CS 498TC, Spring 2018
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Assignment 2 (due Feb 23 Friday 2pm (in class))

You may work in a group of at most 3 students. Carefully read `https://courses.engr.illinois.edu/cs498tc3/policies.html` and `https://courses.engr.illinois.edu/cs498tc3/integrity.html`.

1. [*30 pts*] A *box* (or a *hyper-rectangle*) is the higher-dimensional generalization of a rectangle. In 3D, a box has 8 vertices, 12 edges, and 6 faces, with edges parallel to the x-, y-, and z-axes. Let S be a set of n boxes in 3D, where each box has the origin as a vertex, i.e., each box is of the form $[0, a_i] \times [0, b_i] \times [0, c_i]$. The union U of the n boxes in S is a nonconvex polyhedron. We are interested in computing this polyhedron U. (This problem, in 3 dimensions and higher, has many applications.)

   (a) [*5 pts*] In 3D, show that U has $O(n)$ vertices, edges, and faces. What does a face look like?

   (b) [*7 pts*] Consider an *incremental* approach to computing U. Prove that if we randomize the order of insertion of the boxes, the expected total number of vertices, edges, and faces created and destroyed is $O(n)$.

   (c) [*13 pts*] Again consider an incremental approach. This time, describe a simple insertion order, without randomization, for which the total number of vertices, edges, and faces created and destroyed is guaranteed to be $O(n)$. Using this insertion order (and appropriate data structures), describe a deterministic $O(n \log n)$-time algorithm to compute the polyhedron U in 3D.

   (Note: alternatively, such an algorithm can be obtained by a *sweep* approach. What direction should the sweep be in?)

   (d) [*5 pts*] If we do not assume that each box has the origin as a vertex, does U still have $O(n)$ vertices, edges, and faces? Explain.

2. [*15 pts*] Given n circular disks of the same radius, give an $O(n \log n)$-time algorithm to compute the area of the union of these disks. (Note: the union may be disconnected and may have holes, but we do not need to output the union explicitly, just its area. Hint: what does the union look like inside a Voronoi cell?)