CS 498 TC, Fall 2024
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Homework 1 (due Sep 18 Wed 5pm)

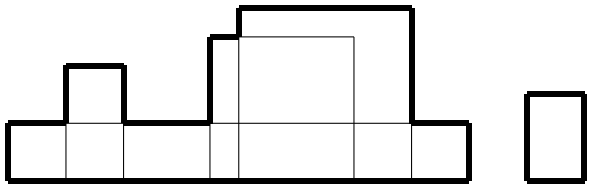
Instructions: You may work individually or in groups of at most 3; submit one set of solutions per group. Always acknowledge discussions you have with other people and any sources you have used (although most homework problems should be doable without using outside sources). In any case, *solutions must be written in your own words.*

See `https://courses.grainger.illinois.edu/cs498tcu/fa2024/policies.html`.

1. [*35 pts*] Let P be a convex polygon with n vertices (stored in an array in counterclockwise order). Let q be a point outside P. Present an $O(\log n)$-time algorithm to compute the closest point on the boundary of P to q.

   (Hint: it may be helpful to first find the leftmost and rightmost point of P, to split P into an upper and a lower part.)

2. [*45 pts*] We consider the following problem of computing a "skyline" of buildings each represented as a rectangle. More precisely, we are given a set R of n (axis-aligned) rectangles in $\mathbb{R}^2$ where all the bottom edges lie on the x-axis. (Each rectangle R is of the form $[a_1, a_2] \times [0, b]$.) We want to compute the union U of these rectangles. (U may be a polygon or consist of multiple polygons.)



   (a) [*17 pts*] Give an $O(n \log n)$-time algorithm for this problem.

   (b) [*11 pts*] Prove that the problem requires $\Omega(n \log n)$ worst-case time for comparison-based algorithms. To be precise, assume the version of the problem where the vertices of each polygon in U must be outputted in left-to-right order, and the polygons themselves (if there are multiple ones) may be outputted in arbitrary order.

   (c) [*17 pts*] Give an $O(nh)$-time algorithm where h is the output size.

   (d) [*Bonus, up to 7 pts*] Give an $O(n \log h)$-time algorithm where h is the output size.

3. [*20 pts*]

(a) [*10 pts*] Consider the following problem: given a set P of n red points and a set Q of n blue points in $\mathbb{R}^2$, find a line ℓ such that all red points of P are below ℓ and all blue points of Q are above ℓ. Convert this problem into an equivalent one in dual space.

(b) [*10 pts*] Consider the following problem: given a set P of n points and a point q in $\mathbb{R}^2$, find a line that passes through q and is tangent to the upper hull of P. Convert this problem into an equivalent one in dual space.