

# “CS 374” Fall 2014 — Homework 0

Due Tuesday, September 2, 2014 at noon

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## ••• Some important course policies •••

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- **Each student must submit individual solutions for this homework.** You may use any source at your disposal—paper, electronic, or human—but you *must* cite *every* source that you use. See the academic integrity policies on the course web site for more details. For all future homeworks, groups of up to three students will be allowed to submit joint solutions.
- **Submit your solutions on standard printer/copier paper.** At the top of each page, please clearly print your name and NetID, and indicate your registered discussion section. Use both sides of the paper. If you plan to write your solutions by hand, please print the last three pages of this homework as templates. If you plan to typeset your homework, you can find a  $\LaTeX$  template on the course web site; well-typeset homework will get a small amount of extra credit.
- **Submit your solutions in the drop boxes outside 1404 Siebel.** There is a separate drop box for each numbered problem. Don't staple your entire homework together. Don't give your homework to Jeff in class; he is fond of losing important pieces of paper.
- **Avoid the Three Deadly Sins!** There are a few dangerous writing (and thinking) habits that will trigger an automatic zero on any homework or exam problem. Yes, we are completely serious.
  - Give complete solutions, not just examples.
  - Declare all your variables.
  - Never use weak induction.
- Answering any homework or exam problem (or subproblem) in this course with “I don't know” *and nothing else* is worth 25% partial credit. We will accept synonyms like “No idea” or “WTF”, but you must write *something*.

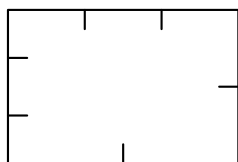
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**See the course web site for more information.**

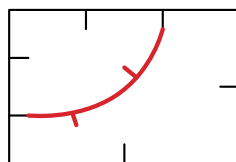
If you have any questions about these policies,  
please don't hesitate to ask in class, in office hours, or on Piazza.

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- The Terminal Game is a two-person game played with pen and paper. The game begins by drawing a rectangle with  $n$  “terminals” protruding into the rectangles, for some positive integer  $n$ , as shown in the figure below. On a player’s turn, she selects two terminals, draws a simple curve from one to the other without crossing any other curve (or itself), and finally draws a new terminal on each side of the curve. A player loses if it is her turn and no moves are possible, that is, if no two terminals may be connected without crossing at least one other curve.



The initial setup.



The first turn.



No more moves.

Analyze this game, answering the following questions (and any more that you determine the answers to): When is it better to play first, and when it is better to play second? Is there always a winning strategy? What is the smallest number of moves in which you can defeat your opponent? Prove your answers are correct.

- Herr Professor Doktor Georg von den Dschungel has a 23-node binary tree, in which each node is labeled with a unique letter of the German alphabet, which is just like the English alphabet with four extra letters: **Ä, Ö, Ü, and ß**. (Don’t confuse these with **A, O, U, and B!**) Preorder and postorder traversals of the tree visit the nodes in the following order:

- Preorder: **B K Ü E H L Z I Ö R C ß T S O A Ä D F M N U G**
- Postorder: **H I Ö Z R L E C Ü S O T A ß K D M U G N F Ä B**

- List the nodes in George’s tree in the order visited by an inorder traversal.
- Draw George’s tree.

- Recursively define a set  $L$  of strings over the alphabet  $\{0, 1\}$  as follows:

- The empty string  $\epsilon$  is in  $L$ .
- For any two strings  $x$  and  $y$  in  $L$ , the string  $0x1y0$  is also in  $L$ .
- These are the only strings in  $L$ .

- Prove that the string **000010101010010100** is in  $L$ .
- Prove by induction that every string in  $L$  has exactly twice as many **0s** as **1s**.
- Give an example of a string with exactly twice as many **0s** as **1s** that is *not* in  $L$ .

Let  $\#(a, w)$  denote the number of times symbol  $a$  appears in string  $w$ ; for example,

$$\#(0, 000010101010010100) = 12 \quad \text{and} \quad \#(1, 000010101010010100) = 6.$$

You may assume without proof that  $\#(a, xy) = \#(a, x) + \#(a, y)$  for any symbol  $a$  and any strings  $x$  and  $y$ .

4. This is an extra credit problem. Submit your solutions in the drop box for problem 1 (but don't staple your solutions for 1 and 4 together.)

A *perfect riffle shuffle*, also known as a *Faro shuffle*, is performed by cutting a deck of cards exactly in half and then *perfectly* interleaving the two halves. There are two different types of perfect shuffles, depending on whether the top card of the resulting deck comes from the top half or the bottom half of the original deck. An *out-shuffle* leaves the top card of the deck unchanged. After an in-shuffle, the original top card becomes the second card from the top. For example:

$$\text{OutShuffle}(A\spadesuit 2\spadesuit 3\spadesuit 4\spadesuit 5\heartsuit 6\heartsuit 7\heartsuit 8\heartsuit) = A\spadesuit 5\heartsuit 2\spadesuit 6\heartsuit 3\spadesuit 7\heartsuit 4\spadesuit 8\heartsuit$$

$$\text{InShuffle}(A\spadesuit 2\spadesuit 3\spadesuit 4\spadesuit 5\heartsuit 6\heartsuit 7\heartsuit 8\heartsuit) = 5\heartsuit A\spadesuit 6\heartsuit 2\spadesuit 7\heartsuit 3\spadesuit 8\heartsuit 4\spadesuit$$

(If you are unfamiliar with playing cards, please refer to the Wikipedia article [https://en.wikipedia.org/wiki/Standard\\_52-card\\_deck](https://en.wikipedia.org/wiki/Standard_52-card_deck).)

Suppose we start with a deck of  $2^n$  distinct cards, for some non-negative integer  $n$ . What is the effect of performing exactly  $n$  perfect in-shuffles on this deck? Prove your answer is correct!

## CS 374 Fall 2014 — Homework 0 Problem 1

Name:	
NetID:	Section: 1 2 3

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Analyze the Terminal Game, answering the following questions (and any more that you determine the answers to): Is it better to play first or second? Is there always a winning strategy? What is the fewest number of moves in which you can defeat your opponent? Prove your answers are correct.

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## CS 374 Fall 2014 — Homework 0 Problem 2

Name:	
NetID:	Section: 1 2 3

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- (a) List the nodes in George's tree in the order visited by an inorder traversal.  
(b) Draw George's tree.
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## CS 374 Fall 2014 — Homework 0 Problem 3

Name:	
NetID:	Section: 1 2 3

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- (a) Prove that the string **000010101010010100** is an element of  $L$ .
- (b) Prove by induction that every string in  $L$  has exactly twice as many **0s** as **1s**.
- (c) Give an example of a string with exactly twice as many **0s** as **1s** that is *not* in  $L$ .
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## CS 374 Fall 2014 — Homework 0 Problem 4

Name:	
NetID:	Section: 1 2 3

**Extra credit. Submit answers in the drop box for problem 1  
(but don't staple problems 1 and 4 together.)**

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Suppose we start with a deck of  $2^n$  cards, for some non-negative integer  $n$ . What is the effect of performing exactly  $n$  perfect in-shuffles on this deck? Prove your answer is correct!

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