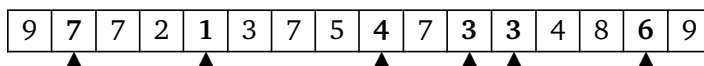


Here are several problems that are easy to solve in $O(n)$ time, essentially by brute force. Your task is to design algorithms for these problems that are significantly faster.

1. (a) Suppose $A[1..n]$ is an array of n distinct integers, sorted so that $A[1] < A[2] < \dots < A[n]$. Each integer $A[i]$ could be positive, negative, or zero. Describe a fast algorithm that either computes an index i such that $A[i] = i$ or correctly reports that no such index exists..
- (b) Now suppose $A[1..n]$ is a sorted array of n distinct **positive** integers. Describe an even faster algorithm that either computes an index i such that $A[i] = i$ or correctly reports that no such index exists. [*Hint: This is really easy.*]
2. Suppose we are given an array $A[1..n]$ such that $A[1] \geq A[2]$ and $A[n-1] \leq A[n]$. We say that an element $A[x]$ is a **local minimum** if both $A[x-1] \geq A[x]$ and $A[x] \leq A[x+1]$. For example, there are exactly six local minima in the following array:



Describe and analyze a fast algorithm that returns the index of one local minimum. For example, given the array above, your algorithm could return the integer 5, because $A[5]$ is a local minimum. [*Hint: With the given boundary conditions, any array must contain at least one local minimum. Why?*]

3. (a) Suppose you are given two sorted arrays $A[1..n]$ and $B[1..n]$ containing distinct integers. Describe a fast algorithm to find the median (meaning the n th smallest element) of the union $A \cup B$. For example, given the input

$$A[1..8] = [0, 1, 6, 9, 12, 13, 18, 20] \quad B[1..8] = [2, 4, 5, 8, 17, 19, 21, 23]$$

your algorithm should return the integer 9. [*Hint: What can you learn by comparing one element of A with one element of B ?*]

- (b) **To think about on your own:** Now suppose you are given two sorted arrays $A[1..m]$ and $B[1..n]$ and an integer k . Describe a fast algorithm to find the k th smallest element in the union $A \cup B$. For example, given the input

$$A[1..8] = [0, 1, 6, 9, 12, 13, 18, 20] \quad B[1..5] = [2, 5, 7, 17, 19] \quad k = 6$$

your algorithm should return the integer 7.