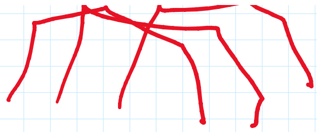
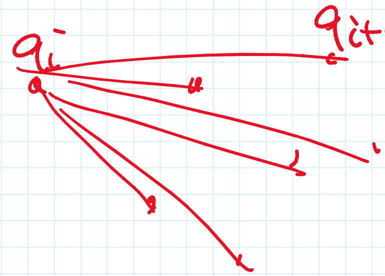
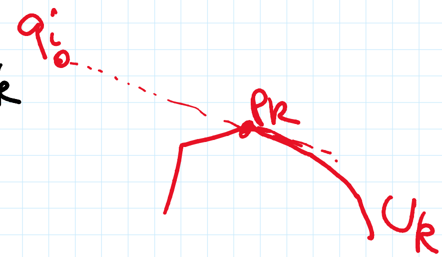
$O(\frac{n}{h} \cdot h \log h)$
$= O(n \log h)$





3. q_1 = leftmost pt
4. for $i = 1$ to h do {
5. if q_i = rightmost pt return $\langle q_1, \ldots, q_i \rangle$
6. q_{i+1} = any pt right of q_i
7. for $k = 1$ to n/h do {

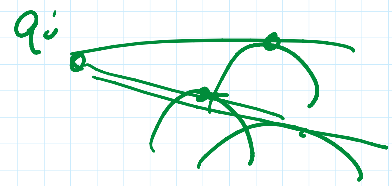
$O(\log h) \rightarrow$ p_k = tangent pt between q_i and U_k



if p_k above $\overleftrightarrow{q_i q_{i+1}}$
$q_{i+1} = p_k$

}

$O(\frac{n}{h} \log h)$



$O(h \cdot \frac{n}{h} \log h) = O(n \log h)$

Total: $\boxed{O(n \log h)}$

issue - don't know h

idea - guess h!

try $h = 2^{2^1}, 2^{2^2}, 2^{2^3}, 2^{2^4}, \ldots$

total time $O\left(\sum_{i=1}^{\lceil \log \log h \rceil} n \log 2^{2^i}\right)$

$= O\left(\sum^{\lceil \log \log h \rceil} n 2^i\right)$

$$= O\left(\sum_{i=1}^{\lceil \log\log h \rceil} n 2^i \right)$$

$$= O\left(n 2^{\lceil \log\log h \rceil} \right)$$

$$\leq O\left(n 2^{\log\log h + 1} \right)$$

$$= \boxed{O(n \log h)}$$

Final Remarks: $\Omega(n \log h)$ lower bd (by Ben-Or's technique).

Afshani, Barbay, C. '09: "instance-optimal" alg'm !!

Alg'm 4 is → "instance-optimal"

C.-Lee '14: # comps $1 \cdot n \log_2 h + O(n\sqrt{\log h})$.

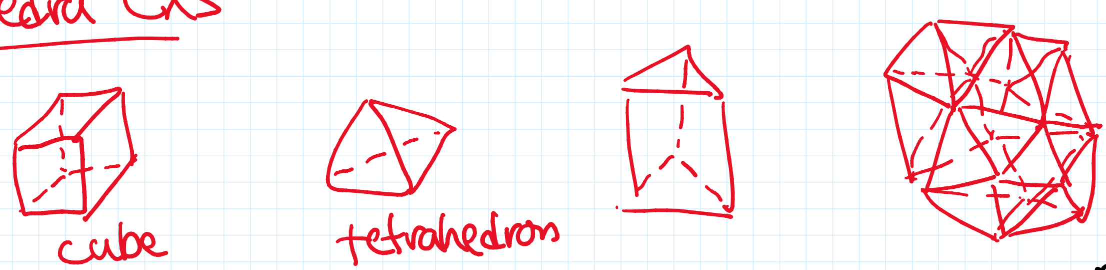
Convex Hull in 3D

Given $P = \{p_1, \ldots, p_n\}$, $p_i = (x_i, y_i, z_i)$,

construct $CH(P)$ = smallest convex set containing P

convex polyhedron

Polyhedra Exs



cube tetrahedron

- boundary consists of vertices, edges, & faces (facets) ← convex polygons

  (2 edges meet at a vertex
  2 faces meet at an edge)

- convexity ⇒ object is topologically nice ...
  (connected boundary, no holes, ...)

- for non-degenerate points, CH is <u>simplicial</u>
  i.e. all faces are triangles

  ↑ no 4 points co-planar
  no 3 points collinear

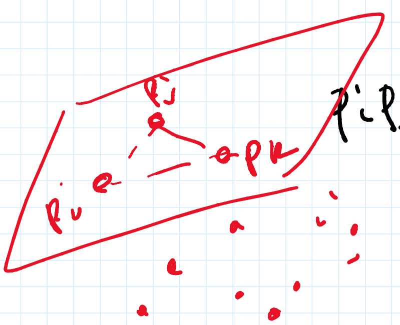
Geometric Properties

Like before:

p_i is a vertex of CH iff $\exists$ plane thru p_i s.t. all pts lie on one side

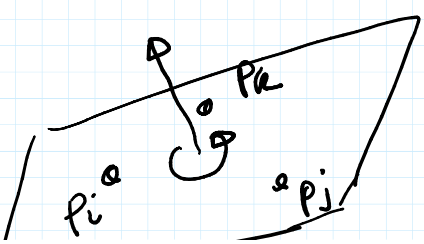
p_ip_j is an edge of CH iff $\exists$ plane thru p_i, p_j s.t. all pts lie on one side

$p_ip_jp_k$ is a <u>face</u> of CH iff all pts lies on one side of unique plane thru p_i, p_j, p_k



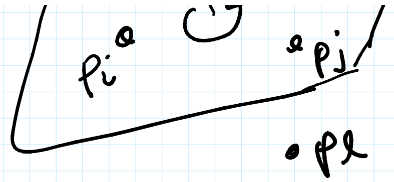
⇒ brute-force $\boxed{O(n^4)}$ time

Primitive ops:



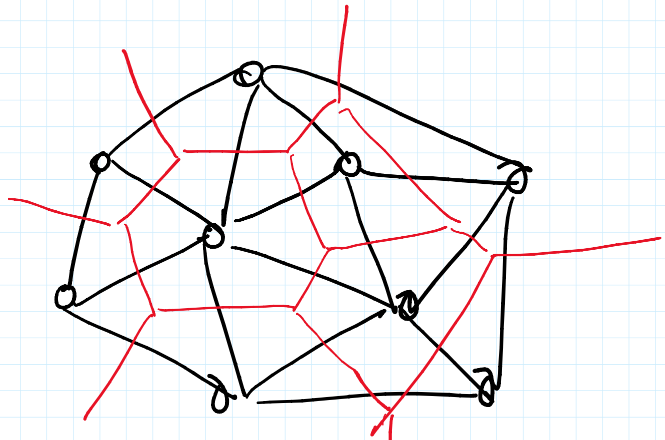
p_l left of "directed" plane thru p_i, p_j, p_k

$$\Longleftrightarrow \begin{vmatrix} 1 & x_i & y_i & z_i \\ 1 & x_j & y_j & z_j \\ 1 & x_k & y_k & z_k \end{vmatrix} > 0$$



$$\Longleftrightarrow \begin{vmatrix} \vdots & \vdots & \vdots & \vdots \\ 1 & x_j & y_j & z_j \\ 1 & x_k & y_k & z_k \\ 1 & x_l & y_l & z_l \end{vmatrix} > 0$$

(9 mults)

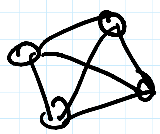


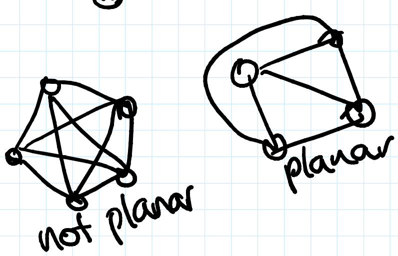
upper hull (top view)

Upper hull - projects to a <u>planar graph</u> G

(graph drawable w/o edge crossings)

Specifically, a <u>triangulation</u>



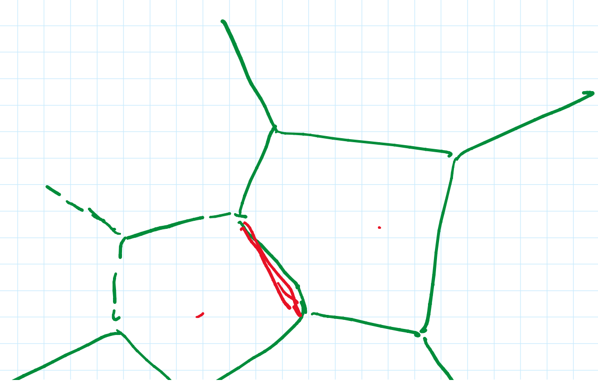


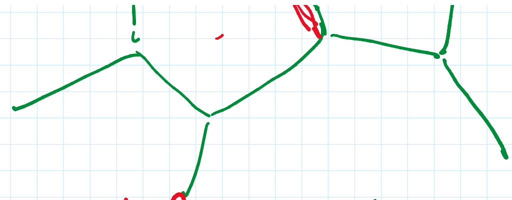
Duality - plane $z = ax + by + c \Longleftrightarrow$ point $(a, b, -c)$

lower envelope $\Longleftrightarrow$ upper hull

OR halfspace $ax + by + cz \leq 1 \Longleftrightarrow$ pt (a, b, c)

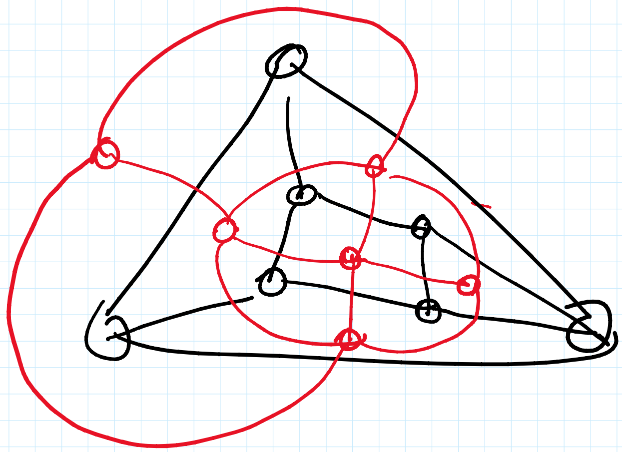
intersection of halfspaces $\Longleftrightarrow$ convex hull





lower env.   upper hull   lower envelope of planes (bottom view)

vertices $\longleftrightarrow$ faces
edges $\longleftrightarrow$ edges
faces $\longleftrightarrow$ pts



planar graph dual

Combinatorial complexity:

let n_v = # vertices $n_v \leq n$
n_e = # edges
n_f = # faces

Thm $n_e, n_f = O(n)$ . . .