

## Example Proof 1

$$\frac{\frac{\frac{B}{A \wedge B} \text{ And I}}{B \Rightarrow (A \wedge B)} \text{ Imp I}}{A \Rightarrow (B \Rightarrow (A \wedge B))} \text{ Imp I}$$

## Example Proof 2

$$\begin{aligned} \Gamma_3 &= \{A \Rightarrow B, B \Rightarrow C, A\} \\ \Gamma_4 &= \{A \Rightarrow B, B \Rightarrow C, A, B\} \\ \Gamma_5 &= \{A \Rightarrow B, B \Rightarrow C, A, B, C\} \end{aligned}$$

$$\frac{\frac{\frac{\text{Hyp}}{\Gamma_3 \vdash A \Rightarrow B} \quad \frac{\text{Hyp}}{\Gamma_3 \vdash A} \quad \frac{\frac{\text{Hyp}}{\Gamma_4 \vdash B \Rightarrow C} \quad \frac{\text{Hyp}}{\Gamma_4 \vdash B} \quad \frac{\text{Hyp}}{\Gamma_5 \vdash C}}{\Gamma_4 \vdash C} \text{ Imp E}}{\Gamma_3 \vdash A \Rightarrow B \quad \Gamma_3 \vdash A} \text{ Imp E}}{\frac{\frac{\{A \Rightarrow B, B \Rightarrow C, A\} \vdash C}{\{A \Rightarrow B, B \Rightarrow C\} \vdash A \Rightarrow C} \text{ Imp I}}{\{A \Rightarrow B\} \vdash (B \Rightarrow C) \Rightarrow (A \Rightarrow C)} \text{ Imp I}} \text{ Imp I}$$

$$\frac{\{A \Rightarrow B\} \vdash (B \Rightarrow C) \Rightarrow (A \Rightarrow C)}{\{\} \vdash (A \Rightarrow B) \Rightarrow ((B \Rightarrow C) \Rightarrow (A \Rightarrow C))} \text{ Imp I}$$