Jeff showed the context-free grammars in class on Tuesday; in each example, the grammar itself is on the left; the explanation for each non-terminal is on the right.

- Properly nested strings of parentheses.

  $$S \to \varepsilon \mid S\texttt{(}S\texttt{)} \qquad \text{properly nested parentheses}$$

  Here is a different grammar for the same language:

  $$S \to \varepsilon \mid \texttt{(}S\texttt{)} \mid SS \qquad \text{properly nested parentheses}$$

- $\{\texttt{0}^m\texttt{1}^n \mid m \neq n\}$. This is the set of all binary strings composed of some number of `0`s followed by a different number of `1`s.

$S \to A \mid B$	all strings $\texttt{0}^m\texttt{1}^n$ where $m \neq n$
$A \to \texttt{0}A \mid \texttt{0}C$	all strings $\texttt{0}^m\texttt{1}^n$ where $m > n$
$B \to B\texttt{1} \mid C\texttt{1}$	all strings $\texttt{0}^m\texttt{1}^n$ where $m < m$
$C \to \varepsilon \mid \texttt{0}C\texttt{1}$	all strings $\texttt{0}^n\texttt{1}^n$ for some integer n

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Give context-free grammars for each of the following languages. For each grammar, describe *in English* the language for each non-terminal, and in the examples above. As usual, we won't get to all of these in section.

1. Binary palindromes: Strings over $\{\texttt{0}, \texttt{1}\}$ that are equal to their reversals. For example: `00111100` and `0100010`, but not `01100`.

2. $\{\texttt{0}^{2n}\texttt{1}^n \mid n \geq 0\}$

3. $\{\texttt{0}^m\texttt{1}^n \mid m \neq 2n\}$

4. $\{\texttt{0}, \texttt{1}\}^* \setminus \{\texttt{0}^{2n}\texttt{1}^n \mid n \geq 0\}$

5. Strings of properly nested parentheses `()`, brackets `[]`, and braces `{}`. For example, the string `([]){}` is in this language, but the string `([)]` is not, because the left and right delimiters don't match.

6. Strings over $\{\texttt{0}, \texttt{1}\}$ where the number of `0`s is equal to the number of `1`s.

7. Strings over $\{\texttt{0}, \texttt{1}\}$ where the number of `0`s is *not* equal to the number of `1`s.