"CS 374" Fall 2014 — Homework 1

Due Tuesday, September 9, 2014 at noon

Groups of up to three students may submit common solutions for each problem in this homework and in all future homeworks. You are responsible for forming you own groups; you are welcome to advertise for group members on Piazza. You need not use the same group for every homework, or even for every problem in a single homework. Please clearly print the names and NetIDs of each of your group members at the top of each submitted solution, along with ***one*** discussion section where we should return your graded work. If you submit hand-written solutions, please use the last three pages of this homework as templates.

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1. Give regular expressions for each of the following languages over the alphabet $\{0, 1\}$. You do not need to prove your answers are correct.

   (a) All strings with an odd number of 1s.

   (b) All strings with at most three 0s.

   (c) All strings that do not contain the substring 010.

   (d) All strings in which every occurrence of the substring 00 occurs before every occurrence of the substring 11.

2. Recall that the ***reversal*** w^R of a string w is defined recursively as follows:

$$w^R = \begin{cases} \varepsilon & \text{if } w = \varepsilon \\ x \bullet a & \text{if } w = ax \text{ for some symbol } a \text{ and some string } x \end{cases}$$

   The reversal L^R of a language L is defined as the set of reversals of all strings in L:

$$L^R := \left\{ w^R \;\middle|\; w \in L \right\}$$

   (a) Prove that $(L^*)^R = (L^R)^*$ for every language L.

   (b) Prove that the reversal of any regular language is also a regular language. (You may assume part (a) even if you haven't proved it yet.)

   You may assume the following facts without proof:

   - $L^* \bullet L^* = L^*$ for every language L.
   - $(w^R)^R = w$ for every string w.
   - $(x \bullet y)^R = y^R \bullet x^R$ for all strings x and y.

   [Hint: Yes, all three proofs use induction, but induction on what? And yes, all ***three*** *proofs.]*

3. Describe context-free grammars for each of the following languages over the alphabet $\{0, 1\}$. Explain *briefly* why your grammars are correct; in particular, describe *in English* the language generated by each non-terminal in your grammars. (We are not looking for full formal proofs of correctness, but convincing evidence that *you* understand why your answers are correct.)

   (a) The set of all strings with more than twice as many 0s as 1s.

   (b) The set of all strings that are *not* palindromes.

   *(c) **[Extra credit]** The set of all strings that are *not* equal to ww for any string w.
   [Hint: $a + b = b + a$.]

CS 374 Fall 2014 — Homework 1 Problem 1

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Give regular expressions for each of the following languages over the alphabet $\{0, 1\}$.

(a) All strings with an odd number of 1s.

(b) All strings with at most three 0s.

(c) All strings that do not contain the substring 010.

(d) All strings in which every occurrence of the substring 00 occurs before every occurrence of the substring 11.

CS 374 Fall 2014 — Homework 1 Problem 2

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Section: 1 2 3	

(a) Prove that $(L^*)^R = (L^R)^*$ for every language L.

(b) Prove that the reversal of any regular language is also a regular language.

CS 374 Fall 2014 — Homework 1 Problem 3

Name:	NetID:
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Section: 1 2 3	

Describe context-free grammars for each of the following languages over the alphabet $\{0, 1\}$.

(a) The set of all strings with more than twice as many 0s as 1s.

(b) The set of all strings that are *not* palindromes.

(c) **[Extra credit]** The set of all strings that are *not* equal to ww for any string w.
