NP Completeness

Lecture 23
April 21, 2015

Part I

NP-Completeness

P and **NP** and Turing Machines

1. **P**: set of decision problems that have polynomial time algorithms.
2. **NP**: set of decision problems that have polynomial time non-deterministic algorithms.

- Many natural problems we would like to solve are in **NP**.
- Every problem in **NP** has an exponential time algorithm
- **P** $\subseteq$ **NP**
- Some problems in **NP** are in **P** (example, shortest path problem)

Big Question: Does every problem in **NP** have an efficient algorithm? Same as asking whether **P** $=$ **NP**.

“Hardest” Problems

Question

What is the hardest problem in **NP**? How do we define it?

Towards a definition

1. Hardest problem must be in **NP**.
2. Hardest problem must be at least as “difficult” as every other problem in **NP**.

NP-Complete Problems

Definition

A problem $\mathbf{X}$ is said to be **NP-Complete** if

1. $\mathbf{X} \in \mathbf{NP}$, and
2. (Hardness) For any $\mathbf{Y} \in \mathbf{NP}$, $\mathbf{Y} \leq_{\mathbf{P}} \mathbf{X}$.

Solving NP-Complete Problems

Proposition

Suppose $\mathbf{X}$ *is* **NP-Complete**. *Then* $\mathbf{X}$ *can be solved in polynomial time if and only if* $\mathbf{P} = \mathbf{NP}$.

Proof.

$\Rightarrow$ Suppose $\mathbf{X}$ can be solved in polynomial time

1. Let $\mathbf{Y} \in \mathbf{NP}$. We know $\mathbf{Y} \leq_{\mathbf{P}} \mathbf{X}$.
2. We showed that if $\mathbf{Y} \leq_{\mathbf{P}} \mathbf{X}$ and $\mathbf{X}$ can be solved in polynomial time, then $\mathbf{Y}$ can be solved in polynomial time.
3. Thus, every problem $\mathbf{Y} \in \mathbf{NP}$ is such that $\mathbf{Y} \in \mathbf{P}$; $\mathbf{NP} \subseteq \mathbf{P}$.
4. Since $\mathbf{P} \subseteq \mathbf{NP}$, we have $\mathbf{P} = \mathbf{NP}$.

$\Leftarrow$ Since $\mathbf{P} = \mathbf{NP}$, and $\mathbf{X} \in \mathbf{NP}$, we have a polynomial time algorithm for $\mathbf{X}$. $\square$

NP-Hard Problems

Definition

A problem $\mathbf{X}$ is said to be **NP-Hard** if

1. (Hardness) For any $\mathbf{Y} \in \mathbf{NP}$, we have that $\mathbf{Y} \leq_{\mathbf{P}} \mathbf{X}$.

An **NP-Hard** problem need not be in **NP**!

Example: Halting problem is **NP-Hard** (why?) but not **NP-Complete**.

Consequences of proving **NP-Complete**ness

If **X** is **NP-Complete**

1. Since we believe **P** $\neq$ **NP**,
2. and solving **X** implies **P** $=$ **NP**.

X is unlikely to be efficiently solvable.

Consequences of proving NP-Completeness

If **X** is **NP-Complete**

1. Since we believe $\mathbf{P} \neq \mathbf{NP}$,
2. and solving **X** implies $\mathbf{P} = \mathbf{NP}$.

X is unlikely to be efficiently solvable.

At the very least, many smart people before you have failed to find an efficient algorithm for **X**.

Consequences of proving **NP-Complete**ness

If **X** is **NP-Complete**

1. Since we believe **P** $\neq$ **NP**,
2. and solving **X** implies **P** $=$ **NP**.

X is unlikely to be efficiently solvable.

At the very least, many smart people before you have failed to find an efficient algorithm for **X**.

Consequences of proving **NP-Complete**ness

If **X** is **NP-Complete**

1. Since we believe **P** $\neq$ **NP**,
2. and solving **X** implies **P** $=$ **NP**.

X is unlikely to be efficiently solvable.

At the very least, many smart people before you have failed to find an efficient algorithm for **X**.
(This is proof by mob opinion — take with a grain of salt.)

NP-Complete Problems

Question

Are there any problems that are **NP-Complete**?

Answer

Yes! Many, many problems are **NP-Complete**.

Cook-Levin Theorem

Theorem (Cook-Levin)

SAT *is* **NP-Complete***.*

Cook-Levin Theorem

Theorem (Cook-Levin)

SAT *is* **NP-Complete***.*

Need to show

1. **SAT** is in **NP**.
2. *every* **NP** problem **X** reduces to **SAT**.

Will see proof in next lecture.

Steve Cook won the Turing award for his theorem.

Proving that a problem **X** is **NP-Complete**

To prove **X** is **NP-Complete**, show

1. Show that **X** is in **NP**.
2. Give a polynomial-time reduction *from* a known **NP-Complete** problem such as **SAT** *to* **X**

Proving that a problem **X** is **NP-Complete**

To prove **X** is **NP-Complete**, show

1. Show that **X** is in **NP**.
2. Give a polynomial-time reduction *from* a known **NP-Complete** problem such as **SAT** *to* **X**

SAT $\leq_P$ **X** implies that every **NP** problem **Y** $\leq_P$ **X**. Why?

Proving that a problem **X** is **NP-Complete**

To prove **X** is **NP-Complete**, show

1. Show that **X** is in **NP**.
2. Give a polynomial-time reduction *from* a known **NP-Complete** problem such as **SAT** *to* **X**

SAT $\leq_P$ **X** implies that every **NP** problem **Y** $\leq_P$ **X**. Why?
Transitivity of reductions:

Y $\leq_P$ **SAT** and **SAT** $\leq_P$ **X** and hence **Y** $\leq_P$ **X**.

3-SAT is NP-Complete

- **3-SAT** is in **NP**
- **SAT** $\leq_P$ **3-SAT** as we saw

NP-Completeness via Reductions

1. **SAT** is **NP-Complete** due to Cook-Levin theorem
2. **SAT** $\leq_P$ **3-SAT**
3. **3-SAT** $\leq_P$ **Independent Set**
4. **Independent Set** $\leq_P$ **Vertex Cover**
5. **Independent Set** $\leq_P$ **Clique**
6. **3-SAT** $\leq_P$ **3-Color**
7. **3-SAT** $\leq_P$ **Hamiltonian Cycle**

NP-Completeness via Reductions

1. **SAT** is **NP-Complete** due to Cook-Levin theorem
2. **SAT** $\leq_P$ **3-SAT**
3. **3-SAT** $\leq_P$ **Independent Set**
4. **Independent Set** $\leq_P$ **Vertex Cover**
5. **Independent Set** $\leq_P$ **Clique**
6. **3-SAT** $\leq_P$ **3-Color**
7. **3-SAT** $\leq_P$ **Hamiltonian Cycle**

Hundreds and thousands of different problems from many areas of science and engineering have been shown to be **NP-Complete**.

A surprisingly frequent phenomenon!

NP-Completeness via Reductions

Part II

Reducing **3-SAT** to **Independent Set**

Independent Set

Problem: Independent Set

Instance: A graph G, integer $\mathbf{k}$.
Question: Is there an independent set in G of size $\mathbf{k}$?

3SAT $\leq_P$ Independent Set

The reduction 3SAT $\leq_P$ Independent Set

Input: Given a $3\mathrm{CNF}$ formula φ
Goal: Construct a graph $\mathbf{G}_\varphi$ and number $\mathbf{k}$ such that $\mathbf{G}_\varphi$ has an independent set of size $\mathbf{k}$ if and only if φ is satisfiable.

3SAT $\leq_P$ Independent Set

The reduction 3SAT $\leq_P$ Independent Set

Input: Given a $3\mathrm{CNF}$ formula φ
Goal: Construct a graph $\mathbf{G}_\varphi$ and number $\mathbf{k}$ such that $\mathbf{G}_\varphi$ has an independent set of size $\mathbf{k}$ if and only if φ is satisfiable.
$\mathbf{G}_\varphi$ should be constructable in time polynomial in size of φ

3SAT $\leq_P$ Independent Set

The reduction 3SAT $\leq_P$ Independent Set

Input: Given a $3\mathrm{CNF}$ formula φ
Goal: Construct a graph $\mathbf{G}_\varphi$ and number $\mathbf{k}$ such that $\mathbf{G}_\varphi$ has an independent set of size $\mathbf{k}$ if and only if φ is satisfiable.
$\mathbf{G}_\varphi$ should be constructable in time polynomial in size of φ

Importance of reduction: Although **3SAT** is much more expressive, it can be reduced to a seemingly specialized Independent Set problem.

Notice: We handle only $3\mathrm{CNF}$ formulas – reduction would not work for other kinds of boolean formulas.

Interpreting **3SAT**

There are two ways to think about **3SAT**

Interpreting **3SAT**

There are two ways to think about **3SAT**

1. Find a way to assign 0/1 (false/true) to the variables such that the formula evaluates to true, that is each clause evaluates to true.

Interpreting **3SAT**

There are two ways to think about **3SAT**

1. Find a way to assign 0/1 (false/true) to the variables such that the formula evaluates to true, that is each clause evaluates to true.
2. Pick a literal from each clause and find a truth assignment to make all of them true

Interpreting **3SAT**

There are two ways to think about **3SAT**

1. Find a way to assign 0/1 (false/true) to the variables such that the formula evaluates to true, that is each clause evaluates to true.
2. Pick a literal from each clause and find a truth assignment to make all of them true. You will fail if two of the literals you pick are in conflict, i.e., you pick $\mathbf{x_i}$ and $\neg\mathbf{x_i}$

We will take the second view of **3SAT** to construct the reduction.

The Reduction

1. $\mathbf{G}_\varphi$ will have one vertex for each literal in a clause

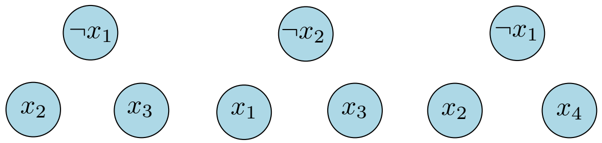


Figure : Graph for
$\varphi = (\neg x_1 \lor x_2 \lor x_3) \land (x_1 \lor \neg x_2 \lor x_3) \land (\neg x_1 \lor x_2 \lor x_4)$

The Reduction

1. $\mathbf{G}_{\varphi}$ will have one vertex for each literal in a clause
2. Connect the 3 literals in a clause to form a triangle; the independent set will pick at most one vertex from each clause, which will correspond to the literal to be set to true

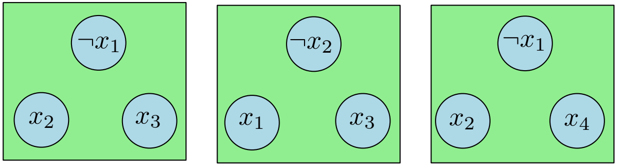


Figure : Graph for
$\varphi = (\neg x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_4)$

The Reduction

1. $\mathbf{G}_\varphi$ will have one vertex for each literal in a clause
2. Connect the 3 literals in a clause to form a triangle; the independent set will pick at most one vertex from each clause, which will correspond to the literal to be set to true

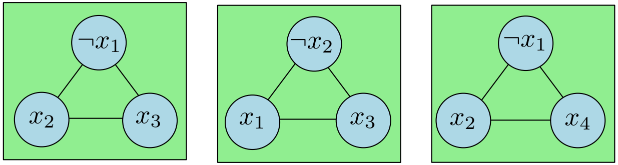


Figure : Graph for
$\varphi = (\neg x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_4)$

The Reduction

1. $\mathbf{G}_\varphi$ will have one vertex for each literal in a clause
2. Connect the 3 literals in a clause to form a triangle; the independent set will pick at most one vertex from each clause, which will correspond to the literal to be set to true
3. Connect 2 vertices if they label complementary literals; this ensures that the literals corresponding to the independent set do not have a conflict

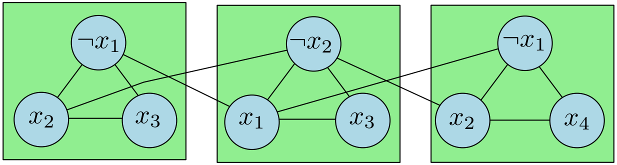


Figure : Graph for
$\varphi = (\neg x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_4)$

The Reduction

1. $\mathbf{G}_\varphi$ will have one vertex for each literal in a clause
2. Connect the 3 literals in a clause to form a triangle; the independent set will pick at most one vertex from each clause, which will correspond to the literal to be set to true
3. Connect 2 vertices if they label complementary literals; this ensures that the literals corresponding to the independent set do not have a conflict
4. Take **k** to be the number of clauses

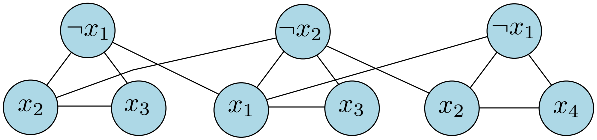


Figure : Graph for
$\varphi = (\neg x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_4)$

Correctness

Proposition

φ is satisfiable iff $\mathbf{G}_\varphi$ has an independent set of size $\mathbf{k}$ ($=$ number of clauses in φ).

Proof.

$\Rightarrow$ Let $\mathbf{a}$ be the truth assignment satisfying φ

Correctness

Proposition

φ is satisfiable iff $\mathbf{G}_\varphi$ has an independent set of size $\mathbf{k}$ ($=$ number of clauses in φ).

Proof.

$\Rightarrow$ Let $\mathbf{a}$ be the truth assignment satisfying φ

1. Pick one of the vertices, corresponding to true literals under $\mathbf{a}$, from each triangle. This is an independent set of the appropriate size. Why? □

Correctness (contd)

Proposition

φ is satisfiable iff $\mathbf{G}_\varphi$ has an independent set of size $\mathbf{k}$ ($=$ number of clauses in φ).

Proof.

$\Leftarrow$ Let $\mathbf{S}$ be an independent set of size $\mathbf{k}$

1. $\mathbf{S}$ must contain *exactly* one vertex from each clause
2. $\mathbf{S}$ cannot contain vertices labeled by conflicting literals
3. Thus, it is possible to obtain a truth assignment that makes in the literals in $\mathbf{S}$ true; such an assignment satisfies one literal in every clause $\square$

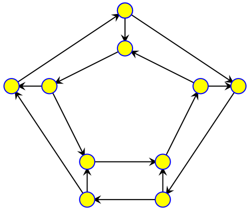
Part III

NP-Completeness of Hamiltonian Cycle

Directed Hamiltonian Cycle

Input Given a directed graph $\mathbf{G} = (\mathbf{V}, \mathbf{E})$ with $\mathbf{n}$ vertices

Goal Does $\mathbf{G}$ have a Hamiltonian cycle?

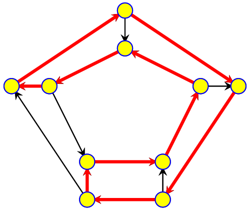


Directed Hamiltonian Cycle

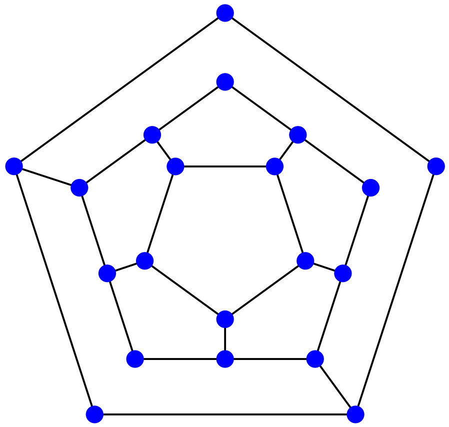
Input Given a directed graph $\mathbf{G = (V, E)}$ with $\mathbf{n}$ vertices

Goal Does $\mathbf{G}$ have a Hamiltonian cycle?

- A Hamiltonian cycle is a cycle in the graph that visits every vertex in $\mathbf{G}$ exactly once



Is the following graph Hamiltonianan?



(A) Yes.

(B) No.

Directed Hamiltonian Cycle is **NP-Complete**

- Directed Hamiltonian Cycle is in **NP**: exercise
- Hardness: We will show
  3-SAT $\leq_P$ Directed Hamiltonian Cycle

Reduction

Given 3-SAT formula φ create a graph $\mathbf{G}_\varphi$ such that

- $\mathbf{G}_\varphi$ has a Hamiltonian cycle if and only if φ is satisfiable
- $\mathbf{G}_\varphi$ should be constructible from φ by a polynomial time algorithm $\mathcal{A}$

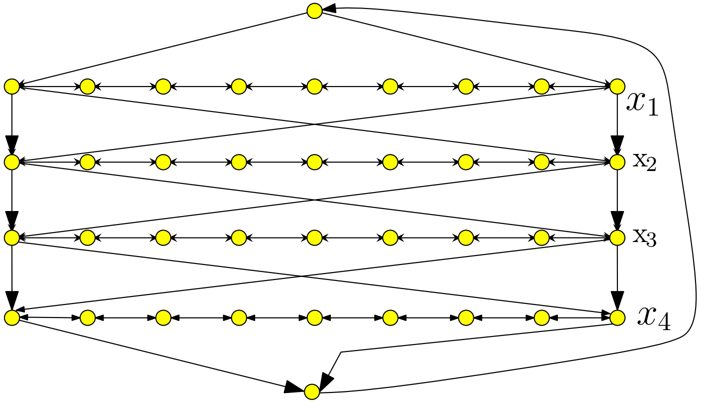
Notation: φ has $\mathbf{n}$ variables $\mathbf{x_1, x_2, \ldots, x_n}$ and $\mathbf{m}$ clauses $\mathbf{C_1, C_2, \ldots, C_m}$.

Reduction: First Ideas

- Viewing SAT: Assign values to $\mathbf{n}$ variables, and each clauses has 3 ways in which it can be satisfied.
- Construct graph with $\mathbf{2^n}$ Hamiltonian cycles, where each cycle corresponds to some boolean assignment.
- Then add more graph structure to encode constraints on assignments imposed by the clauses.

The Reduction: Phase I

- Traverse path $\mathbf{i}$ from left to right iff $\mathbf{x_i}$ is set to true
- Each path has $\mathbf{3(m+1)}$ nodes where $\mathbf{m}$ is number of clauses in φ; nodes numbered from left to right ($\mathbf{1}$ to $\mathbf{3m+3}$)

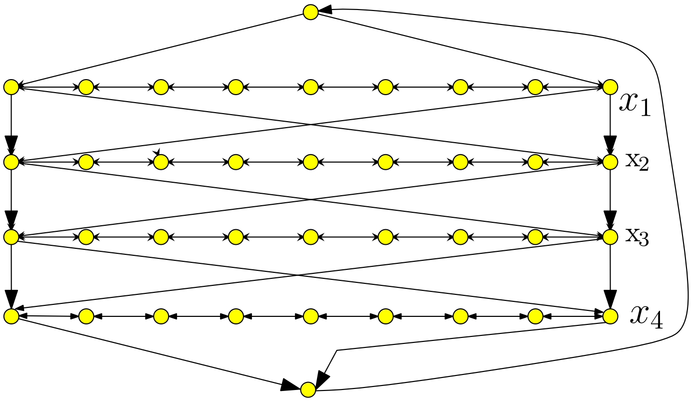


The Reduction: Phase II

- Add vertex $\mathbf{c_j}$ for clause $\mathbf{C_j}$. $\mathbf{c_j}$ has edge *from* vertex $\mathbf{3j}$ and *to* vertex $\mathbf{3j+1}$ on path $\mathbf{i}$ if $\mathbf{x_i}$ appears in clause $\mathbf{C_j}$, and has edge *from* vertex $\mathbf{3j+1}$ and *to* vertex $\mathbf{3j}$ if $\mathbf{\neg x_i}$ appears in $\mathbf{C_j}$.

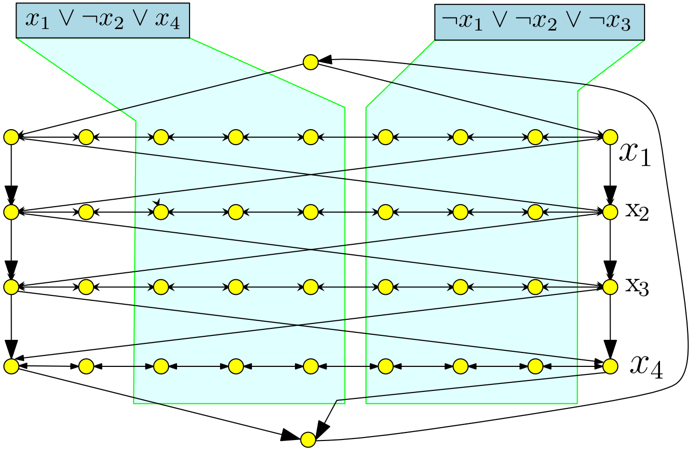
$x_1 \vee \neg x_2 \vee x_4$

$\neg x_1 \vee \neg x_2 \vee \neg x_3$



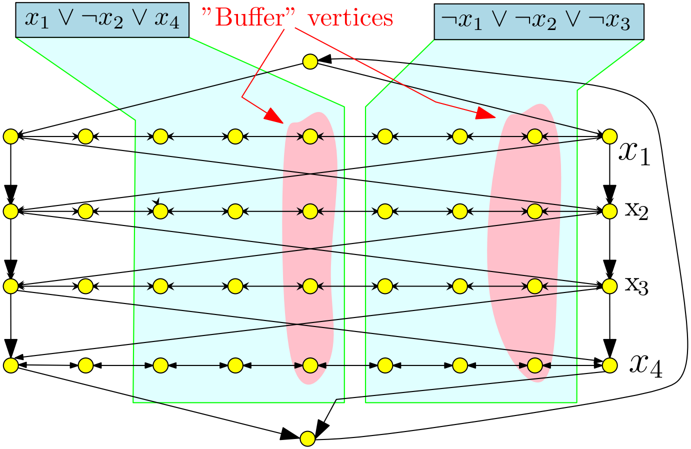
The Reduction: Phase II

- Add vertex $\mathbf{c_j}$ for clause $\mathbf{C_j}$. $\mathbf{c_j}$ has edge *from* vertex $\mathbf{3j}$ and *to* vertex $\mathbf{3j+1}$ on path $\mathbf{i}$ if $\mathbf{x_i}$ appears in clause $\mathbf{C_j}$, and has edge *from* vertex $\mathbf{3j+1}$ and *to* vertex $\mathbf{3j}$ if $\mathbf{\neg x_i}$ appears in $\mathbf{C_j}$.


$x_1 \vee \neg x_2 \vee x_4$
$\neg x_1 \vee \neg x_2 \vee \neg x_3$
x_1
x_2
x_3
x_4

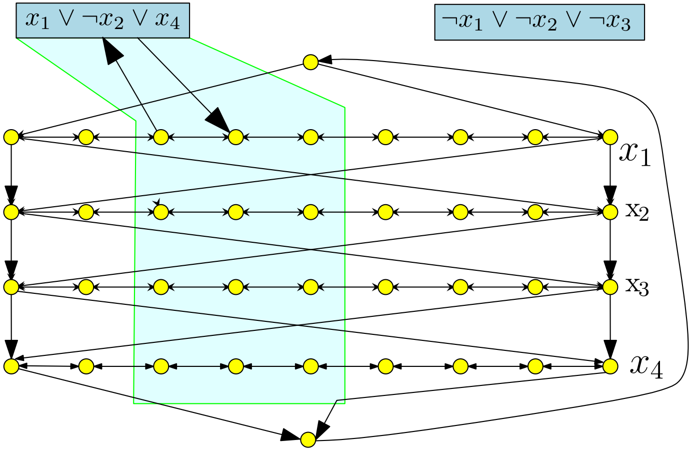
The Reduction: Phase II

- Add vertex $\mathbf{c_j}$ for clause $\mathbf{C_j}$. $\mathbf{c_j}$ has edge *from* vertex $\mathbf{3j}$ and *to* vertex $\mathbf{3j+1}$ on path $\mathbf{i}$ if $\mathbf{x_i}$ appears in clause $\mathbf{C_j}$, and has edge *from* vertex $\mathbf{3j+1}$ and *to* vertex $\mathbf{3j}$ if $\mathbf{\neg x_i}$ appears in $\mathbf{C_j}$.


$x_1 \vee \neg x_2 \vee x_4$
"Buffer" vertices
$\neg x_1 \vee \neg x_2 \vee \neg x_3$
x_1
x_2
x_3
x_4

The Reduction: Phase II

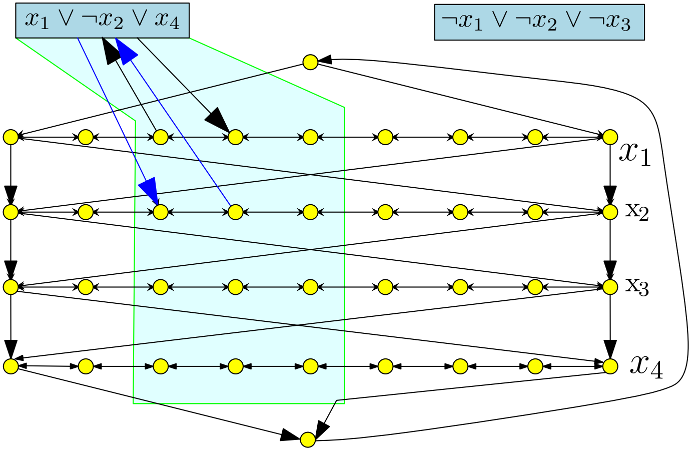
- Add vertex $\mathbf{c_j}$ for clause $\mathbf{C_j}$. $\mathbf{c_j}$ has edge *from* vertex $\mathbf{3j}$ and *to* vertex $\mathbf{3j+1}$ on path $\mathbf{i}$ if $\mathbf{x_i}$ appears in clause $\mathbf{C_j}$, and has edge *from* vertex $\mathbf{3j+1}$ and *to* vertex $\mathbf{3j}$ if $\mathbf{\neg x_i}$ appears in $\mathbf{C_j}$.



$x_1 \vee \neg x_2 \vee x_4$
$\neg x_1 \vee \neg x_2 \vee \neg x_3$
x_1
x_2
x_3
x_4

The Reduction: Phase II

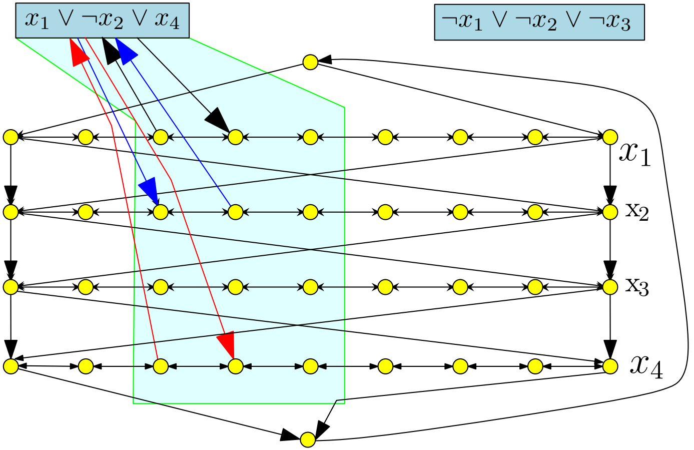
- Add vertex $\mathbf{c_j}$ for clause $\mathbf{C_j}$. $\mathbf{c_j}$ has edge *from* vertex $\mathbf{3j}$ and *to* vertex $\mathbf{3j+1}$ on path $\mathbf{i}$ if $\mathbf{x_i}$ appears in clause $\mathbf{C_j}$, and has edge *from* vertex $\mathbf{3j+1}$ and *to* vertex $\mathbf{3j}$ if $\mathbf{\neg x_i}$ appears in $\mathbf{C_j}$.



$x_1 \lor \neg x_2 \lor x_4$
$\neg x_1 \lor \neg x_2 \lor \neg x_3$
x_1
x_2
x_3
x_4

The Reduction: Phase II

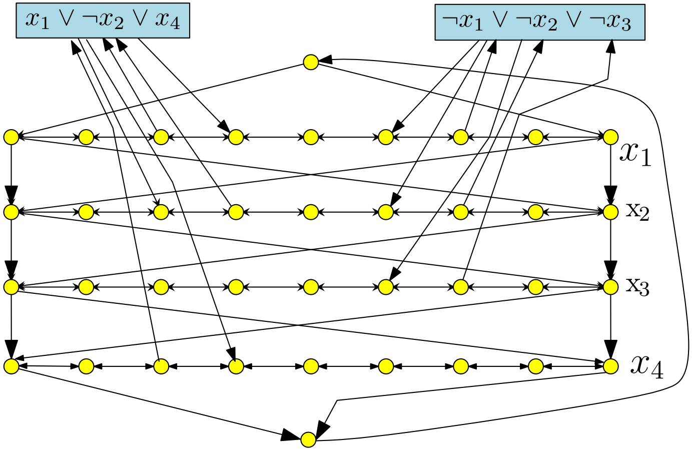
- Add vertex $\mathbf{c_j}$ for clause $\mathbf{C_j}$. $\mathbf{c_j}$ has edge *from* vertex $\mathbf{3j}$ and *to* vertex $\mathbf{3j+1}$ on path $\mathbf{i}$ if $\mathbf{x_i}$ appears in clause $\mathbf{C_j}$, and has edge *from* vertex $\mathbf{3j+1}$ and *to* vertex $\mathbf{3j}$ if $\mathbf{\neg x_i}$ appears in $\mathbf{C_j}$.



$x_1 \vee \neg x_2 \vee x_4$
$\neg x_1 \vee \neg x_2 \vee \neg x_3$
x_1
x_2
x_3
x_4

The Reduction: Phase II

- Add vertex $\mathbf{c_j}$ for clause $\mathbf{C_j}$. $\mathbf{c_j}$ has edge *from* vertex $\mathbf{3j}$ and *to* vertex $\mathbf{3j+1}$ on path $\mathbf{i}$ if $\mathbf{x_i}$ appears in clause $\mathbf{C_j}$, and has edge *from* vertex $\mathbf{3j+1}$ and *to* vertex $\mathbf{3j}$ if $\mathbf{\neg x_i}$ appears in $\mathbf{C_j}$.



$x_1 \vee \neg x_2 \vee x_4$
$\neg x_1 \vee \neg x_2 \vee \neg x_3$
x_1
x_2
x_3
x_4

Correctness Proof

Proposition

φ has a satisfying assignment iff $\mathbf{G}_\varphi$ has a Hamiltonian cycle.

Proof.

$\Rightarrow$ Let $\mathbf{a}$ be the satisfying assignment for φ. Define Hamiltonian cycle as follows

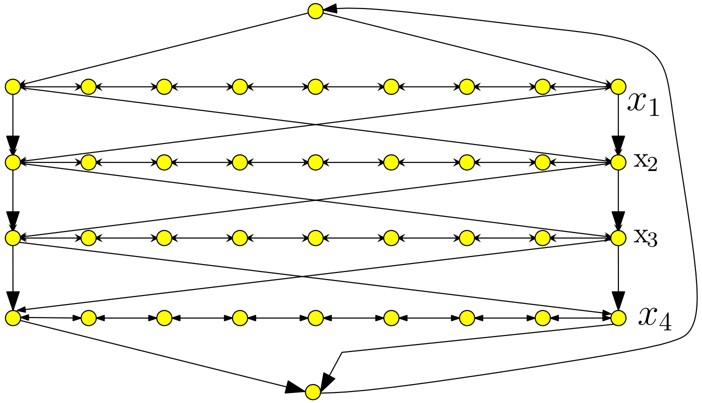
- If $\mathbf{a(x_i) = 1}$ then traverse path $\mathbf{i}$ from left to right
- If $\mathbf{a(x_i) = 0}$ then traverse path $\mathbf{i}$ from right to left
- For each clause, path of at least one variable is in the "right" direction to splice in the node corresponding to clause □

Hamiltonian Cycle $\Rightarrow$ Satisfying assignment

Suppose Π is a Hamiltonian cycle in G_φ

- If Π enters c_j (vertex for clause C_j) from vertex $3j$ on path i then it must leave the clause vertex on edge to $3j+1$ on the *same path* i
  - If not, then only unvisited neighbor of $3j+1$ on path i is $3j+2$
  - Thus, we don't have two unvisited neighbors (one to enter from, and the other to leave) to have a Hamiltonian Cycle
- Similarly, if Π enters c_j from vertex $3j+1$ on path i then it must leave the clause vertex c_j on edge to $3j$ on path i

Example



Hamiltonian Cycle $\Longrightarrow$ Satisfying assignment (contd)

- Thus, vertices visited immediately before and after $\mathbf{C_i}$ are connected by an edge
- We can remove $\mathbf{c_j}$ from cycle, and get Hamiltonian cycle in $\mathbf{G - c_j}$
- Consider Hamiltonian cycle in $\mathbf{G - \{c_1, \ldots c_m\}}$; it traverses each path in only one direction, which determines the truth assignment

Hamiltonian Cycle

Problem

Input *Given undirected graph* $\mathbf{G} = (\mathbf{V}, \mathbf{E})$

Goal *Does* $\mathbf{G}$ *have a Hamiltonian cycle? That is, is there a cycle that visits every vertex exactly one (except start and end vertex)?*

NP-Completeness

Theorem

Hamiltonian cycle *problem for* **undirected** *graphs is* **NP-Complete***.*

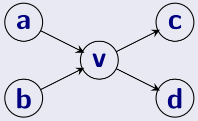
Proof.

- The problem is in **NP**; proof left as exercise.
- Hardness proved by reducing Directed Hamiltonian Cycle to this problem □

Reduction Sketch

Goal: Given directed graph **G**, need to construct undirected graph **G′** such that **G** has Hamiltonian Path iff **G′** has Hamiltonian path

Reduction

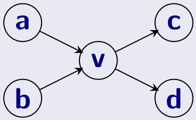


Reduction Sketch

Goal: Given directed graph $\mathbf{G}$, need to construct undirected graph $\mathbf{G}'$ such that $\mathbf{G}$ has Hamiltonian Path iff $\mathbf{G}'$ has Hamiltonian path

Reduction

- Replace each vertex $\mathbf{v}$ by 3 vertices: $\mathbf{v_{in}}$, $\mathbf{v}$, and $\mathbf{v_{out}}$

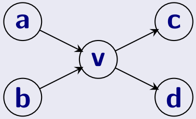


Reduction Sketch

Goal: Given directed graph $\mathbf{G}$, need to construct undirected graph $\mathbf{G}'$ such that $\mathbf{G}$ has Hamiltonian Path iff $\mathbf{G}'$ has Hamiltonian path

Reduction

- Replace each vertex $\mathbf{v}$ by 3 vertices: $\mathbf{v_{in}}$, $\mathbf{v}$, and $\mathbf{v_{out}}$
- A directed edge $(\mathbf{a}, \mathbf{b})$ is replaced by edge $(\mathbf{a_{out}}, \mathbf{b_{in}})$

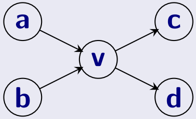


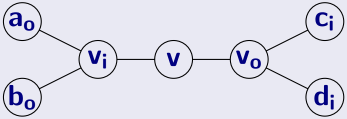
Reduction Sketch

Goal: Given directed graph $\mathbf{G}$, need to construct undirected graph $\mathbf{G'}$ such that $\mathbf{G}$ has Hamiltonian Path iff $\mathbf{G'}$ has Hamiltonian path

Reduction

- Replace each vertex $\mathbf{v}$ by 3 vertices: $\mathbf{v_{in}}$, $\mathbf{v}$, and $\mathbf{v_{out}}$
- A directed edge $(\mathbf{a}, \mathbf{b})$ is replaced by edge $(\mathbf{a_{out}}, \mathbf{b_{in}})$





Reduction: Wrapup

- The reduction is polynomial time (exercise)
- The reduction is correct (exercise)