CS498 MP
Logic in Computer Science
Fall 2020

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Lectures: Tue, Thu: 11am-12:15pm (Online/Zoom)

Zoom: http://piazza.com/illinois/fall2020/cs498mp

Website: *http://courses.engr.illinois.edu/cs498mp3/*

Piazza: http://piazza.com/illinois/fall2020/cs498mp Access code: herbrand

Prerequisites:

Mathematical maturity; discrete math (CS173).
Theory of computation (as covered in CS374) is preferred.
Talk to me if you don't have this background.

What's logic?

- Logic is the study of the principles of valid inference and demonstration.
- Logic is the study of the structure of arguments.
- Study of formal inference.
- Logic: Syntax + Semantics + Inference
  (form) (meaning) (reasoning)
- *"Contrariwise," … "if it was so, it might be; and if it were so, it would be; but as it isn't, it ain't. That's logic."*
  - Tweedledum - Through the Looking Glass, Lewis Carroll

Aims of this course

- Primary aims:
  - Mathematical and logical maturity; understanding how humans and computers reason formally
  - Formal reasoning and formal arguments
  - Various ways in which logic is a foundational aspect of computer science

Aims of this course

- More concretely:
  - Fundamentals of mathematical logic
    - Propositional logic and predicate logic
    - Model theory and proof theory
    - Sound and complete proof systems
    - Gödel's completeness and incompleteness theorems
  - Logic and computability
    - Computability ~ Proofs
    - Finite model theory (computational complexity using logic)
  - Automated reasoning
    - Propositional reasoning using SAT solvers
    - Quantifier elimination
    - Decidability using interpretations
    - Decidable theories supported by SMT solvers
    - Decidability of combined theories
    - Complete reasoning using systematic quantifier instantiation
    - Reasoning using induction

A brief history of logic

- Beginnings
  - Aristotle (~300 B CE)
    - Earliest formal study of logic
    - Rhetoric, syllogism, philosophy, geometry
    - Furthered by Islamic logicians
  - Logic in India
    - Nyaya and Vaisheshika (2 CE)
    - Ontology and epistemiology; obtaining valid knowledge
  - Logic in China
    - Mozi school (~400 B CE)

A brief history of logic

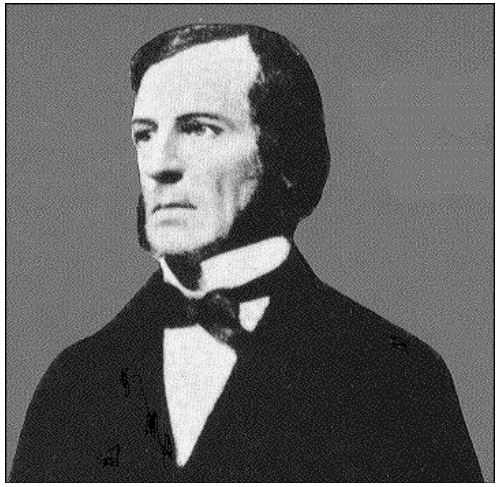
- Syllogism
  - Every man is mortal
  - Socrates is a man
  - Conclude: Socrates is mortal

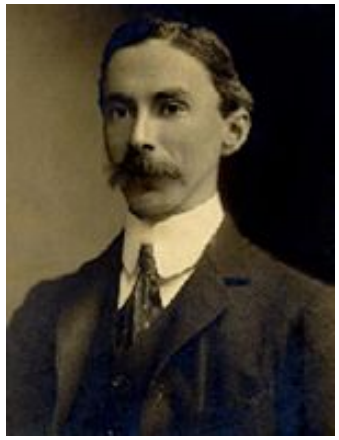
- "man", "mortal", "Socrates" – not important
  - Every M is D
  - S is an M
  - Conclude: S is D

Forall x. M(x) -> D(x)
M(S)
So, D(S).

A brief history of logic

- The algebraic school
  - George Boole and others (end of 19th century)
  - Algebraic structure of formulas; Boolean algebra





- The logicist school

  (Russel,Wittgenstein,Frege)
  - Frege developed an elaborate formal language

  Quantifiers, “predicate calculus”, number-theory -> logic
  - Russel’s paradox
  - Principia Mathematica (Russel-Whitehead)

    (1910-1913)

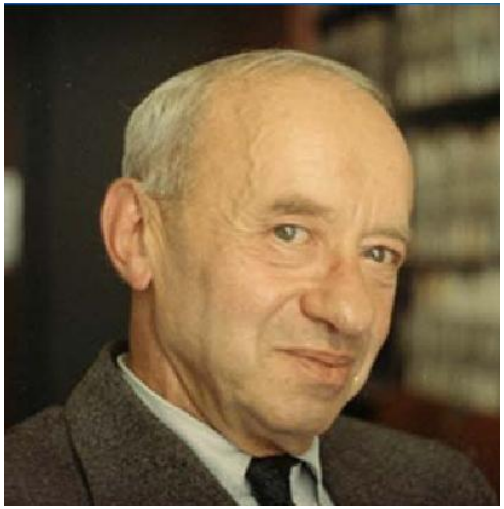
A brief history of logic

- The mathematical school (early 20th century)
  - Dedekind, Peano, Hilbert, Heyton, Zermelo, Tarski
  - Axiomatize branches of math (geometry, arithmetic, set-theory)
  - Zermelo: Axiomatization of set-theory
  - The Hilbert program: formalization of all of mathematics in axiomatic form, with a proof of consistency using "finitary" methods.
  - Godel's theorems:
    - Every FO sentence can be deduced in common deductive systems.

      Godel's completeness theorem
    - Any axiomatization that includes arithmetic must either be unsound or incomplete (i.e. there is a sentence neither provable nor disprovable)

      Godel's incompleteness theorem

A brief history of logic



- The mathematical school (20th century)
  - Tarski: Logician extraoradinaire (~ 2500 papers!)
  - Completeness, truth, definability, decidability
  - Alonzo Church, and students Kleene and Henkin
    - Church's thesis of computability
    - First-order logic is undecidable

Post WW-II

- model theory
  - Study of mathematical structures (groups, etc.) using logic
- proof theory
  - Study of axiom systems, inference systems, proofs
- computability theory
  - Relationship of logic to computability/complexity
- set theory
  - foundations of mathematics, axiomatic set theory
  - Cohen: independence of continuum hypo and independence of axiom of choice from ZF axioms.
  - constructivism
- applications of logic in computer science
  - applied logic to AI, databases, verification, aided by tools like automated and semi-automated theorem provers, and ML

Logic and computability

- Turing's machines (1941):
  - Formal mechanical notion of computation using "finite" means
  - Church-Turing thesis:

    All computable languages are Turing-machine computable
  - Gödel's incompleteness result is about non-existence of proofs using diagonalization:
    - A proof system is only a particular form of computation!
    - Existence of a sound and complete proof system shows a theory is recursively enumerable (computed by TM that may not halt on all inputs)
    - A deep connection between logic and computability

Logic and computational complexity

- Fundamental idea (Fagin): Descriptive complexity
  - An algorithmic problem can be logically described.
  - The logic used determines the computational complexity of the problem!
  - Eg. Existential SO ~ NP

    Least-fix-point over ordered structures ~ P
  - Field: Finite model theory

Logic in Computer Science

Unusual (and unreasonable!) effectiveness in computer science.

- Computability and proofs
- Computational complexity and descriptive complexity
- Semantics of programming languages (types, type inference, domains and fixpoints, linear logic, etc.)
- Automatic reasoning, theorem provers (CoQ, PVS).
- Specifications for hardware systems (decidable temporal logics LTL/CTL)
- Logics to prove programs correct (Hoare-logic, pre-post conditions)
  - Huge impact in verifying systems
  - Modern software validation tools rely on decidable logics for abstraction and verification
  - Large number of decidable theories; combining decidable theories; automatic theorem provers (eg. Z3 from MSR).
- Proof-carrying code
- Database theory: Queries are simply formulas! SQL ~ FOL
- Logic Programming

Unusual effectiveness of logic in CS

- E.P. Wigner's Nobel prize talk, 1963:

  "On the Unreasonable Effectiveness of Mathematics in the Natural Sciences"

- On the Unusual Effectiveness of Logic in Computer Science

  - Halpern, Harper, Immerman, Kolaitis, Vardi, Vianu

Logic and AI

- AI in the past (till about 2000) was obsessed with logic, but was too early.
- We need to first be able to detect cats in a picture, before understanding more complex thinking!
- Daniel Kahneman's "Thinking, Fast and Slow"
- Advent of Machine Learning --- captures "quick"/intuitive/unexplainable thinking
- But humans think symbolically too!

- How does symbolic thinking happen?

- Interface between symbolic and "neural" thinking is still a mystery.

Logic in Computer Science: Themes

Theme 1. Classical propositional and predicate logic.

(proof systems, axioms, soundness, completeness, compactness, Gödel's theorems)

Theme 2. Decidable logics.

(QE for reals, finite-model property, decidability of Presburger arithmetic, interpretations, combining decision procedures, SAT/SMT solvers, tools for logic)

Theme 3. Finite model theory.

(descriptive complexity, computational complexity, characterizing complexity classes using logic)

Theme 4. Other logics

Higher-order logics, modal and temporal logics, logics for querying (databases), relational logics, etc.

Administrivia

- What's "trivia"?
  - Greek education system had "trivium" (three ways):
    - Grammar (mechanics of language)
    - Logic (mechanics of thought and analysis)
    - Rhetoric (mechanics of persuasion)

  followed by the "quadrivium":
    - arithmetic, geometry, music, and astronomy

Administrivia

- Textbooks: None!

  (No text covers the topics we cover)

  The following can be useful, however:

  - Enderton: A Mathematical Introduction to Logic
  - Melvin Fitting: First-order logic and Automated Theorem Proving
    - 1 copy in Engg lib; placed on reserve
  - Borger, Gradel, Gurevich: The Classical Decision Problem
  - Huth, Ryan: Logic in Computer Science
    - 2 copies (soon) in Engg lib; placed on reserve

- More resources (book chapters, papers, notes) will be handed out in class and made available online on the course page.
- Lecture notes (handwritten) will be posted online too.
- I'm writing lecture notes for some aspects of the course.

Administrivia

- Homework
  - Roughly one homework set every two weeks

    (assigned on Tuesdays and due the following Tuesday); may be more often.

- Office hours:
  - To be decided. What suits you?

Projects (for 4 credits)

We could have projects:

Starts somewhere mid-term and ends with the semester.

Either

- A project to read a set of papers and present it in class briefly and write a report

- A project that works on a research-ish problem and tries to build a solution

  (work in groups, work in something that's close to your research, I can work with you, you can even send it for publication if you get good results.)

Exams

- One midterm and a Final Exam
- Written; logistics of proctoring later.

Other tools

- We will probably use Gradescope for homework
- And Piazza for discussions.
- Sign up for Piazza!
- Look out for emails on these topics.

Survey

Email me:

1) What do you expect to learn in this course?

2) What are your general interests?

$\mathbb{N}$

$\forall$ forall

$\exists$ exists

$\wedge$ and

$\vee$ or

$\Rightarrow$ implies

$\forall x \in \mathbb{N} \; \exists y \in \mathbb{N} \;\; y > x$