Image Warping

Computational Photography

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Many slides from Alyosha Efros + Steve Seitz

Photo by Sean Carroll

Reminder: Proj 2 due monday

- Much more difficult than project 1 – get started asap if not already (could take >10 hours)
- Must compute SSD cost for every pixel (slow but not horribly slow using filtering method; see tips at end of project page)
- Learn how to debug visual algorithms: imshow, plot, dbstop if error, keyboard and break points are your friends
  - Suggestion: For “quilt_simple”, first set upper-left patch to be upper-left patch in source and iteratively find minimum cost patch and overlay --- should reproduce original source image, at least for part of the output
- Remember office hours: Amin on Thurs, Derek on Mon

Resuming from last class… Poisson Blending

A good blend should preserve gradients of source region without changing the background

Perez et al. 2003

Method 3: Poisson Blending

A good blend should preserve gradients of source region without changing the background

Treat pixels as variables to be solved

– Minimize squared difference between gradients of foreground region and gradients of target region

– Keep background pixels constant

$$\mathbf{v} = \underset{\mathbf{v}}{\operatorname{argmin}} \sum_{i \in S, j \in N_i \cap S} ((v_i - v_j) - (s_i - s_j))^2 + \sum_{i \in S, j \in N_i \cap \neg S} ((v_i - t_j) - (s_i - s_j))^2$$

Perez et al. 2003

Examples

2. Gradient domain processing

$$\mathbf{v} = \underset{\mathbf{v}}{\operatorname{argmin}} \sum_{i \in S, j \in N_i \cap S} ((v_i - v_j) - (s_i - s_j))^2 + \sum_{i \in S, j \in N_i \cap \neg S} ((v_i - t_j) - (s_i - s_j))^2$$

source image

1 **20**	5 **20**	9 **20**	13 **20**
2 **20**	6 **80**	10 **20**	14 **20**
3 **20**	7 **20**	11 **80**	15 **20**
4 **20**	8 **20**	12 **20**	16 **20**

background image

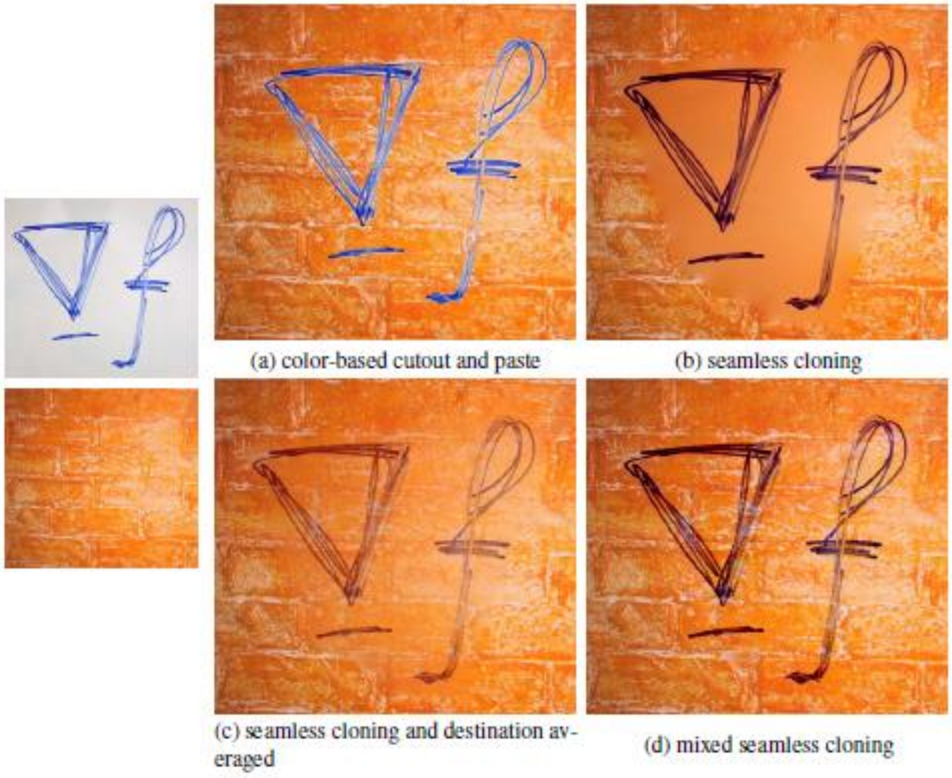
1 **10**	5 **10**	9 **10**	13 **10**
2 **10**	6 **10**	10 **10**	14 **10**
3 **10**	7 **10**	11 **10**	15 **10**
4 **10**	8 **10**	12 **10**	16 **10**

target image

1 **10**	5 **10**	9 **10**	13 **10**
2 **10**	6 $\mathbf{v}_1$	10 $\mathbf{v}_3$	14 **10**
3 **10**	7 $\mathbf{v}_2$	11 $\mathbf{v}_4$	15 **10**
4 **10**	8 **10**	12 **10**	16 **10**

Blending with Mixed Gradients

- Use foreground or background gradient with larger magnitude as the guiding gradient



Perez et al. 2003

Gradient-domain editing

Many image processing applications can be thought of as trying to manipulate gradients or intensities:

- Contrast enhancement
- Denoising
- Poisson blending
- HDR to RGB
- Color to Gray
- Recoloring
- Texture transfer

See Perez et al. 2003 and GradientShop for many examples

Gradient-domain processing

Saliency-based Sharpening

http://www.gradientshop.com

Gradient-domain processing

Non-photorealistic rendering

http://www.gradientshop.com

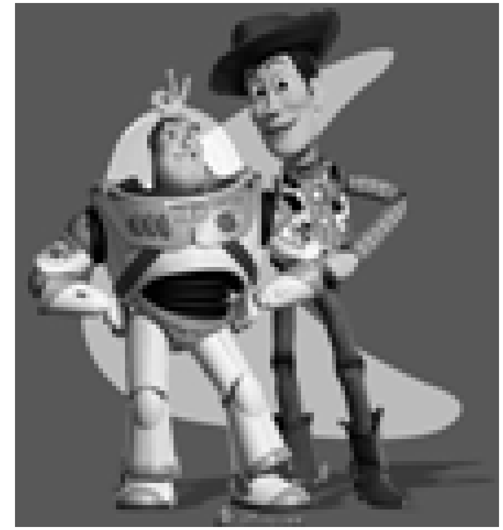
Project 3: Gradient Domain Editing

General concept: Solve for pixels of new image that satisfy constraints on the gradient and the intensity

- Constraints can be from one image (for filtering) or more (for blending)

Project 3: Reconstruction from Gradients

1. Preserve x-y gradients
2. Preserve intensity of one pixel



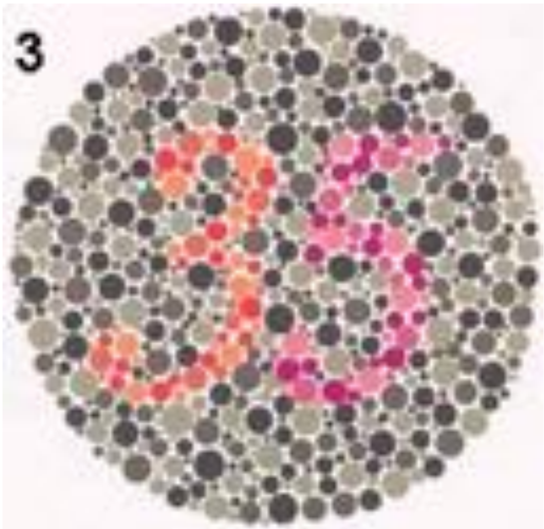
Source pixels: s

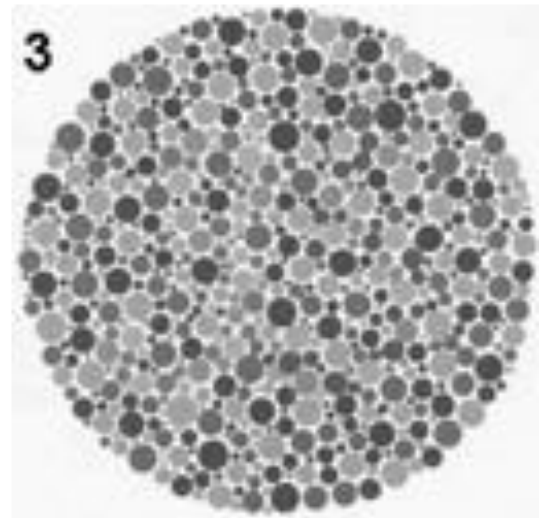
Variable pixels: v

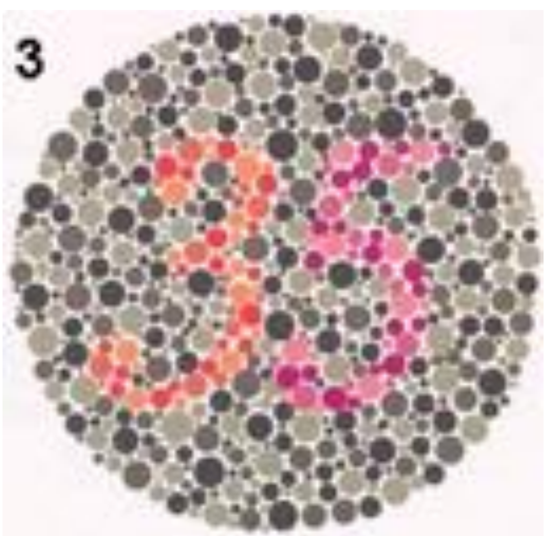
1. minimize (v(x+1,y)-v(x,y) - (s(x+1,y)-s(x,y))^2
2. minimize (v(x,y+1)-v(x,y) - (s(x,y+1)-s(x,y))^2
3. minimize (v(1,1)-s(1,1))^2

Project 3 (extra): Color2Gray

rgb2gray







Gradient-domain editing

?

Project 3 (extra): NPR

- Preserve gradients on edges
  - e.g., get canny edges with edge(im, ‘canny’)
- Reduce gradients not on edges
- Preserve original intensity

Perez et al. 2003

Summary of last class

- Three ways to blend/composite
  1. Alpha compositing
     - Need nice cut (intelligent scissors)
     - Should **feather**
  2. Laplacian pyramid blending
     - **Smooth blending at low frequencies, sharp at high frequencies**
     - Usually used for stitching
  3. Gradient domain editing
     - Also called **Poisson Editing**
     - Explicit control over what to preserve
     - Changes foreground color (for better or worse)
     - Applicable for many things besides blending

Take-home questions

1) I am trying to blend this bear into this pool. What problems will I have if I use:
   a) Alpha compositing with feathering
   b) Laplacian pyramid blending
   c) Poisson editing?

Take-home questions

2) How would you make a sharpening filter using gradient domain processing? What are the constraints on the gradients and the intensities?

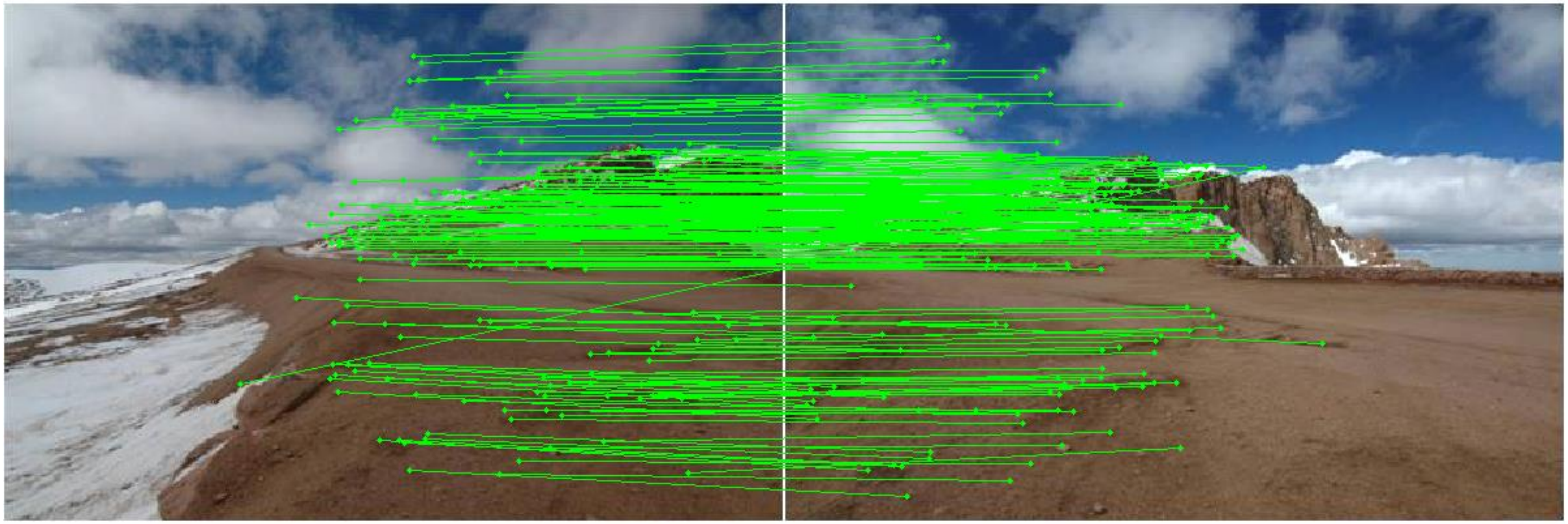
Next two classes

- Today
  - Global coordinate transformations
  - Image alignment

- Tuesday
  - Interpolation and texture mapping
  - Meshes and triangulation
  - Shape morphing

Photo stitching: projective alignment

Find corresponding points in two images



Solve for transformation that aligns the images

Capturing light fields

Estimate light via projection from spherical surface onto image

Morphing

Blend from one object to other with a series of local transformations

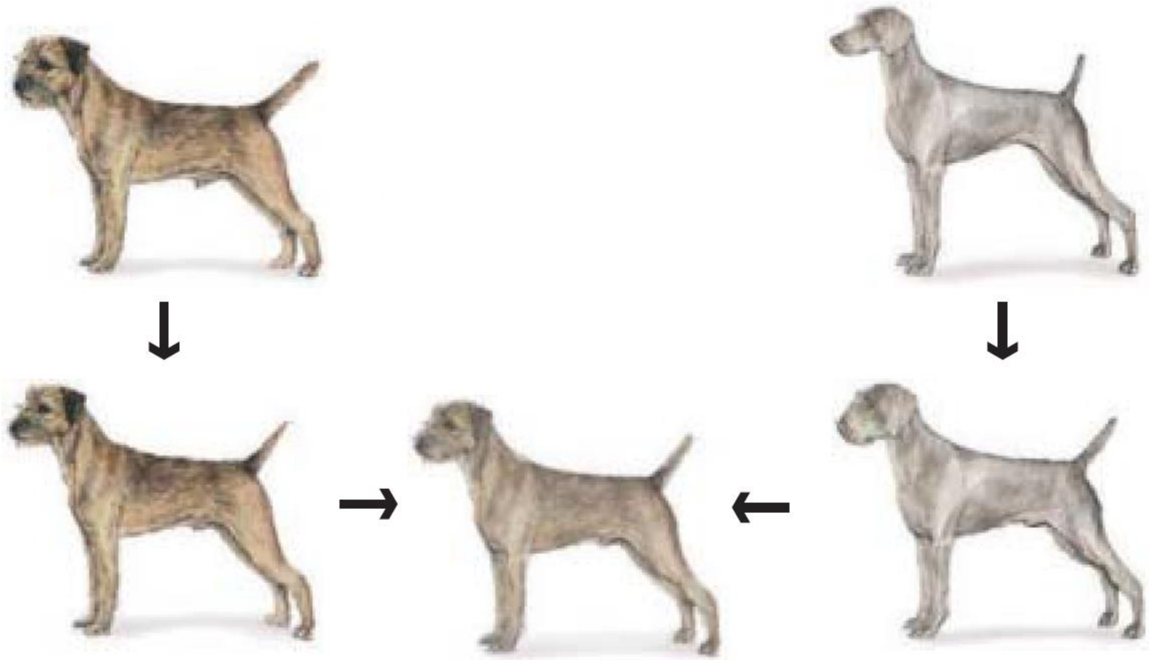


Image Transformations

image filtering: change ***range*** of image

$$g(x) = T(f(x))$$

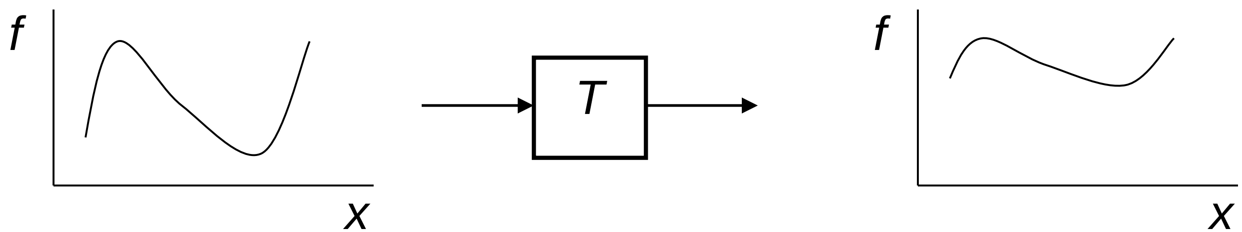


image warping: change ***domain*** of image

$$g(x) = f(T(x))$$

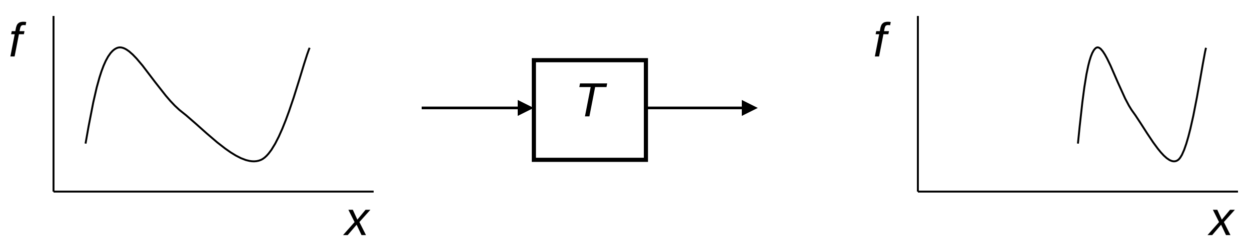


Image Transformations

image filtering: change ***range*** of image

$$g(x) = T(f(x))$$

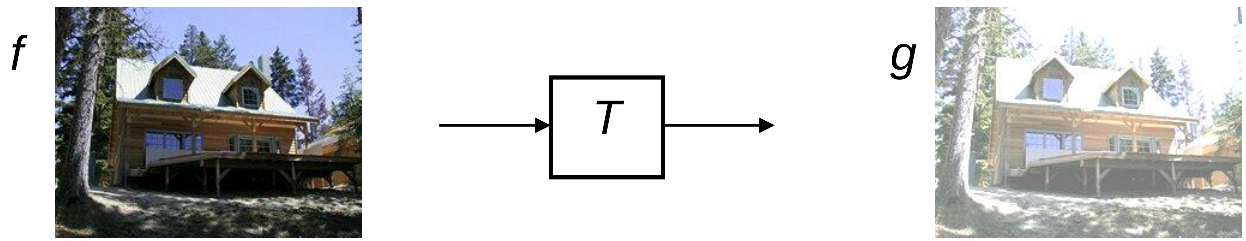
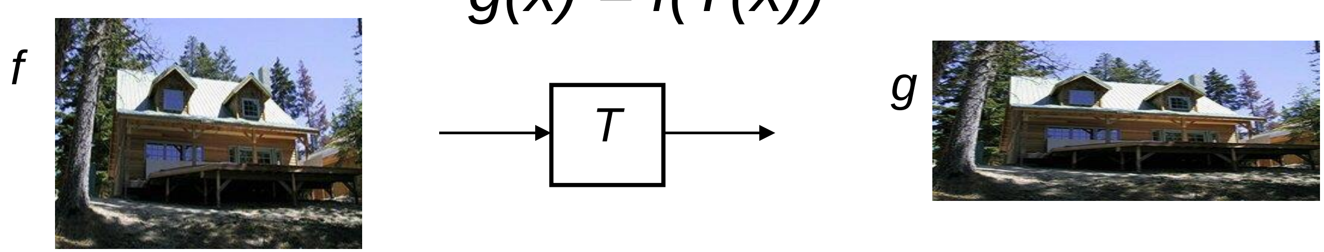
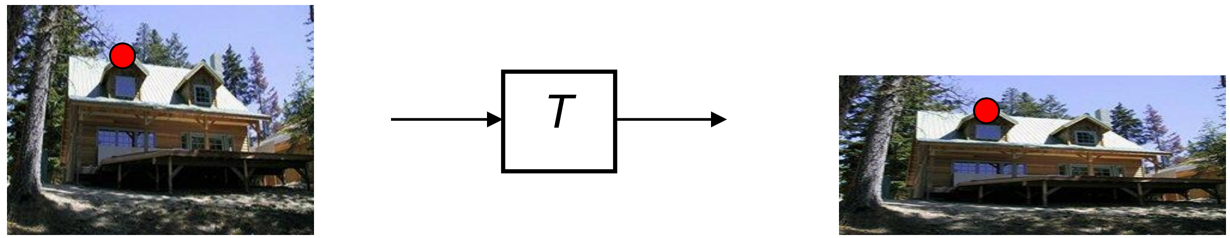


image warping: change ***domain*** of image

$$g(x) = f(T(x))$$



Parametric (global) warping



p = (x,y) **p’** = (x’,y’)

Transformation T is a coordinate-changing machine:

p’ = *T*(p)

What does it mean that *T* is global?

– Is the same for any point p
– can be described by just a few numbers (parameters)

For linear transformations, we can represent T as a matrix

p’ = **M**p

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{M} \begin{bmatrix} x \\ y \end{bmatrix}$$

Parametric (global) warping

Examples of parametric warps:

translation

rotation

aspect

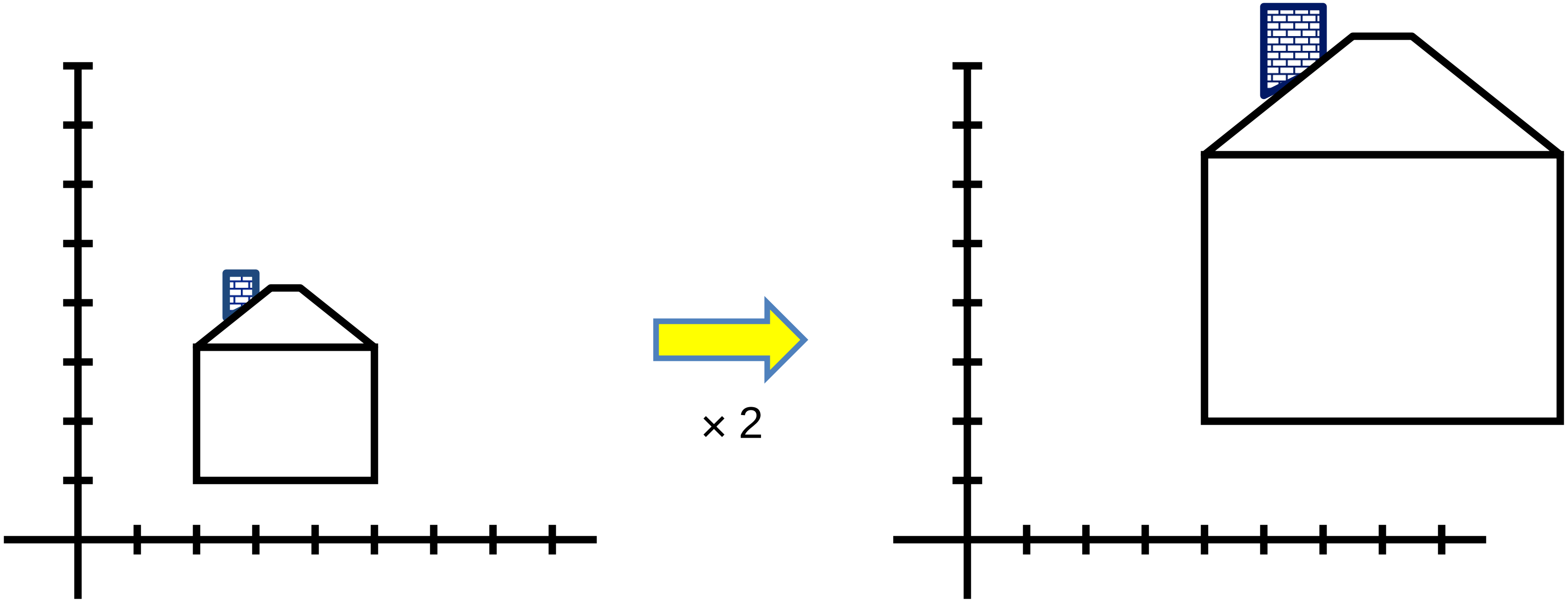
affine

perspective

cylindrical

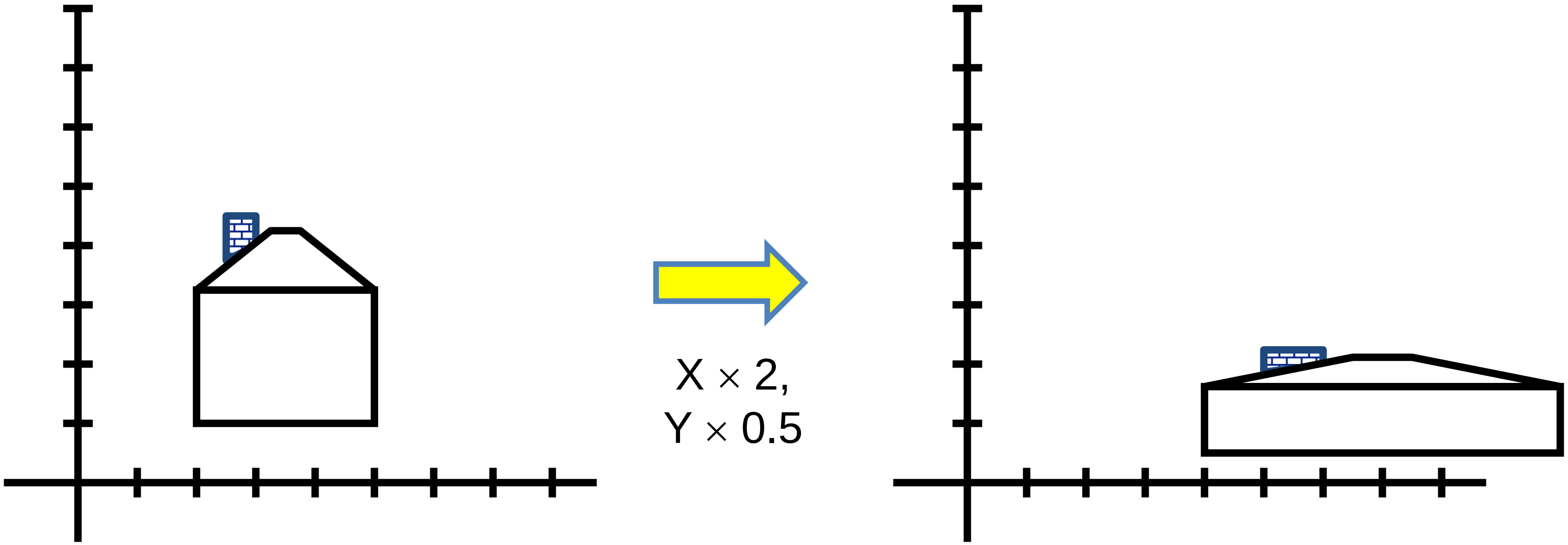
Scaling

- *Scaling* a coordinate means multiplying each of its components by a scalar
- *Uniform scaling* means this scalar is the same for all components:



Scaling

- *Non-uniform scaling*: different scalars per component:



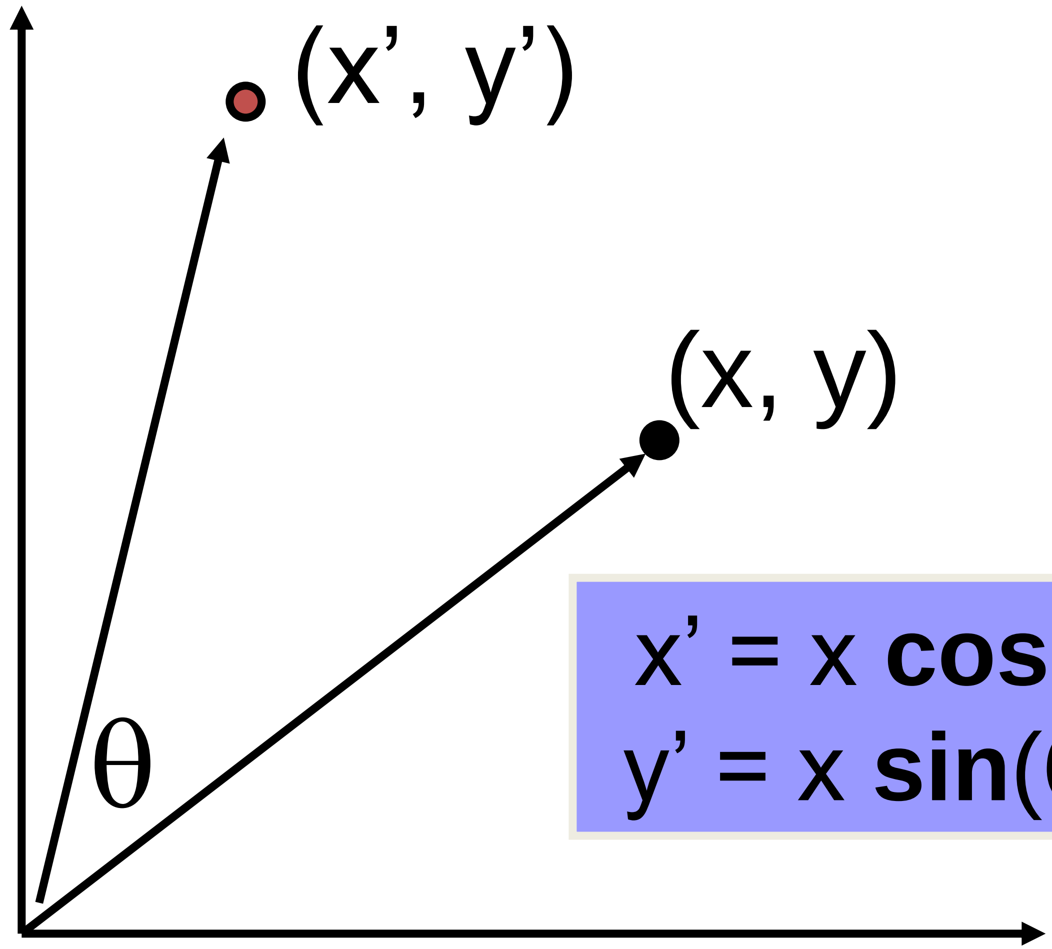
Scaling

- Scaling operation: $$x' = ax$$ $$y' = by$$

- Or, in matrix form: $$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}}_{\textit{scaling matrix } S} \begin{bmatrix} x \\ y \end{bmatrix}$$

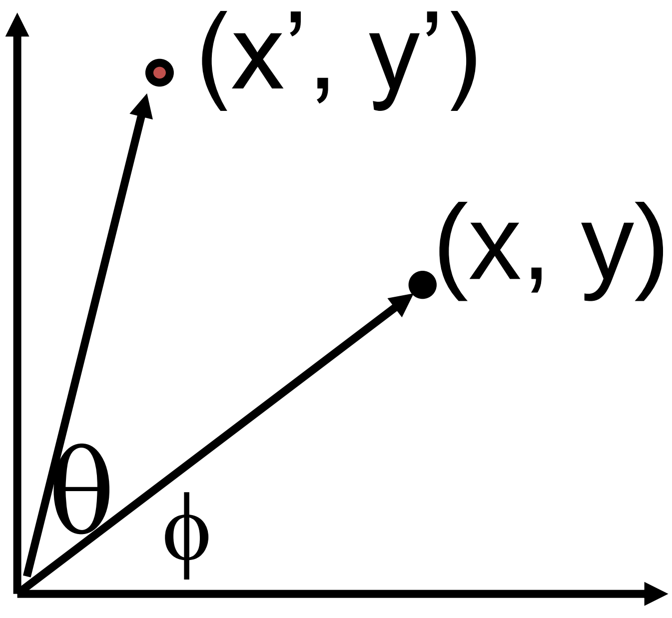
What is the transformation from (x', y') to (x, y)?

2-D Rotation



$$x' = x\,\mathbf{cos}(\theta) - y\,\mathbf{sin}(\theta)$$
$$y' = x\,\mathbf{sin}(\theta) + y\,\mathbf{cos}(\theta)$$

2-D Rotation



Polar coordinates…

$x = r \cos(\phi)$

$y = r \sin(\phi)$

$x' = r \cos(\phi + \theta)$

$y' = r \sin(\phi + \theta)$

Trig Identity…

$x' = r \cos(\phi) \cos(\theta) - r \sin(\phi) \sin(\theta)$

$y' = r \sin(\phi) \cos(\theta) + r \cos(\phi) \sin(\theta)$

Substitute…

$x' = x \, \mathbf{cos}(\theta) - y \, \mathbf{sin}(\theta)$

$y' = x \, \mathbf{sin}(\theta) + y \, \mathbf{cos}(\theta)$

2-D Rotation

This is easy to capture in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}}_{\mathbf{R}} \begin{bmatrix} x \\ y \end{bmatrix}$$

Even though sin(θ) and cos(θ) are nonlinear functions of θ,

- ***x' is a linear combination of x and y***
- ***y' is a linear combination of x and y***

What is the inverse transformation?

- Rotation by –θ
- For rotation matrices $\mathbf{R}^{-1} = \mathbf{R}^{T}$

2x2 Matrices

What types of transformations can be represented with a 2x2 matrix?

2D Identity?

$$x' = x$$
$$y' = y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Scale around (0,0)?

$$\boldsymbol{x' = s_x * x}$$
$$\boldsymbol{y' = s_y * y}$$

$$\begin{bmatrix} \boldsymbol{x'} \\ \boldsymbol{y'} \end{bmatrix} = \begin{bmatrix} \boldsymbol{s_x} & 0 \\ 0 & \boldsymbol{s_y} \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix}$$

2x2 Matrices

What types of transformations can be represented with a 2x2 matrix?

2D Rotate around (0,0)?

$$x' = \cos\Theta * x - \sin\Theta * y$$
$$y' = \sin\Theta * x + \cos\Theta * y$$

$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix}$$

2D Shear?

$$x' = x + k_x * y$$
$$y' = k_y * x + y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & k_x \\ k_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2x2 Matrices

What types of transformations can be represented with a 2x2 matrix?

2D Mirror about Y axis?

$$x' = -x$$
$$y' = y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Mirror over (0,0)?

$$x' = -x$$
$$y' = -y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2x2 Matrices

What types of transformations can be represented with a 2x2 matrix?

2D Translation?

$$x' = x + t_x$$
$$y' = y + t_y$$

NO!

Only linear 2D transformations can be represented with a 2x2 matrix

All 2D Linear Transformations

- Linear transformations are combinations of …
  - Scale,
  - Rotation,
  - Shear, and
  - Mirror
- Properties of linear transformations:
  - Origin maps to origin
  - Lines map to lines
  - Parallel lines remain parallel
  - Ratios are preserved
  - Closed under composition

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Homogeneous Coordinates

Q: How can we represent translation in matrix form?

$$x' = x + t_x$$
$$y' = y + t_y$$

Homogeneous Coordinates

Homogeneous coordinates

- represent coordinates in 2 dimensions with a 3-vector

$$\begin{bmatrix} x \\ y \end{bmatrix} \xrightarrow{\textbf{homogeneous coords}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Homogeneous Coordinates

2D Points → Homogeneous Coordinates

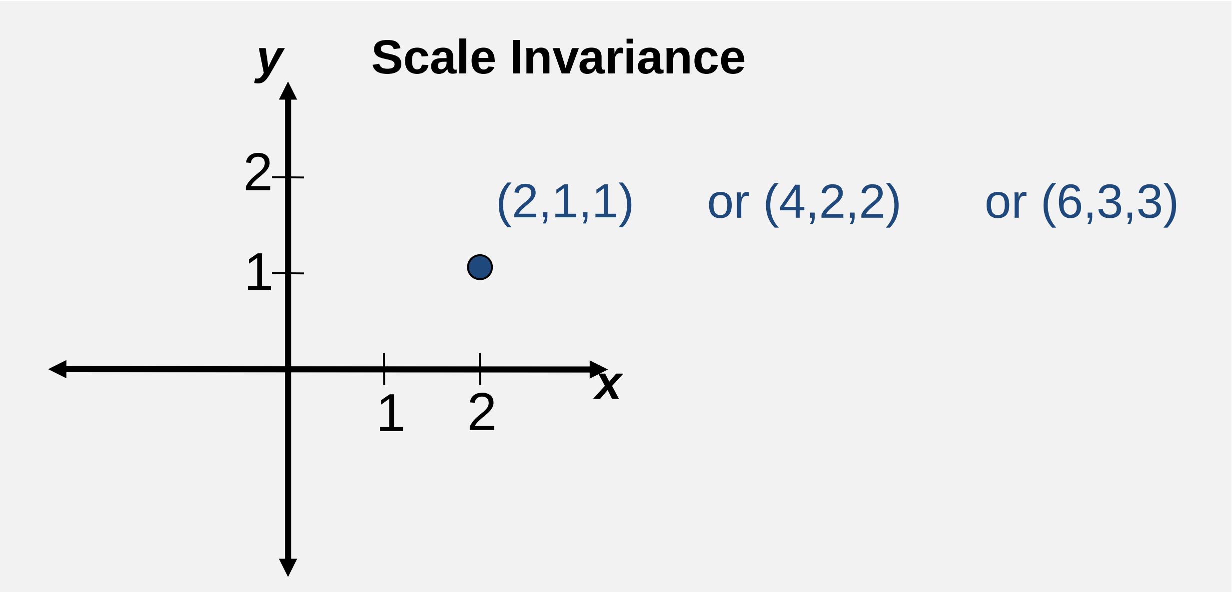
- Append 1 to every 2D point: (x y) → (x y 1)

Homogeneous coordinates → 2D Points

- Divide by third coordinate (x y w) → (x/w y/w)

Special properties

- Scale invariant: (x y w) = k * (x y w)
- (x, y, 0) represents a point at infinity
- (0, 0, 0) is not allowed



Homogeneous Coordinates

Q: How can we represent translation in matrix form?

$$x' = x + t_x$$

$$y' = y + t_y$$

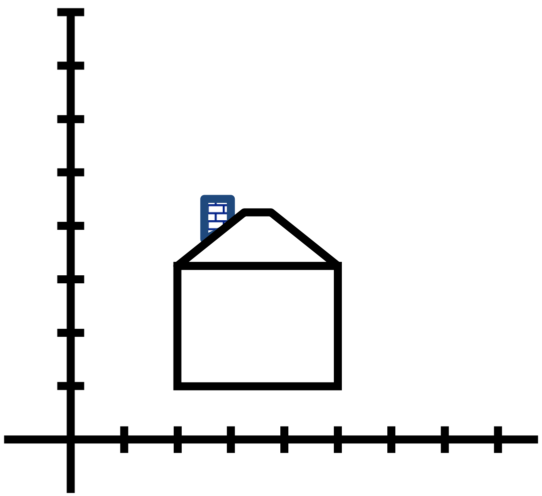
A: Using the rightmost column:

$$\mathbf{Translation} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

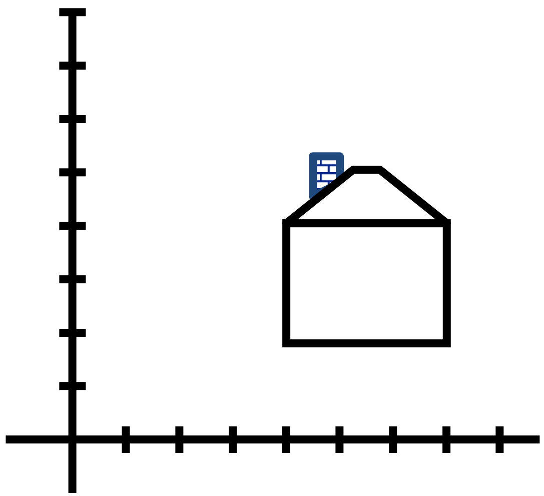
Translation Example

Homogeneous Coordinates

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$



$t_x = 2$
$t_y = 1$



Basic 2D transformations as 3x3 matrices

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{s}_x & 0 & 0 \\ 0 & \boldsymbol{s}_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & \beta_x & 0 \\ \beta_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Shear

Matrix Composition

Transformations can be combined by matrix multiplication

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \left(\begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

$\mathbf{p}'$ = $T(t_x,t_y)$ $R(\Theta)$ $S(s_x,s_y)$ $\mathbf{p}$

Does the order of multiplication matter?

Affine Transformations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Affine transformations are combinations of

- Linear transformations, and
- Translations

Properties of affine transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

Projective Transformations

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

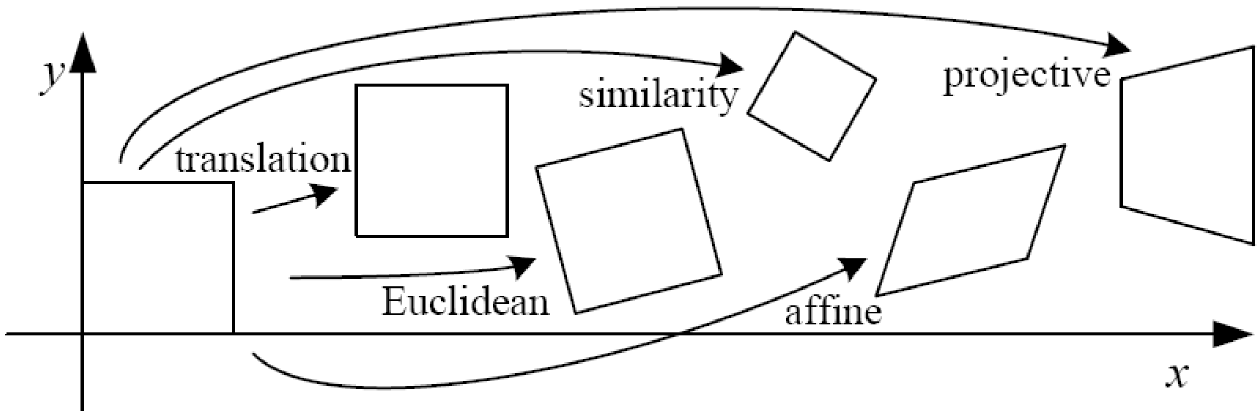
Projective transformations are combos of

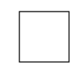
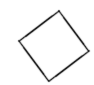
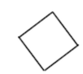
- Affine transformations, and
- Projective warps

Properties of projective transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Ratios are not preserved
- Closed under composition
- Models change of basis
- Projective matrix is defined up to a scale (8 DOF)

2D image transformations

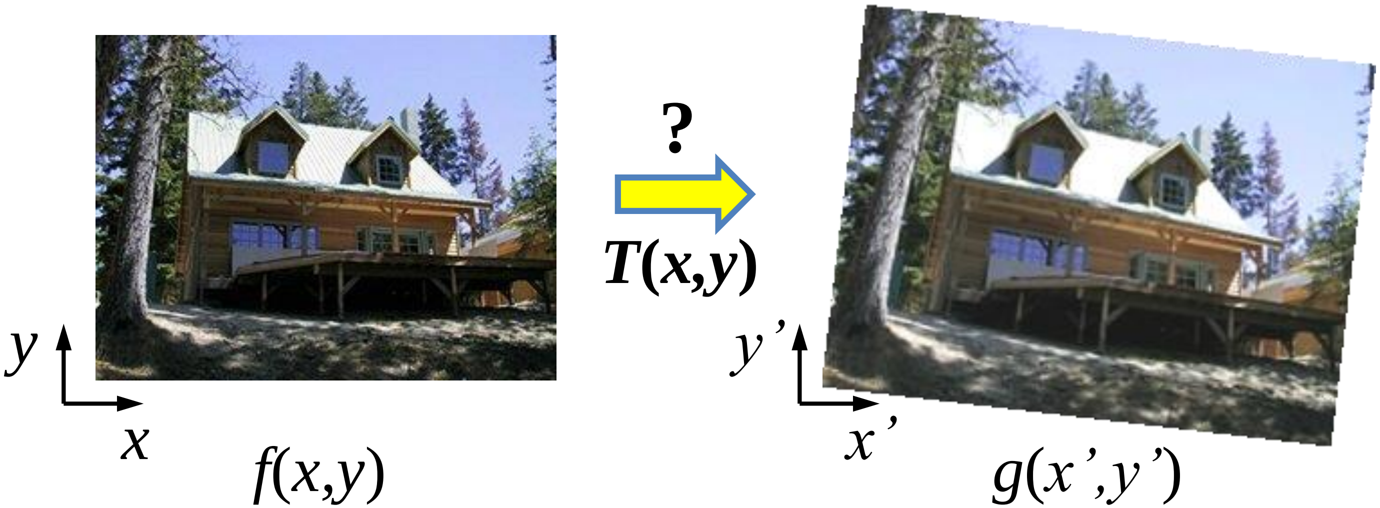


Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} \boldsymbol{I} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$			
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$			
similarity	$\begin{bmatrix} s\boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$			
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{2\times 3}$			
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{3\times 3}$			

These transformations are a nested set of groups

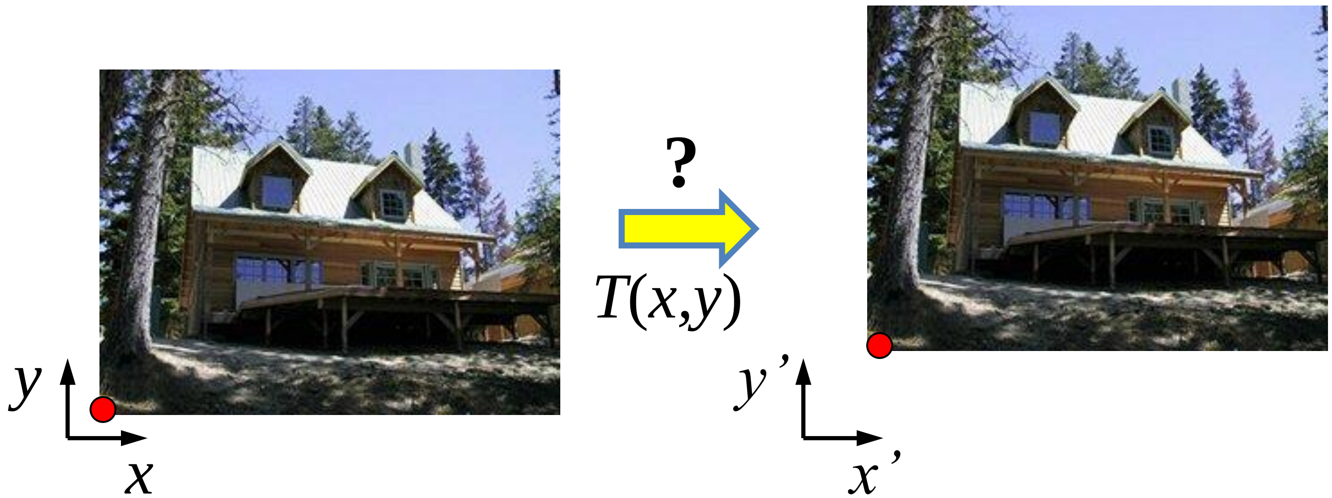
- Closed under composition and inverse is a member

Recovering Transformations



- What if we know *f* and *g* and want to recover the transform T?
  - – willing to let user provide correspondences
    - How many do we need?

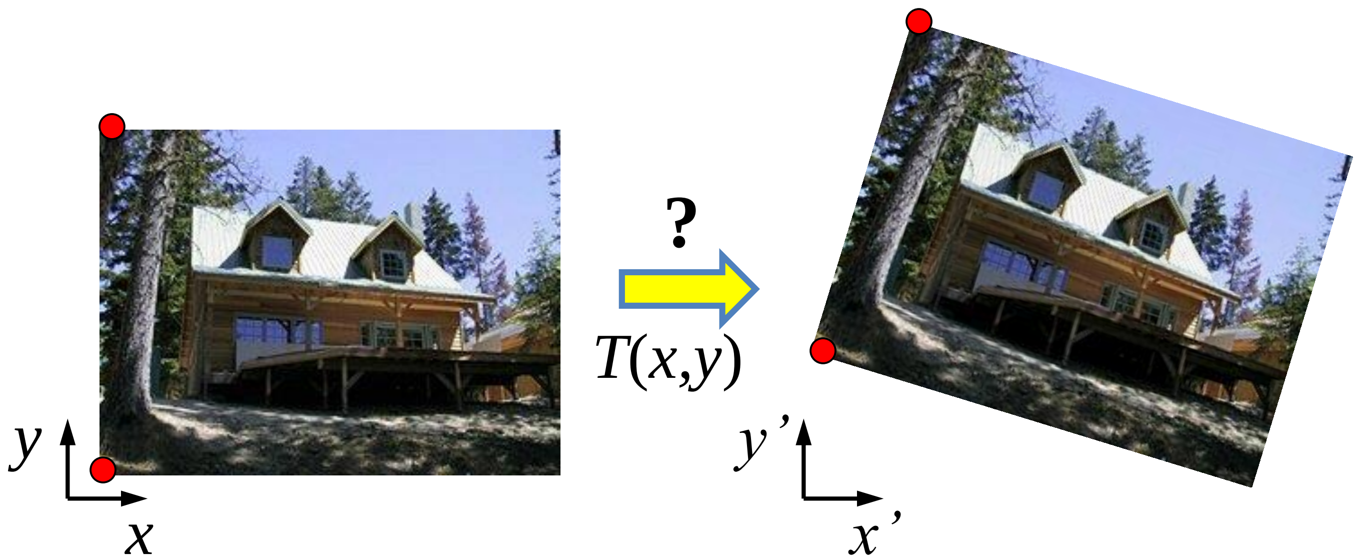
Translation: # correspondences?



- How many Degrees of Freedom?
- How many correspondences needed for translation?
- What is the transformation matrix?

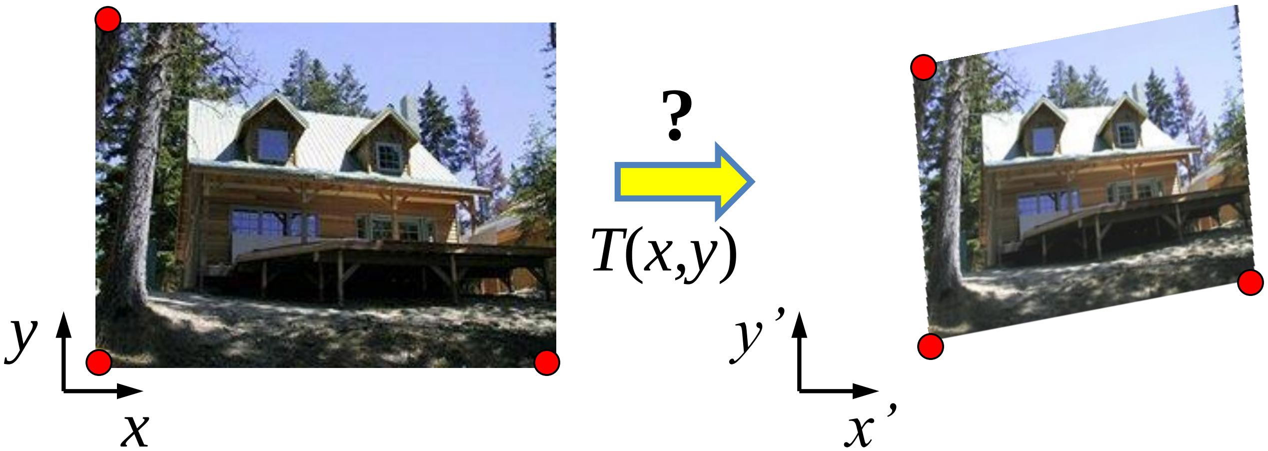
$$\mathbf{M} = \begin{bmatrix} 1 & 0 & p'_x - p_x \\ 0 & 1 & p'_y - p_y \\ 0 & 0 & 1 \end{bmatrix}$$

Euclidian: # correspondences?



- How many DOF?
- How many correspondences needed for translation+rotation?

Affine: # correspondences?

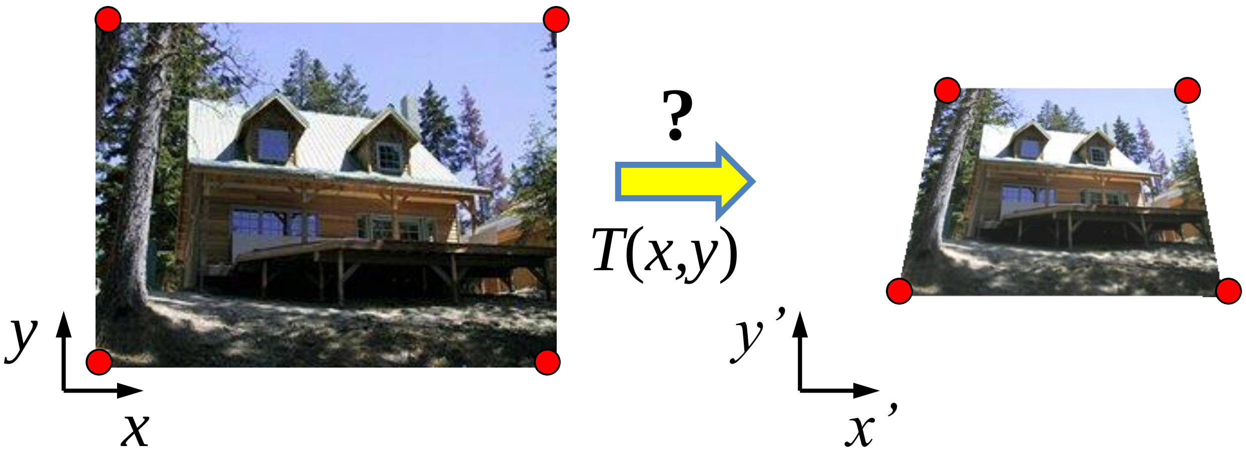


- How many DOF?
- How many correspondences needed for affine?

Affine transformation estimation

- Math
- Matlab demo

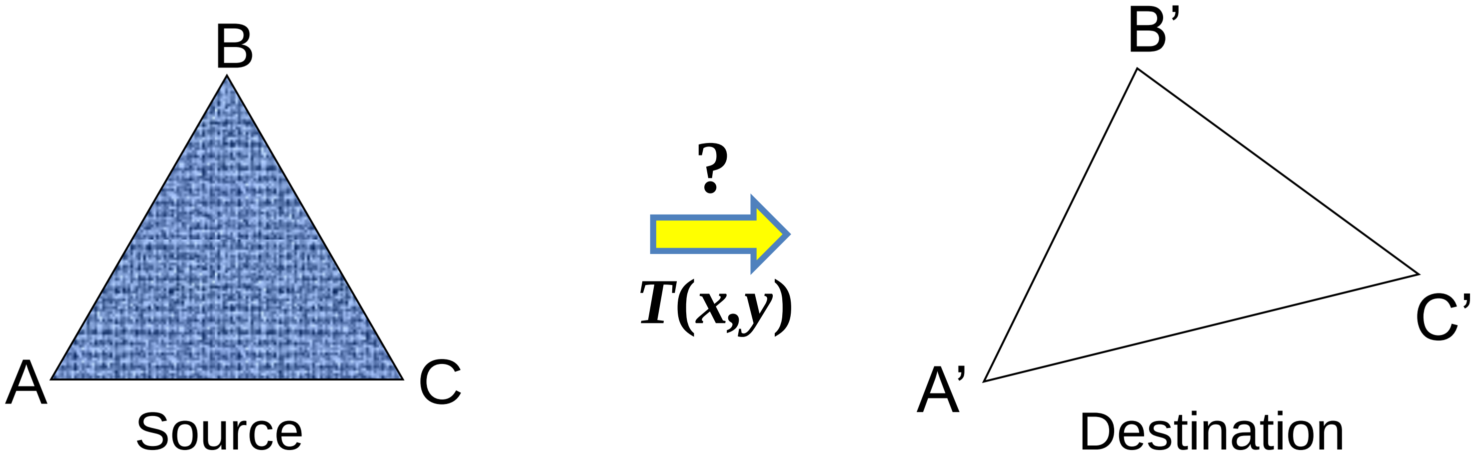
Projective: # correspondences?



- How many DOF?
- How many correspondences needed for projective?

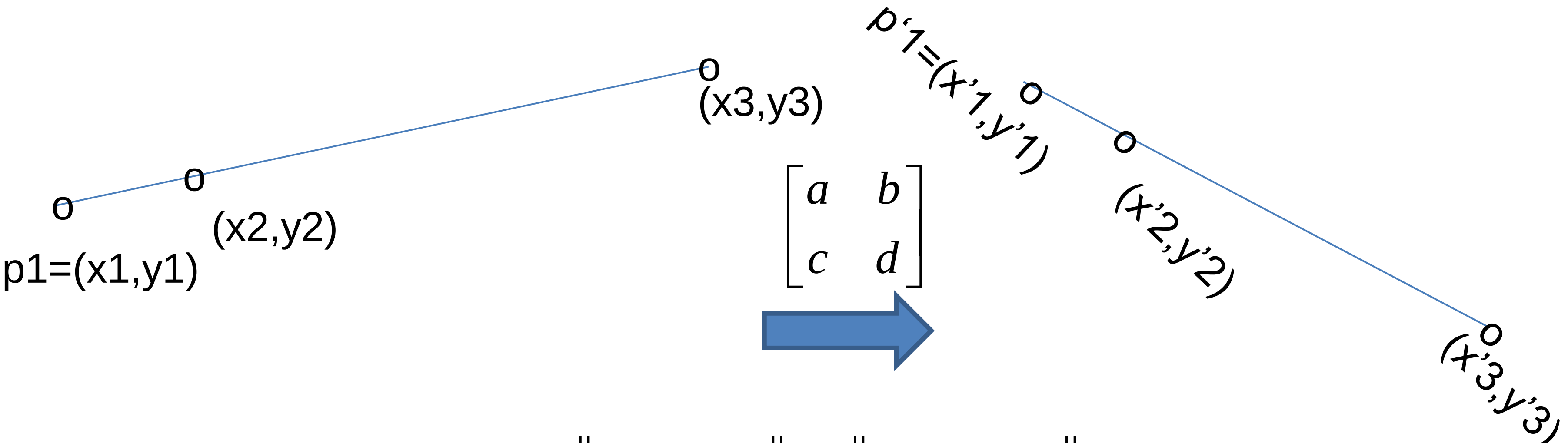
Take-home Question

1) Suppose we have two triangles: ABC and A'B'C'. What transformation will map A to A', B to B', and C to C'? How can we get the parameters?



Take-home Question

2) Show that distance ratios along a line are preserved under 2d linear transformations.



$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\frac{\|p2-p1\|}{\|p3-p1\|}=\frac{\|p'2-p'1\|}{\|p'3-p'1\|}$$

Hint: Write down x2 in terms of x1 and x3, given that the three points are co-linear

Next class: texture mapping and morphing