CS 498 TC, Fall 2024
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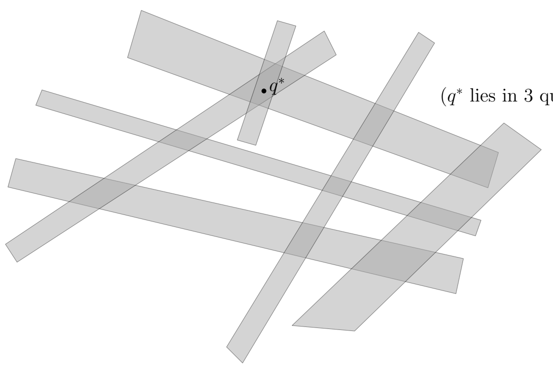
Homework 4 (due Nov 13 Wed 5pm)

Instructions: See previous homework.

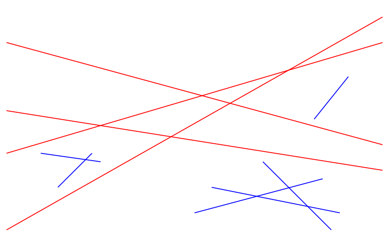
1. [*35 pts*] Remember the eclipse earlier this year… Consider the following question: which location in the US has seen the most occurrences of total solar eclipses over the past century? One way to turn this question into a precise algorithmic problem is as follows (modelling the path of totality of an eclipse as a quadrilateral):

   Given a set Q of n quadrilaterals in 2D, find a point $q^* \in \mathbb{R}^2$ that lies inside the largest number of quadrilaterals. Show that this problem can be solved in $O(n^2)$ time (using a certain algorithm from class).



2. [*65 pts*] Given a set R of r red line segments and a set B of b blue line segments with $r+b=n$, we want to decide whether there exists a red-blue intersection, i.e., an intersection between a red segment and a blue segment. (Note: there are examples with quadratically many red-red and blue-blue intersections but no red-blue intersections, so Bentley and Ottman's algorithm does not work well in the worst case for this problem.)

   (a) [*20 pts*] First show that if the red segments are lines, then the problem can be solved in $O(r^2 + b \log n)$ time.
   (Hint: Use algorithms from class, in particular, point location.)



(b) [*15 pts*] Show that if the red segments are lines, then the problem can be solved in $O(n^{3/2} \log^{1/2} n)$ time.

(Hint: use (a) and divide the red segments into groups of a certain size.[1])

(c) [*30 pts*] Show that the general problem for red line segments and blue line segments can be solved in $O(n^{3/2} \log^{3/2} n)$ time.

(Hint: follow the segment tree approach and use (b) to filter out all long segments before recursing.)

[1]As usual, if you are unable to solve (a) correctly, you can still do (b) using (a) as a black box.