Homework 1
CS 498 MV: Logic in Computer Science

Assigned: September 6, 2018   Due on: September 13, 2018

Instructions: Please do not turn in solutions to the practice problems. Solutions to the homework problems should be turned in as a PDF file on Gradescope. See instructions on Piazza.

Recommended Reading: Lectures 1 through 3: propositional logic, proof systems (Frege and Resolution), soundness and completeness.

Practice Problems

Practice Problem 1. The following sentence is taken from the specification of a telephone system: "If the directory database is opened, then the monitor is put in a closed state, if the system is not in its initial state." Find an equivalent easier to understand specification that involves disjunctions and negations but no implications.

Practice Problem 2. Are the following system specifications consistent? Determine this by encoding the specification in logic and using your favorite proof system to determine if the constructed formula is satisfiable.

1. "The system is in multiuser state if and only if it is operating normally. If the system is operating normally, the kernel is functioning. The kernel is not functioning or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is not in interrupt mode."
2. "If the file system is not locked, then new messages will be queued. If the file system is not locked, then the system is functioning normally, and conversely. If new messages are not queued, then they will be sent to the message buffer. If the file system is not locked, the new messages will be sent to the message buffer. New messages will not be sent to the message buffer."

Homework Problems

Problem 1. A set S is said to be *countable* if there is a function $f : S \to \mathbb{N}$ that is 1-to-1. For a set A, A^* is the set of all finite strings over A. Prove that if A is countable then A^* is countable.

Problem 2. Prove the soundness theorem for the Frege Proof system. That is, show that if $\Gamma \vdash \varphi$ then $\Gamma \models \varphi$. *Hint:* Let $\psi_1, \psi_2, \ldots \psi_m$ be a proof of φ from Γ. Prove by induction on i that, for every i, $\Gamma \models \psi_i$

Problem 3. For the Frege proof system, prove the Deduction Theorem (without assuming the completeness theorem). That is, if $\Gamma \cup \{\varphi\} \vdash \psi$ then $\Gamma \vdash \varphi \to \psi$. *Hint:* Let $\psi_1, \psi_2, \ldots \psi_m$ be a proof of ψ from $\Gamma \cup \{\varphi\}$. Show by induction on i that, for every i, $\Gamma \vdash \varphi \to \psi_i$.

Problem 4. The Davis-Putnam proof showing the completeness of resolution, outlines an algorithm to determine satisfiability of a set of clauses. Use the Davis-Putnam algorithm to determine if the following sets of clauses are satisfiable. Outline all the steps of the algorithm in each case.

1. $\Gamma = \{\{a, b, c\}, \{b, \neg c, \neg f\}, \{\neg b, e\}\}$
2. $\Gamma = \{\{a, b\}, \{a, \neg b\}, \{\neg a, c\}, \{\neg a, \neg c\}\}$