

Objectives

Fenwick Trees

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Your Objectives:

- ▶ Describe and implement a Fenwick Tree
- ▶ Compare a Fenwick Tree to a Segment Tree

Motivating Example

Exam scores = {2, 4, 5, 5, 6, 6, 6, 7, 7, 8, 9}

Motivating Example

Exam scores = {2, 4, 5, 5, 6, 6, 6, 7, 7, 8, 9}

index	1	2	3	4	5	6	7	8	9	10
value	0	1	0	1	2	3	2	1	1	0

Motivating Example

Exam scores = {2, 4, 5, 5, 6, 6, 6, 7, 7, 8, 9}

index	1	2	3	4	5	6	7	8	9	10
value	0	1	0	1	2	3	2	1	1	0
cumulative	0	1	1	2	4	7	9	10	11	11



Fenwick Tree

value	0	1	0	1	2	3	2	1	1	0
index	1	2	3	4	5	6	7	8	9	10



Motivating Example

Exam scores = {2, **3**, 4, 5, 5, 6, 6, 6, 7, 7, 8, 9}

index	1	2	3	4	5	6	7	8	9	10
value	0	1	1	1	2	3	2	1	1	0
cumulative	0	1	2	3	5	8	10	11	12	12



Fenwick Tree

value	0	1	0	1	2	3	2	1	1	0
index	1	2	3	4	5	6	7	8	9	10
index	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010



Fenwick Tree

ft 1	0		0		2		2		1	
value	0	1	0	1	2	3	2	1	1	0
index	1	2	3	4	5	6	7	8	9	10
index	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010



Fenwick Tree

ft 3				2						
ft 2		1				5				1
ft 1	0		0		2		2		1	
value	0	1	0	1	2	3	2	1	1	0
index	1	2	3	4	5	6	7	8	9	10
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Fenwick Tree

ft 2		1				5				1
ft 1	0		0		2		2		1	
value	0	1	0	1	2	3	2	1	1	0
index	1	2	3	4	5	6	7	8	9	10
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Fenwick Tree

ft 4								10		
ft 3				2						
ft 2		1				5				1
ft 1	0		0		2		2		1	
value	0	1	0	1	2	3	2	1	1	0
index	1	2	3	4	5	6	7	8	9	10
index	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010



Queries

ft 4								10		
ft 3				2						
ft 2		1				5				1
ft 1	0		0		2		2		1	
value	0	1	0	1	2	3	2	1	1	0
index	1	2	3	4	5	6	7	8	9	10
index	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010

- ▶ To Query sum for position n , first read n .
- ▶ Then, subtract the lowest order bit, and repeat until $n = 0$.

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ft 3				2						
ft 2		1				5				1
ft 1	0		0		2		2		1	
value	0	1	0	1	2	3	2	1	1	0
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- ▶ E.g.: $5 = 101 \rightarrow 4 = 100 \rightarrow 0$
- ▶ $ft(5) + ft(4) = 2 + 2 = 4$

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ft 4								10		
ft 3				2						
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ft 1	0		0		2		2		1	
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- ▶ $ft(5) + ft(4) = 2 + 2 = 4$
- ▶ E.g.: $10 = 1010 \rightarrow 8 = 1000 \rightarrow 0$
- ▶ $ft(10) + ft(8) = 1 + 10 = 11$

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ft 1	0		0		2		2		1	
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- ▶ E.g.: $5 = 101 \rightarrow 4 = 100 \rightarrow 0$
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 - ▶ Update 5: Visit $5=101 \rightarrow 6=110 \rightarrow 8=1000$
- ▶ E.g.: $10 = 1010 \rightarrow 8 = 1000 \rightarrow 0$
 - ▶ $ft(10) + ft(8) = 1 + 10 = 11$



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ft 2		1				5				1
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value	0	1	0	1	2	3	2	1	1	0
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 - ▶ Update 5: Visit $5=101 \rightarrow 6=110 \rightarrow 8=1000$
- ▶ E.g.: $10 = 1010 \rightarrow 8 = 1000 \rightarrow 0$
 - ▶ $LSOne(n) = n \& -n$
 - ▶ $ft(10) + ft(8) = 1 + 10 = 11$

