HW 1 – Truth and Proof in Propositional Logic

CS 477 – Spring 2020

Revision 1.0

Assigned January 29, 2020
Due February 5, 2020, 9:00 pm
Extension 48 hours (20% penalty)

1 Change Log

1.0 Initial Release.

2 Objectives and Background

The purpose of this HW is to test your understanding of

- validity of propositions in the standard model of propositional logic
- Natural Deduction proofs of propositions in propositional logic

Another purpose of HWs is to provide you with experience answering non-programming written questions of the kind you may experience on the midterm and final.

3 Turn-In Procedure

The pdf for this assignment (`hw1.pdf`) should be found in the `assignments/hw1/` subdirectory of your git directory for this course. Your solution should be put in that same directory. Using your favorite tool(s), you should put your solution in a file named `hw1-submission.pdf`. If you have problems generating a pdf, please seek help from the course staff. Your answers to the following questions are to be submitted electronically from within the `assignments/hw1/` subdirectory by commiting the file as follows:

```
git add hw1-submission.pdf
git commit -m "Turning in hw1"
git push
```

4 Problem

For each of the following propositions, give both all possible valuations of every subformula of the proposition, inclucing the formula itself, in the form of a truth table, and given a Natural Deduction proof of the proposition. For the Natural Deduction proof, you may use the pure style first indtroduced in class, but it must be accompanied by a discription of how each assumption is discharged. Alternatively, you may use the sequent encoding of Natural Deduction proofs.

1. (5pts + 7pts) $(A \wedge B) \Rightarrow (B \wedge A)$

2. (5pts + 6pts) $(A \vee A) \Rightarrow (B \vee A)$

3. (7pts + 7pts) $(A \wedge B) \Rightarrow ((\neg B) \Rightarrow (\neg A))$

4. (7pts + 7pts) $(A \Rightarrow B) \Rightarrow ((\neg B) \Rightarrow (\neg A))$

5. (8pts + 9pts) $((A \wedge B) \Rightarrow C) \Rightarrow (A \Rightarrow (B \Rightarrow C))$

6. (7pts + 14pts) $((\neg B) \vee (\neg A)) \Rightarrow (\neg(A \wedge B))$

7. (8pts + 14pts) $((\neg A) \vee (\neg B)) \Rightarrow (\neg(A \wedge B))$

5 Extra Credit

8. (10 pts)

   Give a detailed, rigorous proof of the following:

   For all propositions P, if there exists a proof of the sequent $\{\ \} \vdash P$ in the sequent encoding of the Natural Deduction system, then there exists a fully discharged proof of P in the Natural Deduction system.

   You will want to prove a more general fact by induction on the structure or height of proofs.