Write your answers in the separate answer booklet.
Please return this question sheet and your cheat sheet with your answers.

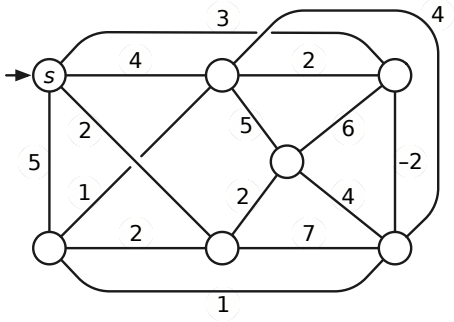
1. ***Clearly*** indicate the edges of the following spanning trees of the weighted graph pictured below. (Pretend that the person grading your exam has bad eyesight.) Some of these subproblems have more than one correct answer. Yes, that edge on the right has negative weight.

   (a) A depth-first spanning tree rooted at s

   (b) A breadth-first spanning tree rooted at s

   (c) A shortest-path tree rooted at s

   (d) A minimum spanning tree



2. An array $A[0..n-1]$ of n distinct numbers is ***bitonic*** if there are unique indices i and j such that $A[(i-1) \bmod n] < A[i] > A[(i+1) \bmod n]$ and $A[(j-1) \bmod n] > A[j] < A[(j+1) \bmod n]$. In other words, a bitonic sequence either consists of an increasing sequence followed by a decreasing sequence, or can be circularly shifted to become so. For example,

4	6	9	8	7	5	1	2	3

   is bitonic, but

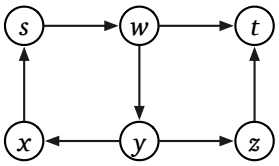
3	6	9	8	7	5	1	2	4

   is *not* bitonic.

   Describe and analyze an algorithm to find the index of the *smallest* element in a given bitonic array $A[0..n-1]$ in $O(\log n)$ time. You may assume that the numbers in the input array are distinct. For example, given the first array above, your algorithm should return 6, because $A[6] = 1$ is the smallest element in that array.

3. Suppose you are given a directed graph $G = (V, E)$ and two vertices s and t. Describe and analyze an algorithm to determine if there is a walk in G from s to t (possibly repeating vertices and/or edges) whose length is divisible by 3.

   For example, given the graph below, with the indicated vertices s and t, your algorithm should return TRUE, because the walk $s \to w \to y \to x \to s \to w \to t$ has length 6.



   [Hint: Build a (different) graph.]

4. The new swap-puzzle game *Candy Swap Saga XIII* involves n cute animals numbered 1 through n. Each animal holds one of three types of candy: circus peanuts, Heath bars, and Cioccolateria Gardini chocolate truffles. You also have a candy in your hand; at the start of the game, you have a circus peanut.

   To earn points, you visit each of the animals in order from 1 to n. For each animal, you can either keep the candy in your hand or exchange it with the candy the animal is holding.

   - If you swap your candy for another candy of the *same* type, you earn one point.
   - If you swap your candy for a candy of a *different* type, you lose one point. (Yes, your score can be negative.)
   - If you visit an animal and decide not to swap candy, your score does not change.

   You *must* visit the animals in order, and once you visit an animal, you can never visit it again.

   Describe and analyze an efficient algorithm to compute your maximum possible score. Your input is an array $C[1..n]$, where $C[i]$ is the type of candy that the ith animal is holding.

5. Let G be a directed graph with weighted edges, and let s be a vertex of G. Suppose every vertex $v \neq s$ stores a pointer $pred(v)$ to another vertex in G. Describe and analyze an algorithm to determine whether these predecessor pointers correctly define a single-source shortest path tree rooted at s. Do ***not*** assume that G has no negative cycles.