1. Design a two-tape TM that computes the function $f(x) = x/2$, assuming x is even. More specifically, when started in the initial state scanning the first 0 in a block of $2k$ consecutive 0s on tape 1, the TM halts scanning the first 0 in a block of exactly k consecutive 0s on tape 2.

   Recall that a transition of a 2-tape machine is of the form $\delta(p, a, b) = (q, a', b', D_1, D_2)$, indicating that if the machine is in state p scanning the symbols a and b on tapes 1 and 2 respectively, then it writes the symbols a' and b' on tapes 1 and 2 respectively, moves the heads on tapes 1 and 2 in directions D_1 and D_2 respectively, and transitions to state q. (Here, each D_i is one of {L,R,S}.)

   When programming, you can use a variable symbol to indicate "any character" as follows. Suppose that regardless of the symbol scanned on tape 2, if M is in state p and is scanning a 0 on tape 1, then it should transition to state q, write a 1 on tape 1, move right on tape 1, and leave tape 2 untouched, then you could write $\delta(p, 0, x) = (q, 1, x, R, S)$ and it is understood that this is describing a set of transitions - one for each possible value of x.

   Think first of a high-level strategy and describe it carefully. Your TM probably should not need more than a few states.

2. Give a detailed description (but not the code) of a multitape TM that computes the function $f(n) = \lceil \log n \rceil$. (For simplicity, assume $n = 2^k$ for some value of k, so that the ceiling function is not needed.) Your TM should start with tape 1 holding $\$0^{2^k}$ and halt with tape 2 holding $\$0^k$ In your description, answer the following questions: How many tapes? What are their uses? What are the main phases of the computation? What are the states needed? What are their main functions? Your description can use phrases such as "copy tape 2 to tape 3", or "move across the tape until you find the first 1, and mark it". Give enough details that a TM programmer could implement it easily.