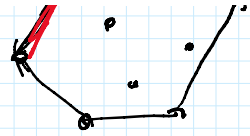


Alg'm 0: Brute force

```
for i = 1 to n do
    for j = 1 to n do {
        flag = true
        for k = 1 to n do   (k ≠ i,j)
            if pk above ←→PiPj then flag = false
        if flag then output pipj
    }
```

$O(n^3)$ time

Implementation issues:

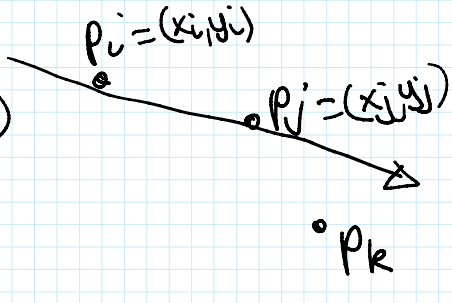
(i) Primitive ops:
how to test p_k above $\overleftrightarrow{p_i p_j}$?

(ii) degeneracies

(iii) precision issues ←

:

(i) say p_i left of p_j $(x_i < x_j)$



p_k below $\overleftrightarrow{p_i p_j}$

iff p_k right of $\overrightarrow{p_i p_j}$

iff $p_i p_j p_k$ in CW order

iff (x_k, y_k) below line with eq'n

$$y - y_i = \frac{y_j - y_i}{x_j - x_i}(x - x_i)$$

iff $y_k - y_i < \left(\frac{y_j - y_i}{x_j - x_i}\right)(x_k - x_i)$

iff $(y_k - y_i)(x_j - x_i) < (y_j - y_i)(x_k - x_i)$

Alternative:

iff $\begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix} < 0$

determinant test

2 signed-area of $\Delta p_i p_j p_k$ =

iff $\begin{vmatrix} x_j - x_i & y_j - y_i \\ x_k - x_i & y_k - x_i \end{vmatrix} < 0.$

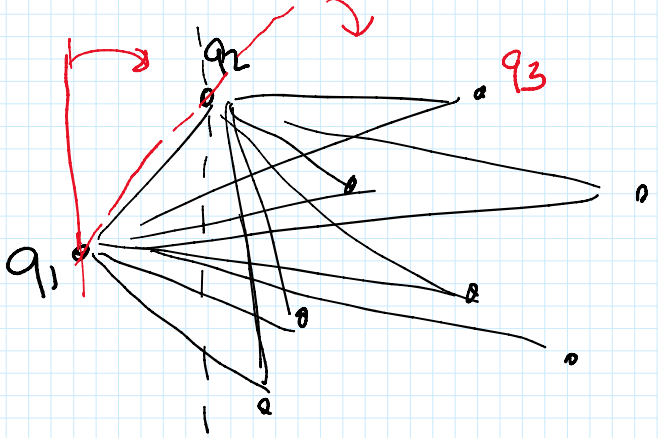
(ii) degeneracies

(iii) precision issues

:

Alg'm 1: Jarvis march (1973) / "gift wrapping"

idea - go from one UH vertex to next



generalization of selection sort

q_1 = leftmost pt ← O(n) time

for $i = 1, 2, \ldots$ do {

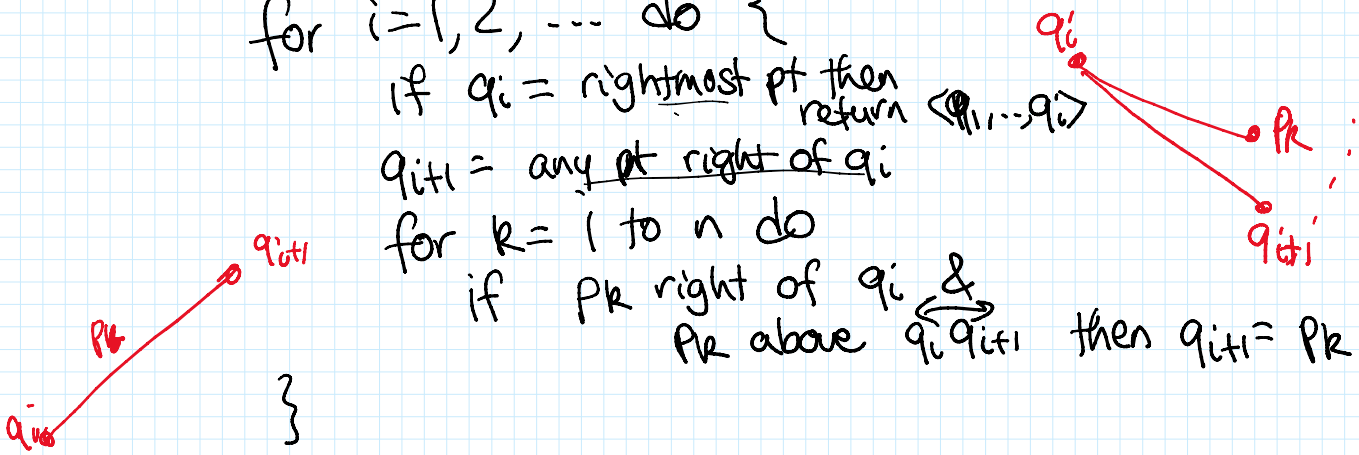
if q_i = rightmost pt then return $\langle q_1 \ldots q_i \rangle$

q_{i+1} = any pt right of q_i

for $k = 1$ to n do

if p_k right of q_i & p_k above $q_i q_{i+1}$ then $q_{i+1} = p_k$

}



Analysis: ≤ n iterations, each O(n) time

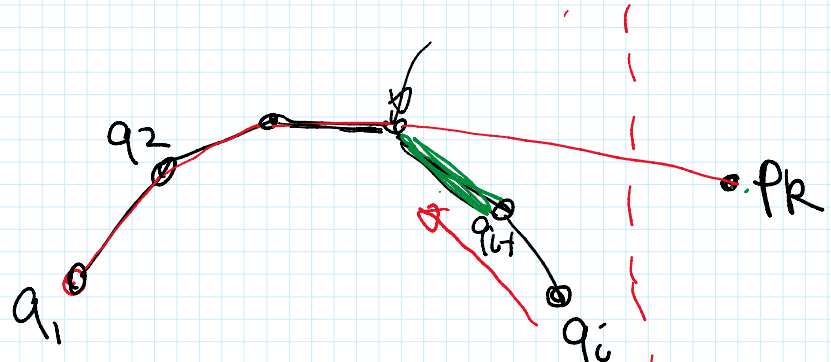
⇒ $O(n^2)$ time

Algm 2: Graham Scan (1972)

idea - add pts one at a time & maintain CH ← "incremental"

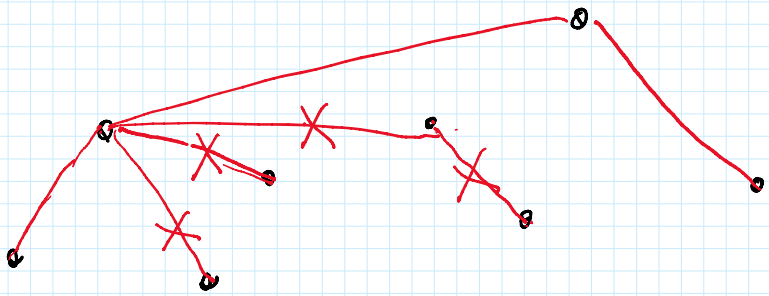
like insertion sort

- go from left to right ← "sweeping"



→ O(n log n) time

1. sort $p_1, \dots, p_n$ by x-coord. ← $O(n \log n)$ time by heapsort, mergesort, ...
   $q_0 = (0, -\infty)$
2. $q_1 = p_1$, $q_2 = p_2$, $i = 2$
3. for $k = 3$ to n do { // insert p_k to CH
4. while p_k above $q_{i-1} q_i$ do $i = i - 1$
5. $q_{i+1} = p_k$, $i = i + 1$
   }
6. return $\langle q_1, q_2, \dots, q_i \rangle$



Rough Analysis: n iterations, each in $O(n)$ time
$\Rightarrow$ $O(n^2)$ time

Better Analysis:
total time for lines 3-5
$= O(n + \#\text{decrements})$
$\leq O(n + \#\text{increments})$
$= O(n)$

$\boxed{O(n \log n)}$ time total