"CS 374" Fall 2014 — Homework 8
Due Tuesday, November 4, 2014 at noon

1. After a grueling algorithms midterm, you decide to take the bus home. Since you planned ahead, you have a schedule that lists the times and locations of every stop of every bus in Champaign-Urbana. Champaign-Urbana is currently suffering from a plague of zombies, so even though the bus stops have fences that *supposedly* keep the zombies out, you'd still like to spend as little time waiting at bus stops as possible. Unfortunately, there isn't a single bus that visits both your exam building and your home; you must transfer between buses at least once.

   Describe and analyze an algorithm to determine a sequence of bus rides from Siebel to your home, that minimizes the total time you spend waiting at bus stops. You can assume that there are b different bus lines, and each bus stops n times per day. Assume that the buses run exactly on schedule, that you have an accurate watch, and that walking between bus stops is too dangerous to even contemplate.

2. It is well known that the global economic collapse of 2017 was caused by computer scientists indiscriminately abusing weaknesses in the currency exchange market. *Arbitrage* was a money-making scheme that takes advantage of inconsistencies in currency exchange rates. Suppose a currency trader with $1,000,000 discovered that 1 US dollar could be traded for 120 Japanese yen, 1 yen could be traded for 0.01 euros, and 1 euro could be traded for 1.2 US dollars. Then by converting his money from dollars to yen, then from yen to euros, and finally from euros back to dollars, the trader could instantly turn his $1,000,000 into $1,440,000! The cycle of currencies $\$ \to \yen \to \euro \to \$$ was called an ***arbitrage cycle***. Finding and exploiting arbitrage cycles before the prices were corrected required extremely fast algorithms. Of course, now that the entire world uses plastic bags as currency, such abuse is impossible.

   Suppose n different currencies are traded in the global currency market. You are given a two-dimensional array $Exch[1..n, 1..n]$ of exchange rates between every pair of currencies; for all indices i and j, one unit of currency i buys $Exch[i, j]$ units of currency j. (Do *not* assume that $Exch[i,j] \cdot Exch[j,i] = 1$.)

   (a) Describe an algorithm that computes an array $Most[1..n]$, where $Most[i]$ is the largest amount of currency i that you can obtain by trading, starting with one unit of currency 1, assuming there are no arbitrage cycles.

   (b) Describe an algorithm to determine whether the given array of currency exchange rates creates an arbitrage cycle.

3. Describe and analyze and algorithm to find the *second smallest spanning tree* of a given undirected graph G with weighted edges, that is, the spanning tree of G with smallest total weight except for the minimum spanning tree. Because the minimum spanning tree is haunted, or something.

CS 374 Fall 2014 — Homework 8 Problem 1

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Describe and analyze an algorithm to determine a sequence of bus rides from Siebel to your home, that minimizes the total time you spend waiting at bus stops.

CS 374 Fall 2014 — Homework 8 Problem 2

Name:	NetID:
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(a) Describe an algorithm that computes an array $Most[1..n]$, where $Most[i]$ is the largest amount of currency i that you can obtain by trading, starting with one unit of currency 1, assuming there are no arbitrage cycles.

(b) Describe an algorithm to determine whether the given array of currency exchange rates creates an arbitrage cycle.

CS 374 Fall 2014 — Homework 8 Problem 3

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Describe and analyze and algorithm to find the *second smallest spanning tree* of a given undirected graph G with weighted edges, that is, the spanning tree of G with smallest total weight except for the minimum spanning tree.