CS 498TC, Spring 2018
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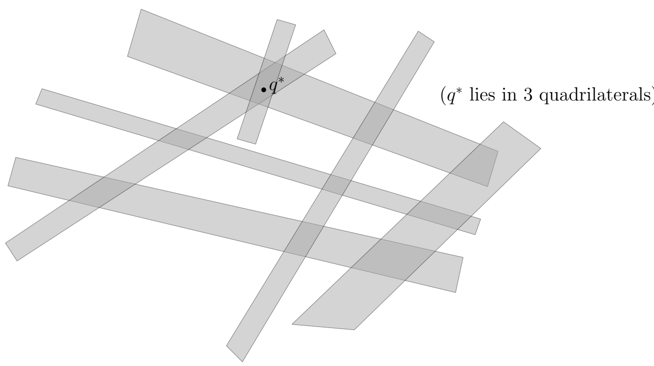
Assignment 4 (due April 13 Friday 2pm (in class))

You may work in a group of at most 3 students. Carefully read `https://courses.engr.illinois.edu/cs498tc3/policies.html` and `https://courses.engr.illinois.edu/cs498tc3/integrity.html`.

1. [*10 pts*] Inspired by events from last summer, someone wonders about which location in the US experienced the most occurrences of total solar eclipses over the past X years. One way to turn this question into a precise algorithmic problem is as follows (if the path of totality of each eclipse is modeled as a quadrilateral):

   Given a set Q of n quadrilaterals in 2D, find a point $q^* \in \mathbb{R}^2$ that lies inside the largest number of quadrilaterals. Show that this problem can be solved in $O(n^2)$ time. [Hint: use a known algorithm from class.]



2. [*25 pts*] Given a set R of r red line segments and a set B of b blue line segments with $r + b = n$, the problem is to detect whether there is an intersection between a red segment and a blue segment. (Note that there are examples with quadratically many red-red and blue-blue intersections but no red-blue intersections, so Bentley and Ottman's algorithm does not work well in the worst case.)

   (a) [*8 pts*] First show that if the red segments are actually lines, then the problem can be solved in $O(r^2 + b \log n)$ time. [Hint: Use (several) results from class.]

   (b) [*8 pts*] Show that if the red segments are lines, then the problem can be solved in $O(n^{3/2} \log^{1/2} n)$ time. [Hint: use (a) and divide the red segments into groups of a certain size.]

   (c) [*9 pts*] Show that in general, the problem can be solved in $O(n^{3/2} \log^{3/2} n)$ time. [Hint: follow the segment tree approach and use (b) to filter out all long segments before recursing.]

3. [*10 pts*] We are given a (nonconvex) polyhedron P with n vertices in 3D (where there are no holes and all the faces are polygons). Show that we can always select a subset G of $\lfloor n/2 \rfloor$ vertices so that every point on the boundary of P is visible from some point of G (where p and q are visible iff the line segment pq lies on the boundary of P).

   [Hint: triangulate all the faces and apply the *4-Color Theorem* (every planar graph can be colored with 4 colors, so that no two adjacent vertices are assigned the same color). Which color classes should we pick?]