CS477 Formal Software Dev Methods

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Büchi Automaton

A Büchi automaton is a 5-tuple $A = (\Sigma, S, \Delta, I, L, F)$ such that

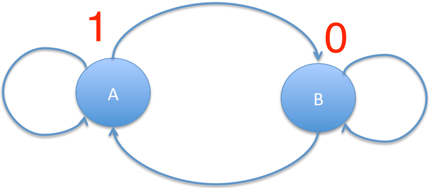
- Σ is a finite *alphabet*.
- S is a finite set of *states*.
- $\Delta \subseteq S \times S$ is the *transition relation*.
- $I \subseteq S$ are the *start states*.
- $L : S \rightarrow \Sigma$ is a *labeling* of the *states*.
- F is a set of sets of *accepting states* f where $f \in F \rightarrow f \subseteq S$.

Path in a Büchi Automaton

- Let $A = (\Sigma, S, \Delta, I, L, F)$ be a Büchi automaton
- A sequence $\rho : \mathbb{N} \rightarrow S$ is a path in A provided that
  - $\rho_0 \in I$
  - For all i, $(\rho_i, \rho_{1+1}) \in \Delta$.
- Let ρ be a path in A. Then $inf(\rho) = \{q \in S | \ |\{n | q = \rho_n\}| = \omega\}$.
- Path ρ is accepted if for all $f \in F$, $inf(\rho) \cap f \neq \emptyset$
- A word of A is a sequence $v : \mathbb{N} \rightarrow \Sigma = \Sigma^\omega$ such that there exists a path $\rho \in S^\omega$ where for all i, $L(\rho_i) = v_i$.
- The language of A, $\mathcal{L}(A) = \{v \in \Sigma^\omega | v \text{ is a word of } A\}$.

Example Büchi 1

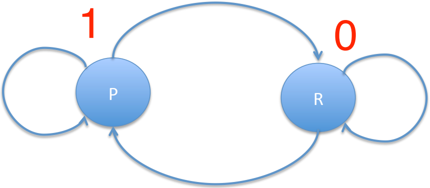
- $S = \{A, B\}$, $I = \{A\}$, $\Delta = \{(A,A), (A,B), (B,B), (B, A)\}$,
  $\Sigma = \{0,1\}$, $L(A) = 1$, $L(B) = 0$, $F_1 = \{\{A, B\}\}$



L(A) = all strings of 1's and '0s that starts with 1

Example Büchi 2

- S = {P, R}, I = {A}, Δ ={(P,P), (P,R), (R,R), (R,P)},
  Σ = {0,1}, L(P) = 1, L(R) = 0, F_1 = {{P}, {Q}}



- *L*(A) = all strings with both infinitely many 1's and infinitely many 0's that starts with 1