"CS 374" Fall 2014 — Homework 10

Due Tuesday, December 2, 2014 at noon

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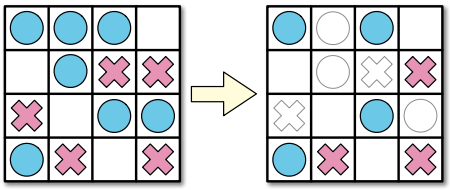
1. Consider the following problem, called BOXDEPTH: Given a set of n axis-aligned rectangles in the plane, how big is the largest subset of these rectangles that contain a common point?

   (a) Describe a polynomial-time reduction from BOXDEPTH to MAXCLIQUE.

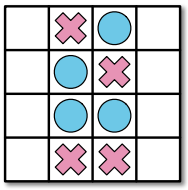
   (b) Describe and analyze a polynomial-time algorithm for BOXDEPTH. *[Hint: Don't try to optimize the running time; $O(n^3)$ is good enough.]*

   (c) Why don't these two results imply that P=NP?

2. Consider the following solitaire game. The puzzle consists of an $n \times m$ grid of squares, where each square may be empty, occupied by a red stone, or occupied by a blue stone. The goal of the puzzle is to remove some of the given stones so that the remaining stones satisfy two conditions: (1) every row contains at least one stone, and (2) no column contains stones of both colors. For some initial configurations of stones, reaching this goal is impossible.



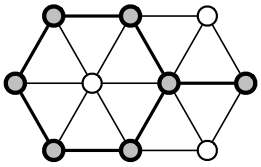
A solvable puzzle and one of its many solutions.



An unsolvable puzzle.

   Prove that it is NP-hard to determine, given an initial configuration of red and blue stones, whether this puzzle can be solved.

3. A subset S of vertices in an undirected graph G is called ***triangle-free*** if, for every triple of vertices $u, v, w \in S$, at least one of the three edges uv, uw, vw is *absent* from G. Prove that finding the size of the largest triangle-free subset of vertices in a given undirected graph is NP-hard.



A triangle-free subset of 7 vertices.
This is ***not*** the largest triangle-free subset in this graph.

   In addition to submitting paper solutions, please also electronically submit your solution to this problem on CrowdGrader.

4. **[Extra credit]** Describe a direct polynomial-time reduction from 4COLOR to 3COLOR. (This is significantly harder than the opposite direction, which you'll see in lab on Wednesday. Don't go through the Cook-Levin Theorem.)

CS 374 Fall 2014 — Homework 10 Problem 1

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(a) Describe a polynomial-time reduction from BOXDEPTH to MAXCLIQUE.

(b) Describe and analyze a polynomial-time algorithm for BOXDEPTH. *[Hint: Don't try to optimize the running time; $O(n^3)$ is good enough.]*

(c) Why don't these two results imply that P=NP?

CS 374 Fall 2014 — Homework 10 Problem 2

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Prove that it is NP-hard to determine, given an initial configuration of red and blue stones, whether the puzzle can be solved.

CS 374 Fall 2014 — Homework 10 Problem 3

Name:	NetID:
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Prove that finding the size of the largest triangle-free subset of vertices in a given undirected graph is NP-hard.

CS 374 Fall 2014 — Homework 10 Problem 4

[Extra Credit]

Name:	NetID:
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Describe a direct polynomial-time reduction from 4COLOR to 3COLOR.
