Build proof tree for Natural Deduction, Sequent Calculus, etc.

Paul Taylor
Department of Computing,
Imperial College,
London SW7 2BZ
+44 71 589 5111 *ext* 5057
<pt@doc.ic.ac.uk>

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Using my Proof Tree macros, you can produce

$$\dfrac{\dfrac{[(\exists y.\,((\exists n.x_n = y) \wedge qy)) \wedge (\forall w.\,((\exists n.x_n = w) \Rightarrow pw))]_\alpha}{\dfrac{\forall w.\,((\exists n.x_n = w) \Rightarrow pw)}{(\exists n.x_n = y) \Rightarrow py}\,\forall\mathcal{E} \qquad \dfrac{[(\exists n.x_n = y) \wedge qy]_\beta}{\exists n.x_n = y}\,\wedge 1\mathcal{E}}{py}\,\Rightarrow\mathcal{E} \qquad [(\exists n.x_n = y) \wedge qy]_\beta}{\dfrac{[(\exists y.\,((\exists n.x_n = y) \wedge qy)) \wedge (\forall w.\,((\exists n.x_n = w) \Rightarrow pw))]_\alpha}{\exists y.\,((\exists n.x_n = y) \wedge qy)}\,\wedge 1\mathcal{E} \qquad \dfrac{py \wedge ((\exists n.x_n = y) \wedge qy)}{\exists z.pz \wedge (((\exists n.x_n = z) \wedge qz))}\,\exists\mathcal{I}}{\exists z.pz \wedge (((\exists n.x_n = z) \wedge qz))}\,\exists\mathcal{E}_\beta}{(\exists y.\,((\exists n.x_n = y) \wedge qy)) \wedge (\forall w.\,((\exists n.x_n = w) \Rightarrow pw)) \Rightarrow (\exists z.pz \wedge (((\exists n.x_n = z) \wedge qz)))}\,\Rightarrow\mathcal{I}_\alpha$$

using the TEX or LATEX code

```
\input prooftree
$$
\begin{prooftree}
\[ \[ [(\A y n)\land(\B w n)]_\alpha
      \andelim1
      \A y n
   \]
   \kern-26em
   \[ \[ \[ \[ \[ [(\A y n)\land(\B w n)]_\alpha
                  \andelim2 \shiftright60pt
                  \B w n
```

```
            \]
            \allelim
            (\E y n)\imp\P y
         \]
         \[
            [(\E y n)\land\Q y]_\beta
            \andelim1
            \E y n
         \]
         \impelim \shiftright50pt
         \P y
      \]
      \kern-25pt
      [(\E y n)\land\Q y]_\beta
      \andintro
      \P y\land((\E y n)\land\Q y)
   \]
   \existsintro
   \C z n
\]
\existselim\beta
\C z n
\]
\impintro\alpha
(\A yn)\land(\B w n)\imp(\C z n)
\end{prooftree}$$
```

In fact the commands `\allintro`, *etc.,* are not primitive; the basic form is

```
\[
A\quad
B
\justifies
A \land B
\thickness=0.08em
\shiftright 2em
\using
{\land}{\cal I}
\]
```

which gives

$$\frac{A \quad B}{A \land B}\,\land\mathcal{I}$$

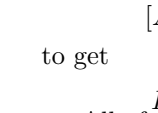
The hypotheses may themselves be proof trees (enclosed in `\[...\]`) and the purpose of the macros is to adjust the length of the horizontal "deduction" line.

When the hypotheses are proof trees, suitable space is put between them, but of course this must be supplied by hand for simple formulae. The `\thickness` and `\shiftright` commands are, of course, optional; they apply to the horizontal line and to the positioning of the conclusion relative to it. For a double line, use `\Justifies` instead of `\justifies`.

Notice the overloading of the `\[...\]`; the outermost proof tree must be enclosed with `\begin{prooftree}` and `\end{prooftree}` or `\prooftree` and `\endprooftree`.

To get a vertical string of dots instead of the proof rule, do

```
\[
[A]
\using
\pi
\proofdotseparation=1.2ex
\proofdotnumber=4
\leadsto
B
\]
```

to get 

All of of the keywords except `\prooftree` and `\endprooftree` are optional and may appear in any order. They may also be combined in `\newcommand`s, for example

```
\newcommand\Cut{\using\sf cut\thickness.08em\justifies}
```

with the abbreviation

```
\[ A \vdash B \qquad
   B \vdash C
  \Cut
   A \vdash C
\]
```

`\thickness` specifies the breadth of the rule in any units, although font-relative units such as ex or em are preferable. It may optionally be followed by =. `\proofrulebreadth=.08em` or `\setlength\proofrulebreadth{.08em}` may also be used either in place of `\thickness` or globally; the default is 0.04em. `\proofdotseparation` and `\proofdotnumber` control the size of the string of dots.

If proof trees and formulae are mixed, some explicit spacing is needed, but don't put anything to the left of the left-most (or the right of the right-most)

hypothesis, or put it in braces, because this will cause the indentation to be lost.

By default the conclusion is centered wrt the left-most and right-most immediate hypotheses (not their proofs); `\shiftright` or `\shiftleft` moves it relative to this position. (Not sure about this specification or how it should affect spreading of proof tree.)