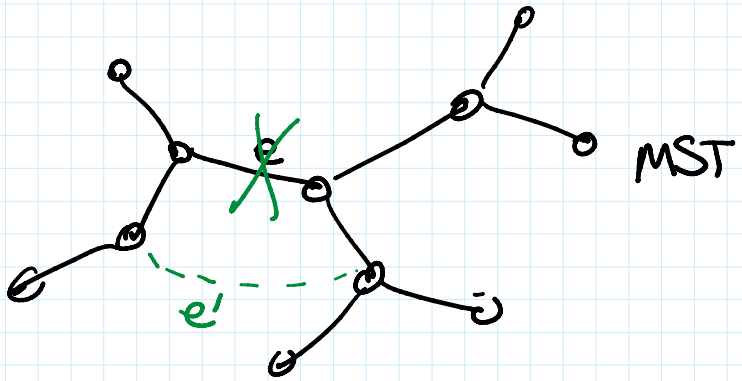
MST for graphs:

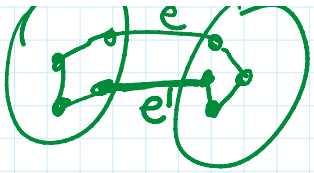
- Kruskal $O(m \log n)$
- Prim $O(n^2)$, $O(m \log n)$, $O(n \log n + m)$.
- → Boruvka $O(m \log n)$
  Chazelle '97 $O(m \alpha(n))$
- → Karger, Klein, Tarjan '94 $O(m)$ rand.

EMST: build complete graph & run Prim's ⇒ $O(n^2)$ time

Fact for any cycle C, the heaviest edge e in C can't be in the MST.

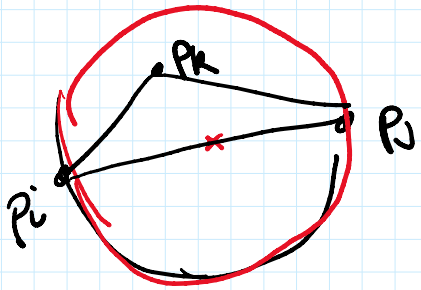
Pf sketch:





Thm $EMST \subseteq DT$.

Pf: if $p_i p_j$ is in EMST,



draw circle thru p_i, p_j with center at midpt

this circle is empty

$d(p_i, p_k) < d(p_i, p_j)$

$d(p_k, p_j) < d(p_i, p_j)$.

by Fact $\Rightarrow$ $p_i p_j$ can't be in MST of complete graph. □

Algm: build DT + run Prim/Kruskal in DT ($m = O(n)$)

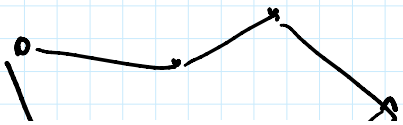
$O(n \log n)$ $O(n \log n)$

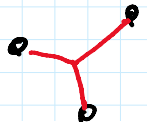
$= \boxed{O(n \log n)}$

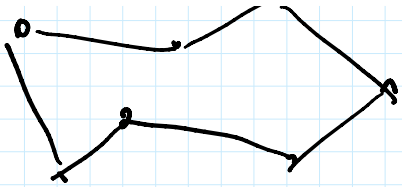
3D: $O(n^{4/3})$

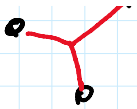
Rmk: approx algms for related NP-hard problems:

TSP min Steiner tree









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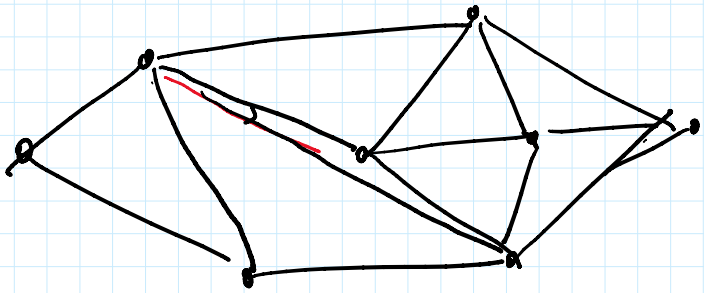
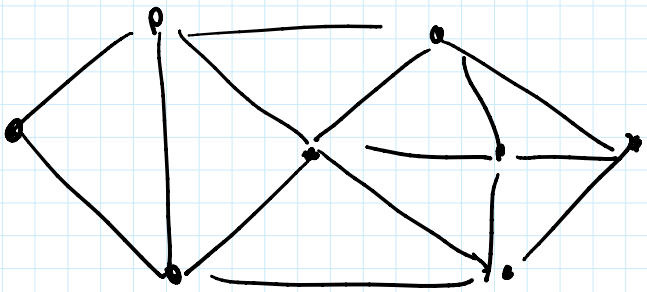
Prob 4 "Best" triangulation of a point set

One way to define "best"

- maximize smallest angle
- in case of tie, maximize next smallest angle, etc.





motivation - Piecewise-linear interpolation
finite-element methods

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