CS477 Formal Software Development Methods

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How to build a Reduced Ordered BDD

- Done by recursion on the structure of the formula
- Order propositional variables
- For each variable, create a three node tree corresponding to (if Var then true else false)
- For formula not P, build ROBDD from ROBDD for P by flipping values in the leaves

How to build a Reduced Ordered BDD

- For P <op> Q
  - Fill in each branch of BDDs for P and Q with all nodes appearing in either BDD, not already on branch
  - For each ordered sequence of values (done in order of variables, greatest to least, false before true) build the branch in partial BDD for (P <op> Q)
  - Value in end leaf is v1 <op> v2 where v1 is value at end of branch in BDD for P, v2 for branch in Q

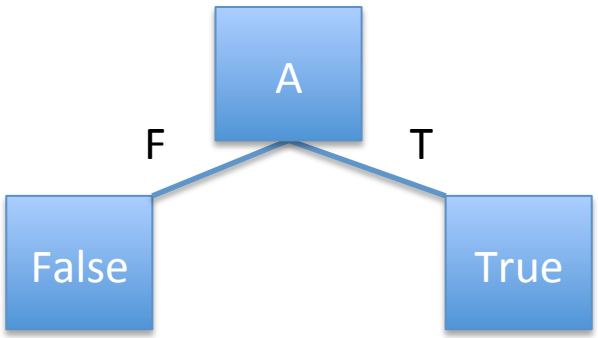
How to build a Reduced Ordered BDD

- When you add a new node
  - If the outedges point to same node, ghost it.
  - If there exists another node with the same label whose outedges of the a given label point the same nodes as its outedges of the given label, remove the new node and move all edges pointing to it to the other node
- Final step when returning final result
  - For each ghost node, move all edges pointing to it to point to what it points to, and remove it

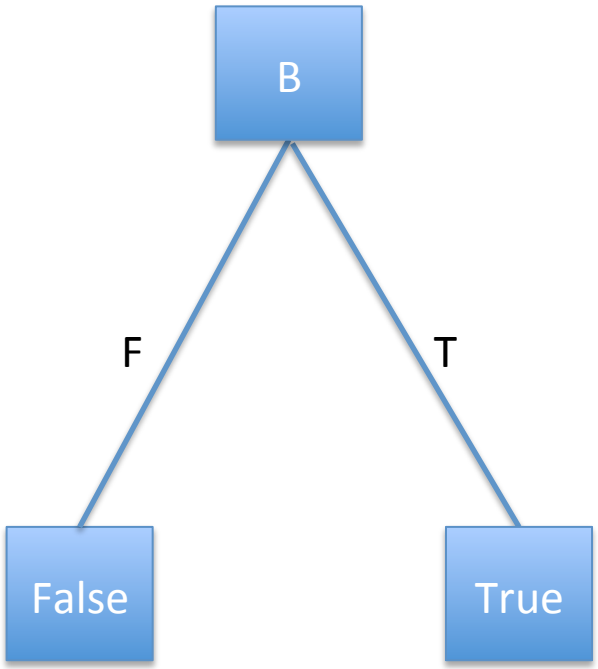
Example

- Find ROBDD for (A $\wedge$B) $\vee$ (not C)

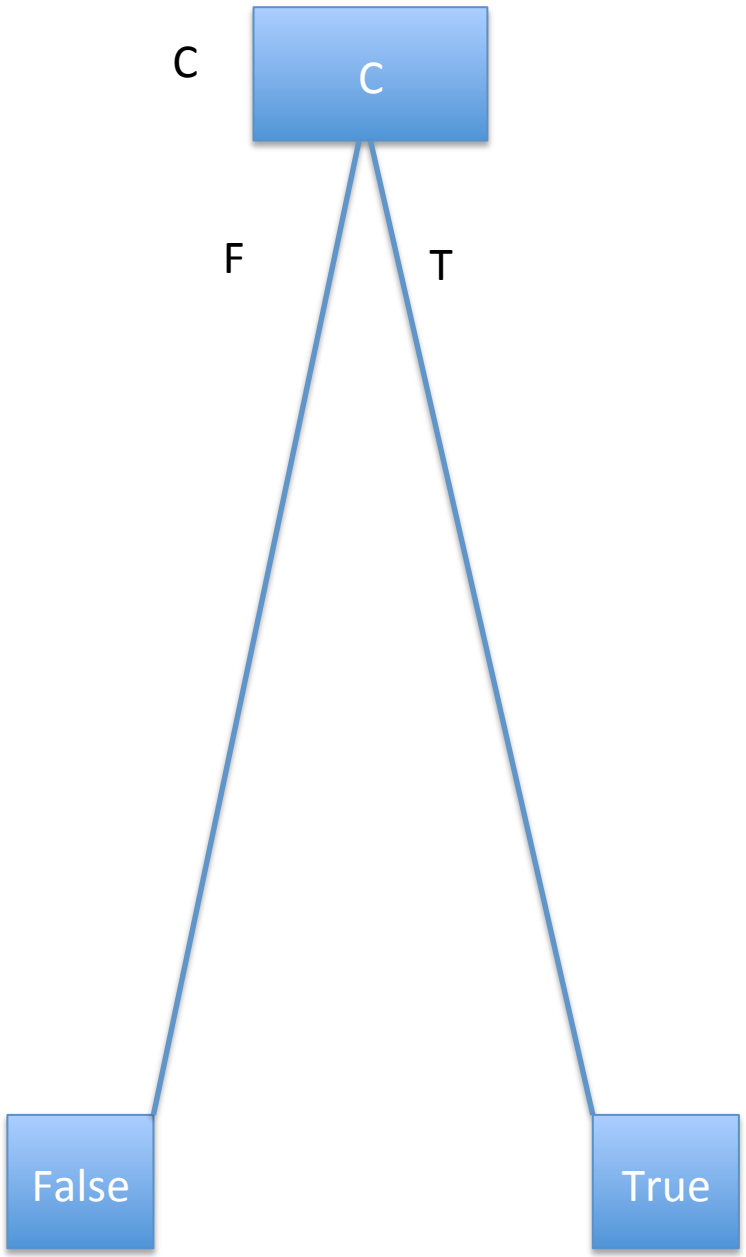
A Variables: A < B < C



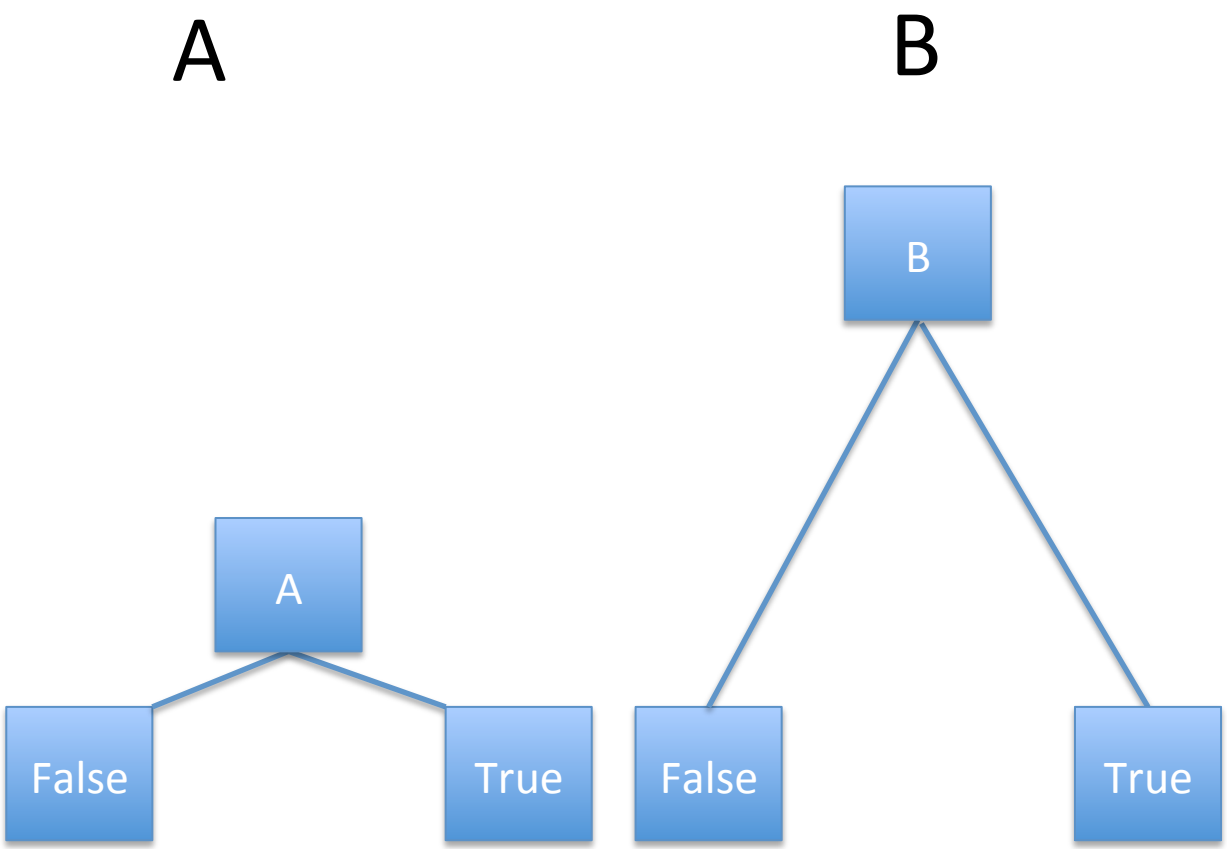
B Variables: A < B < C



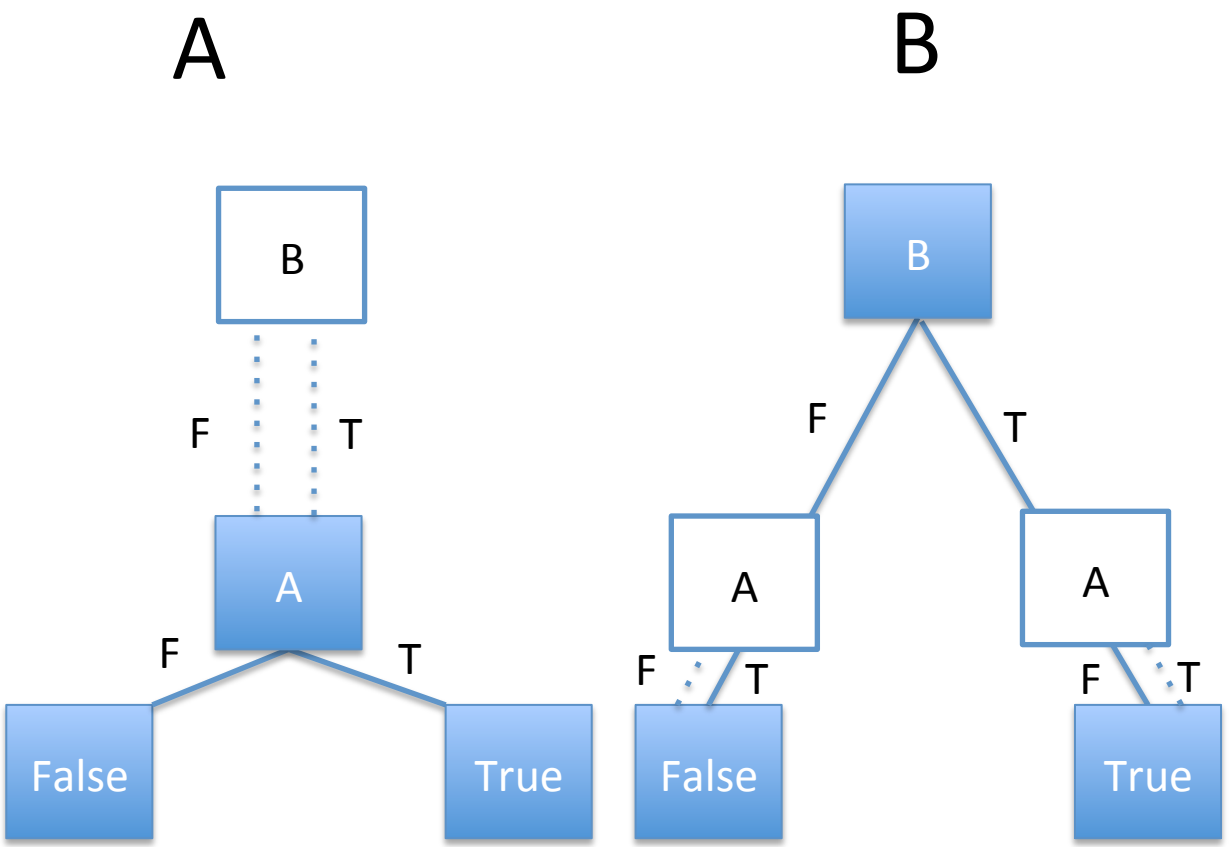
C Variables: C > B > A



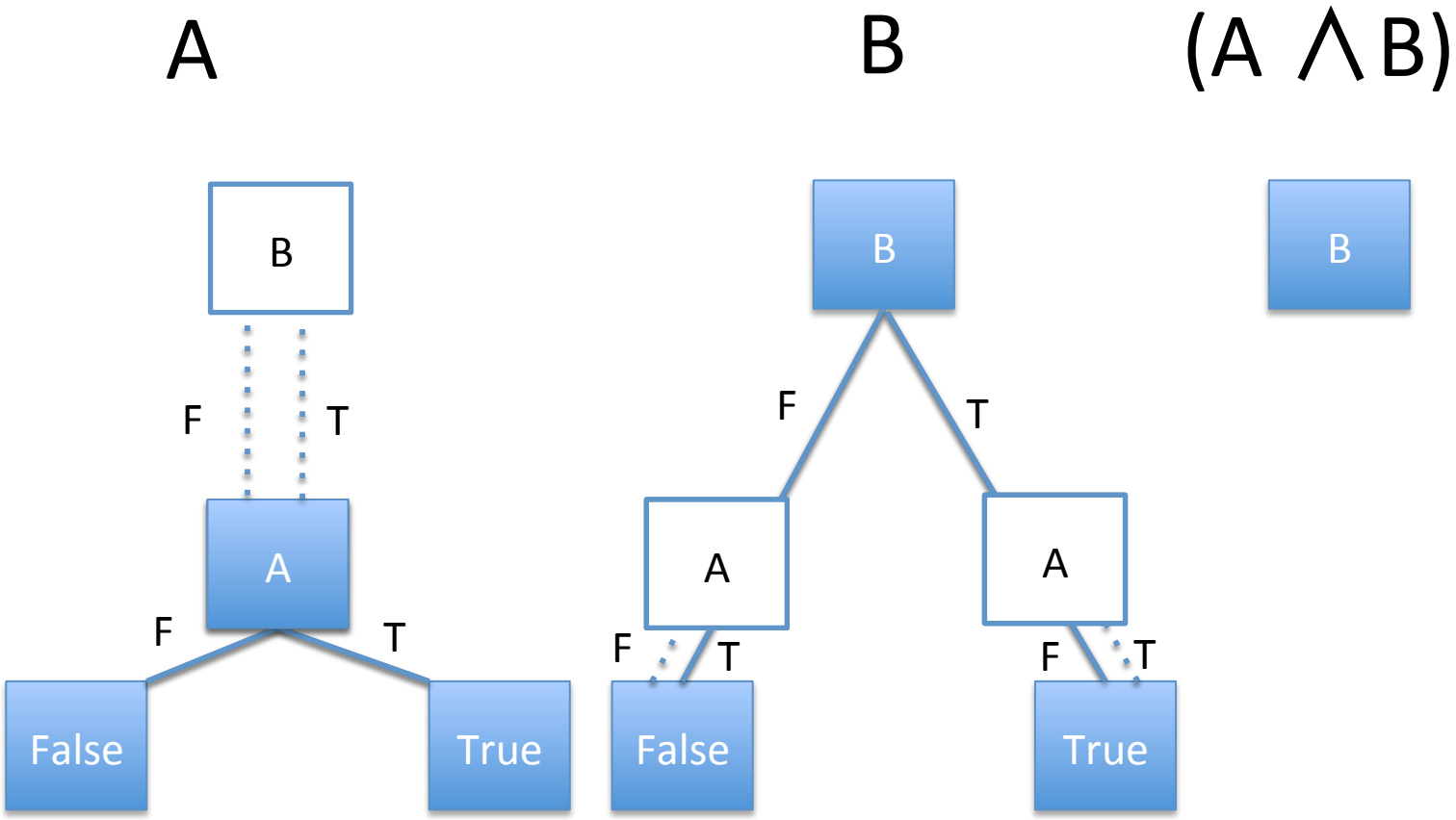
(A ⋀ B) Variables: A < B < C



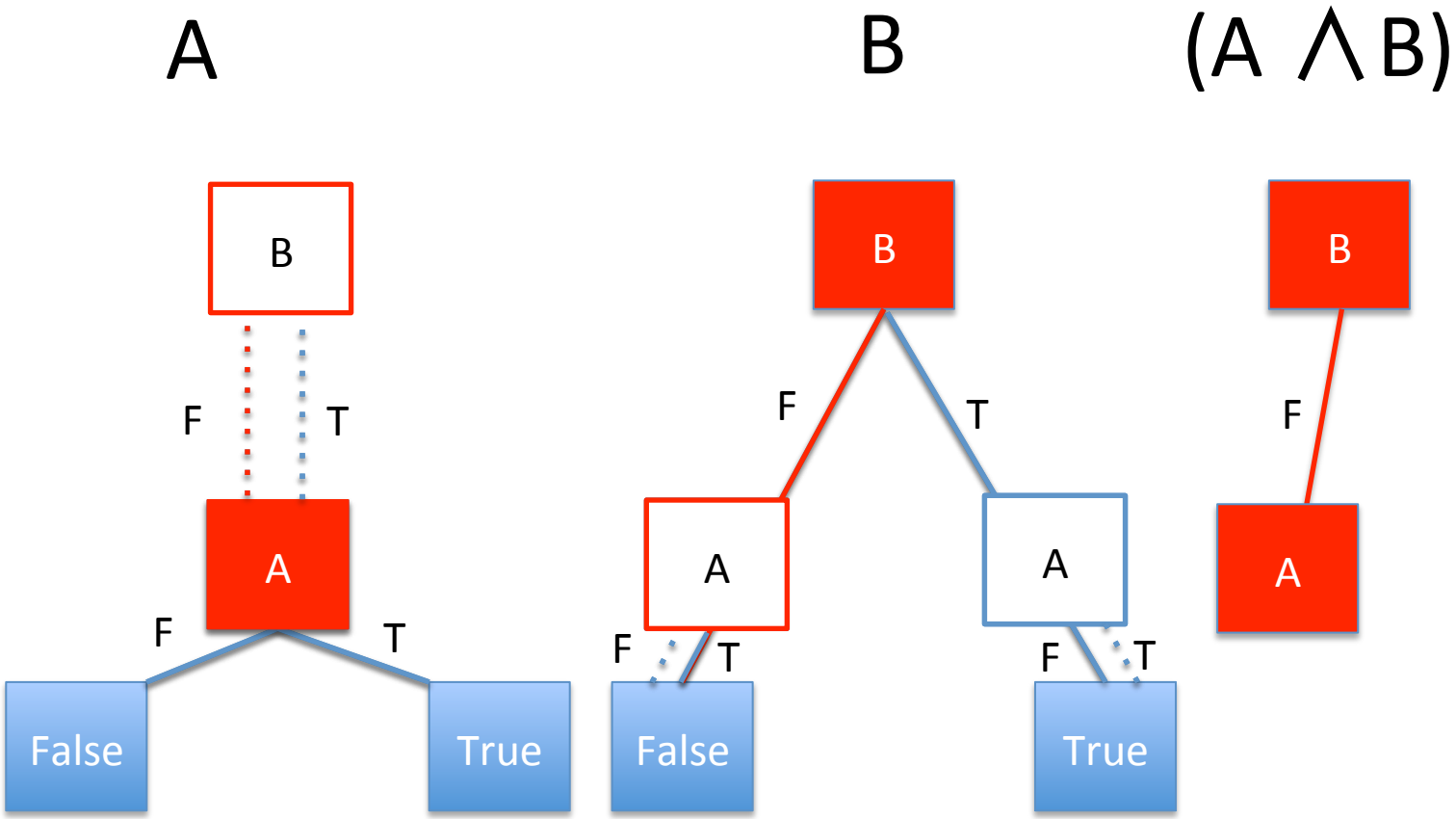
(A ⋀ B) Variables: A < B < C



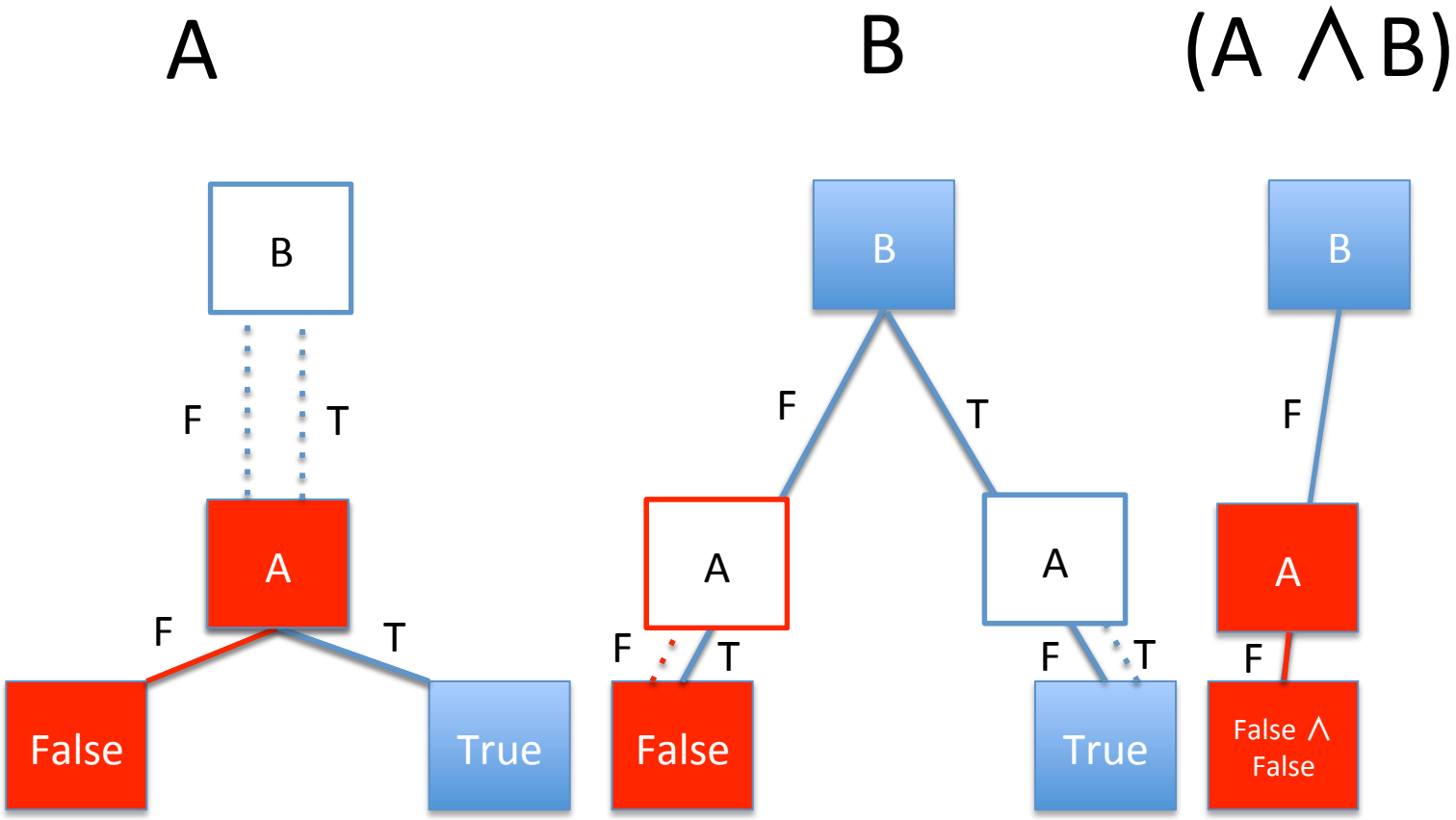
(A ⋀ B) Variables: A < B < C



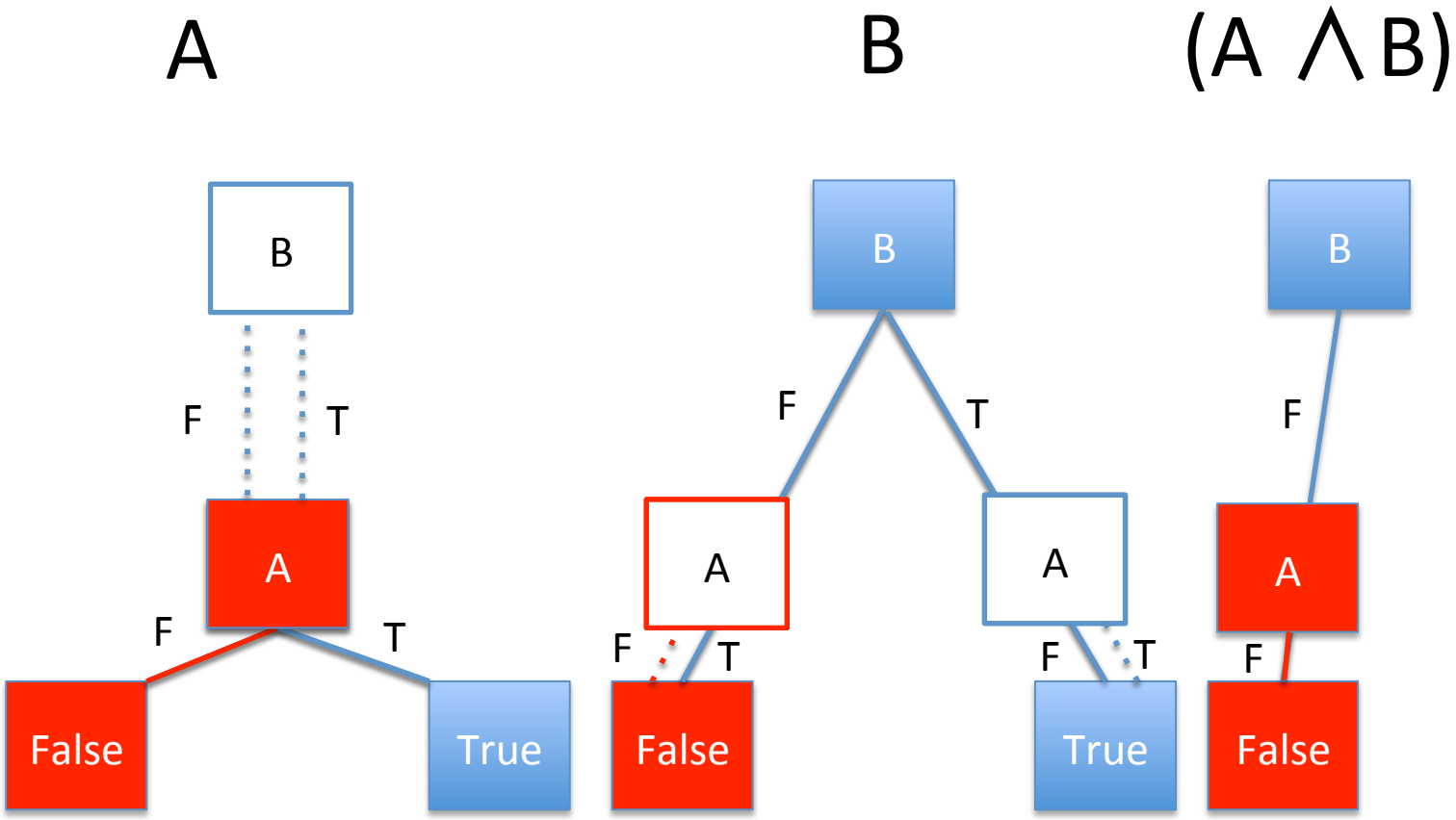
$(A \wedge B)$ Variables: A < B < C



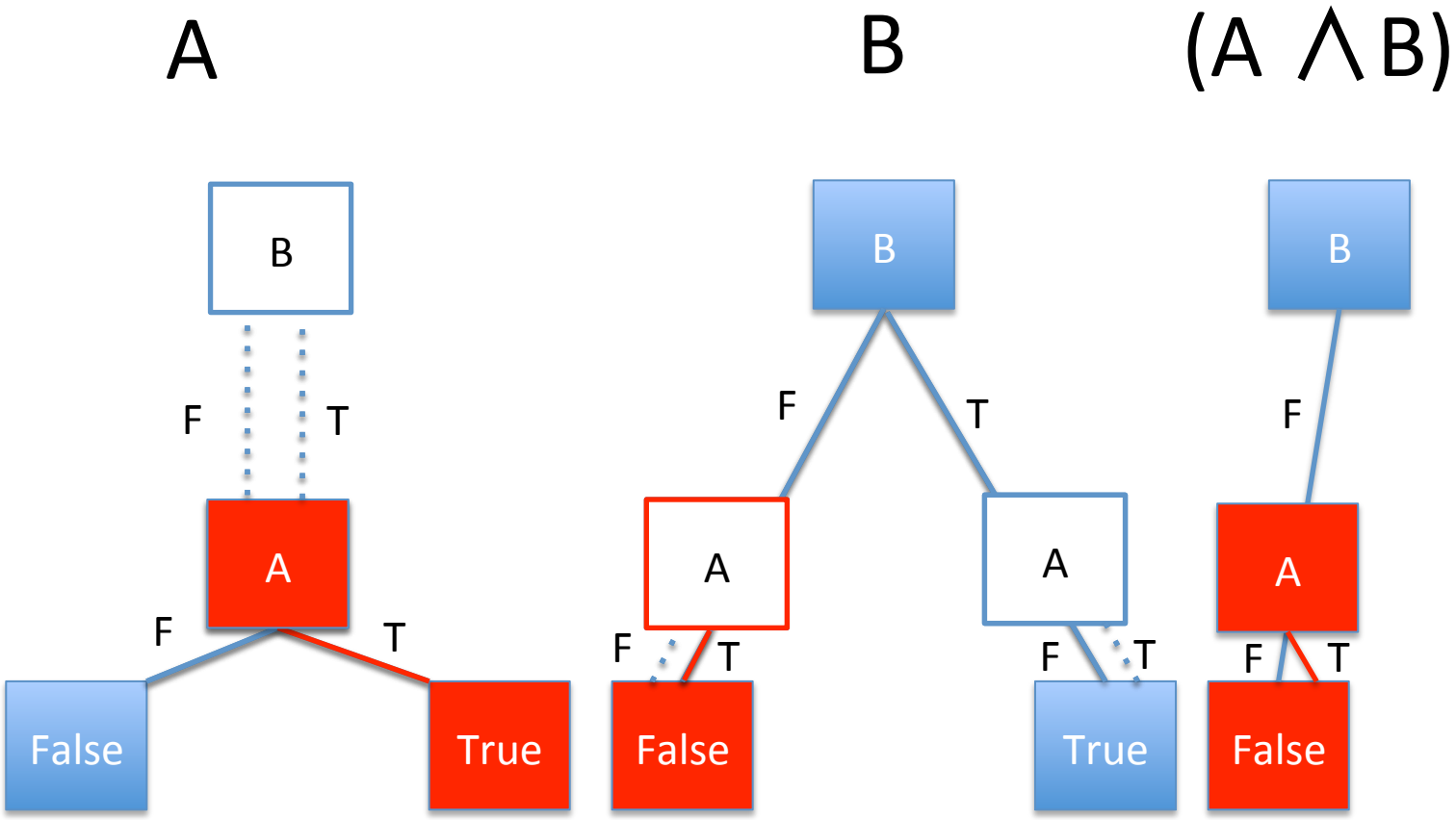
(A ∧ B) Variables: A < B < C



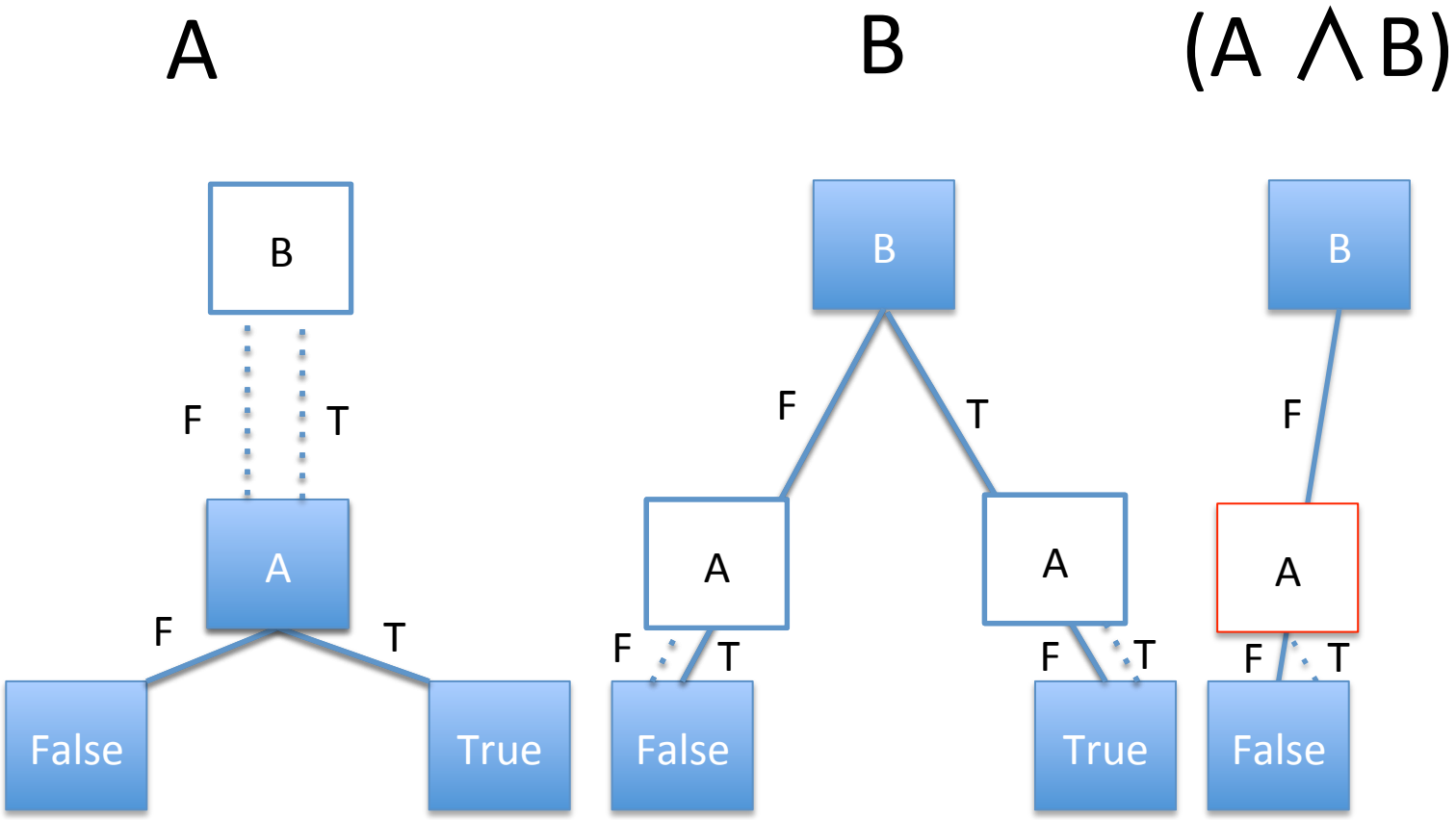
(A ⋀ B) Variables: A < B < C



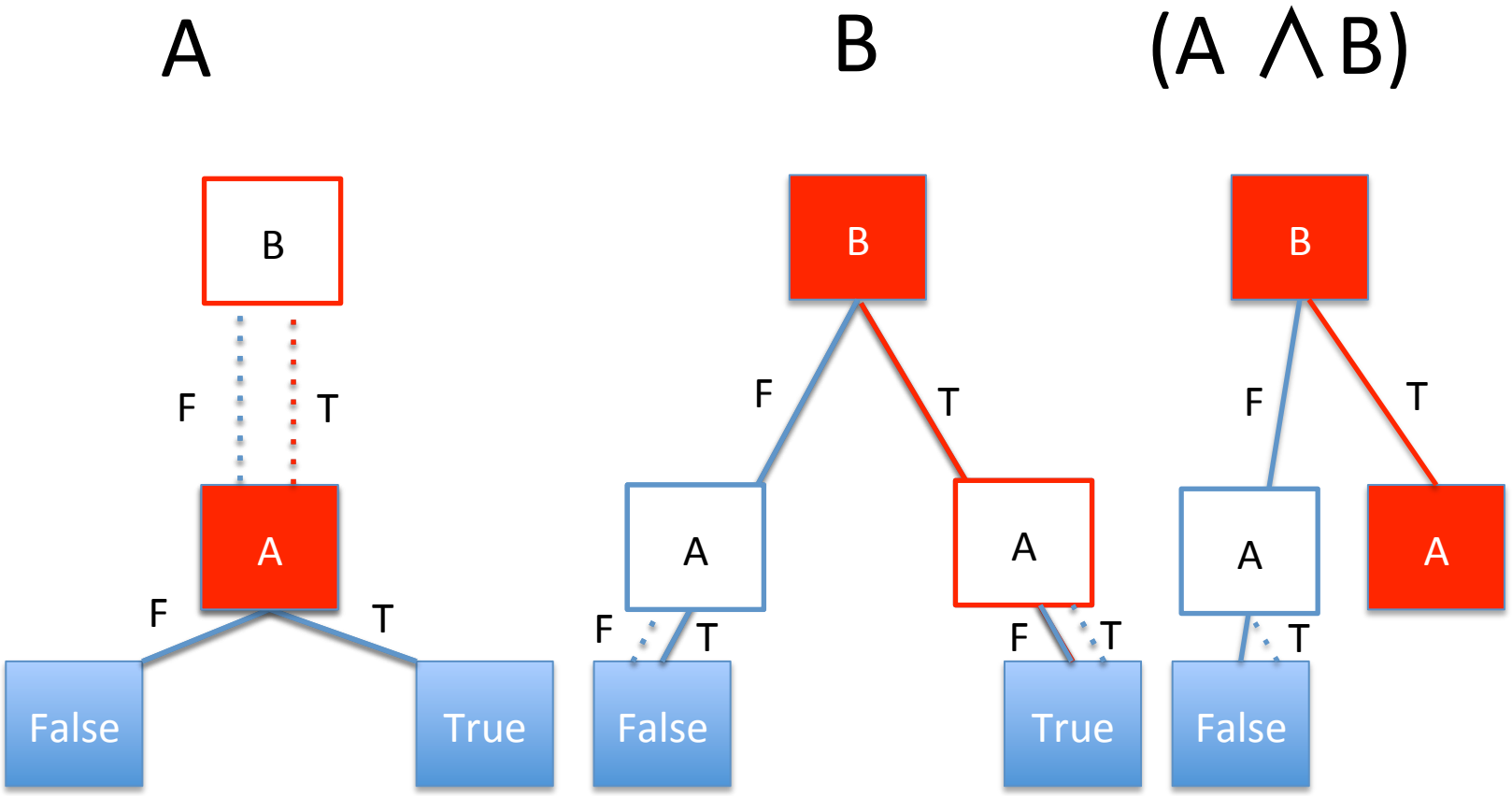
$(A \wedge B)$ Variables: A < B < C



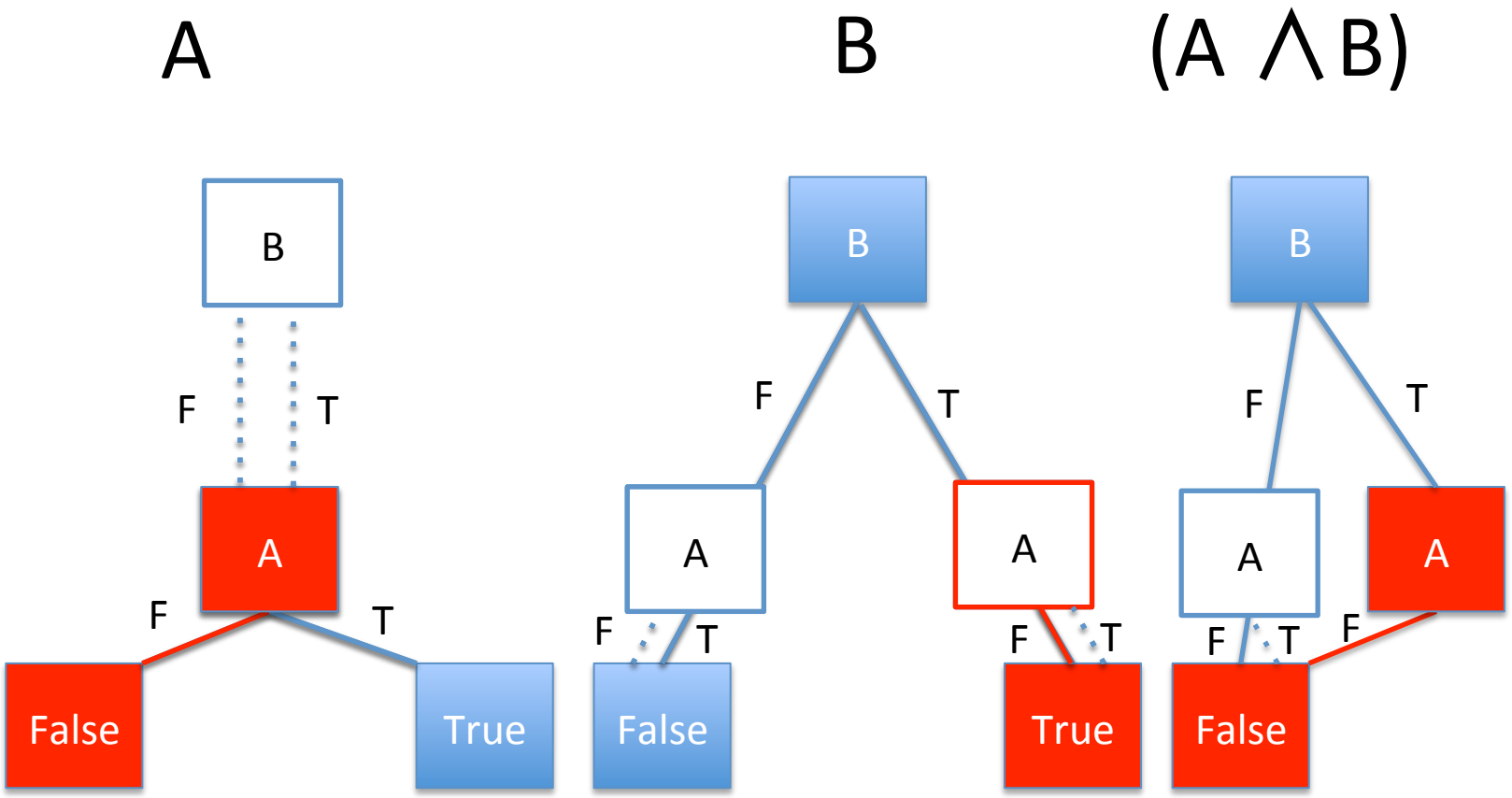
(A ⋀ B) Variables: A < B < C



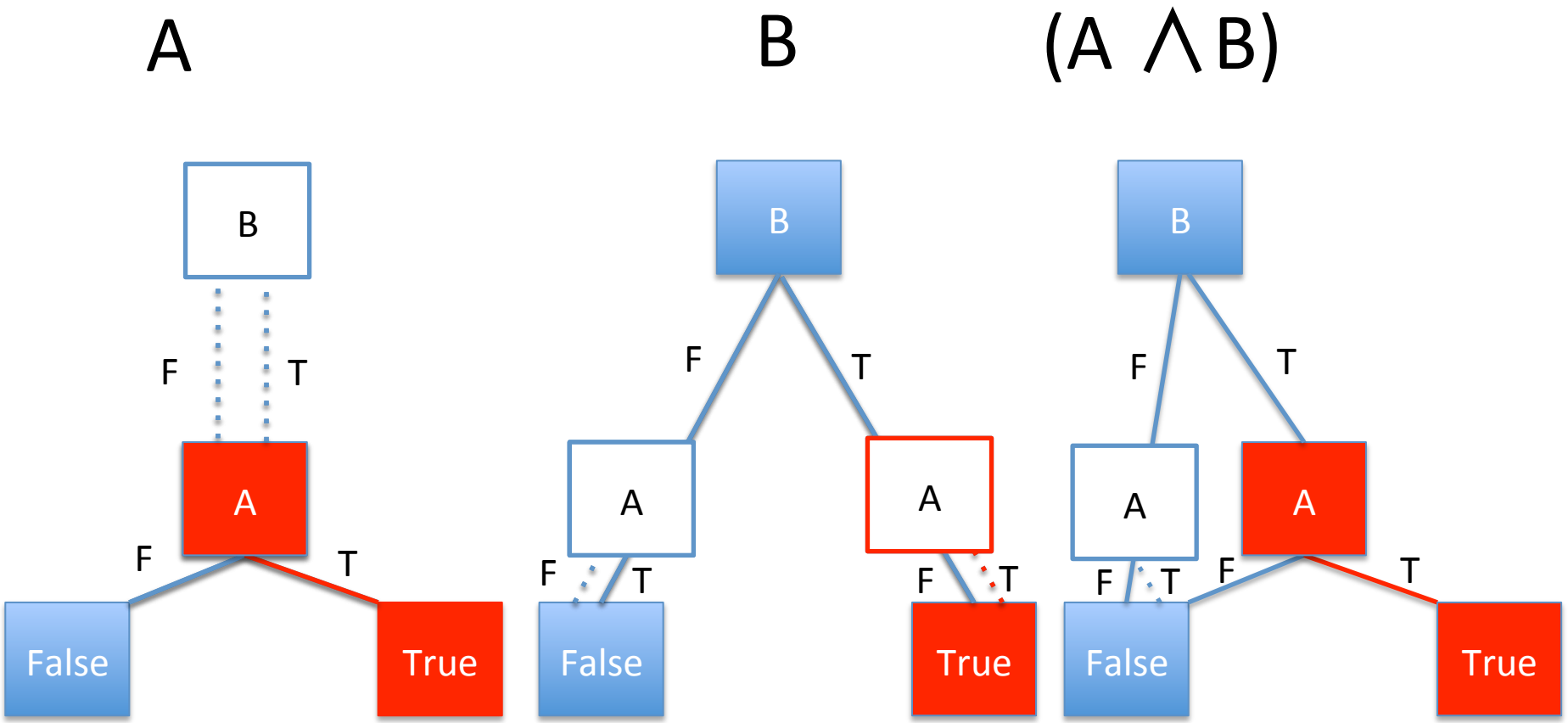
(A ⋀ B) Variables: A < B < C



$(A \wedge B)$ Variables: A < B < C

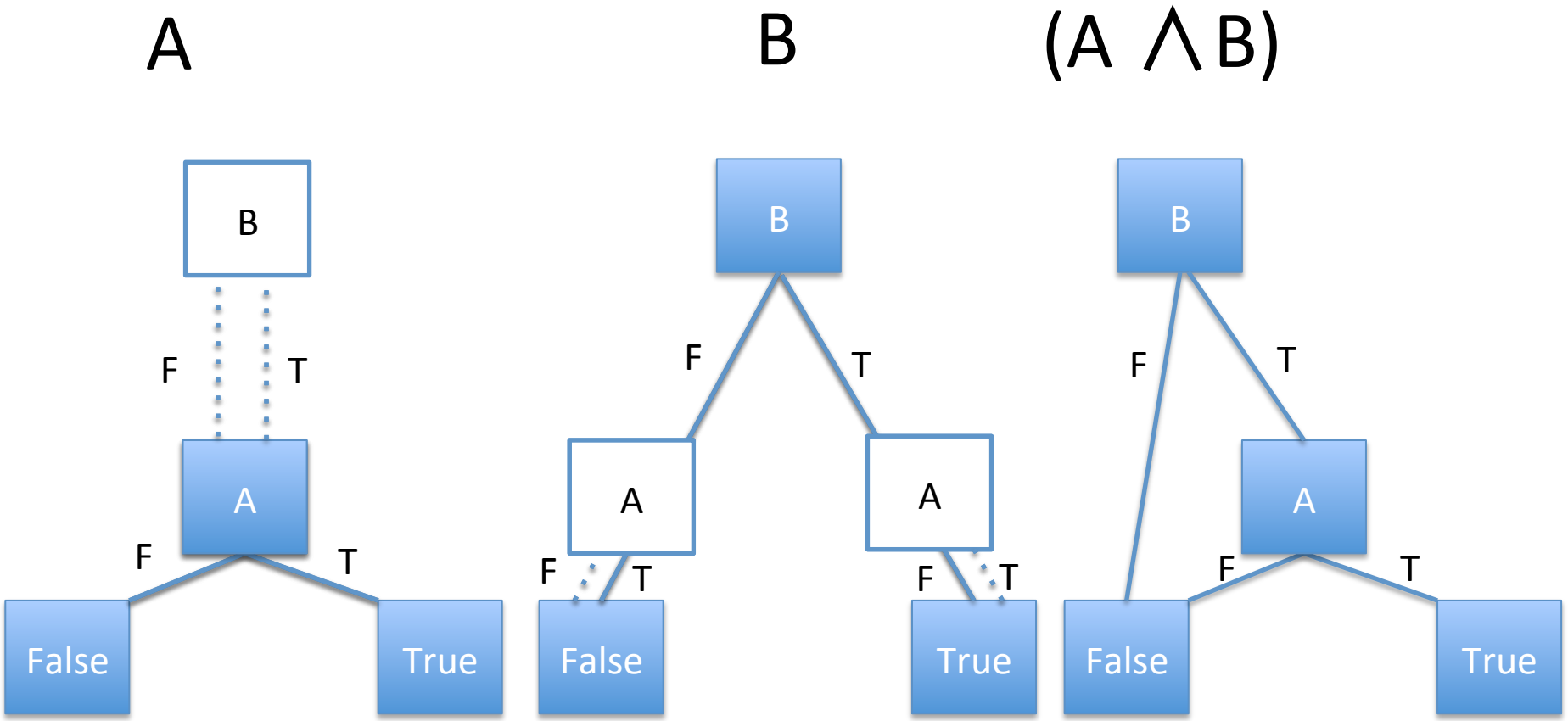


$(A \wedge B)$ Variables: A < B < C



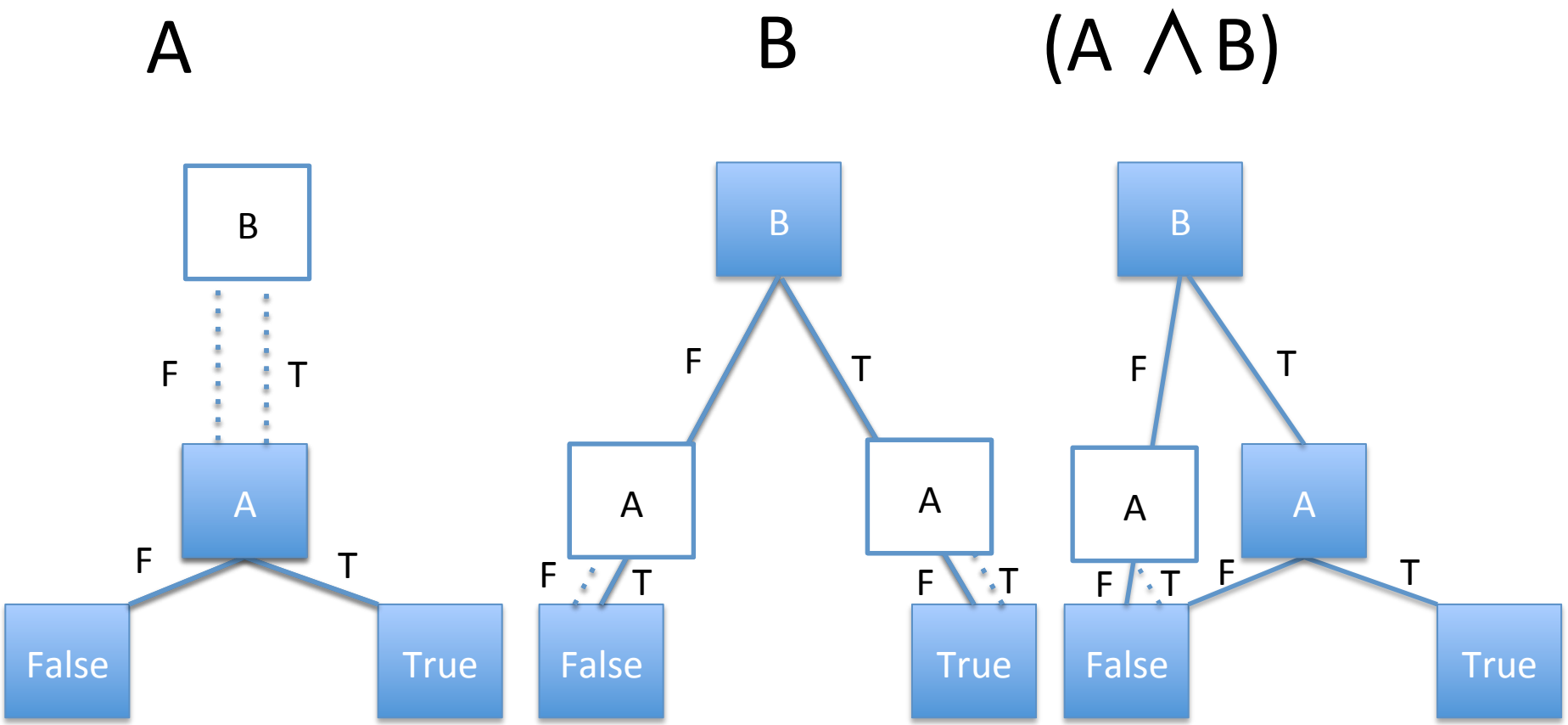
(A ∧ B) Variables: A < B < C

Final result for (A ∧ B)

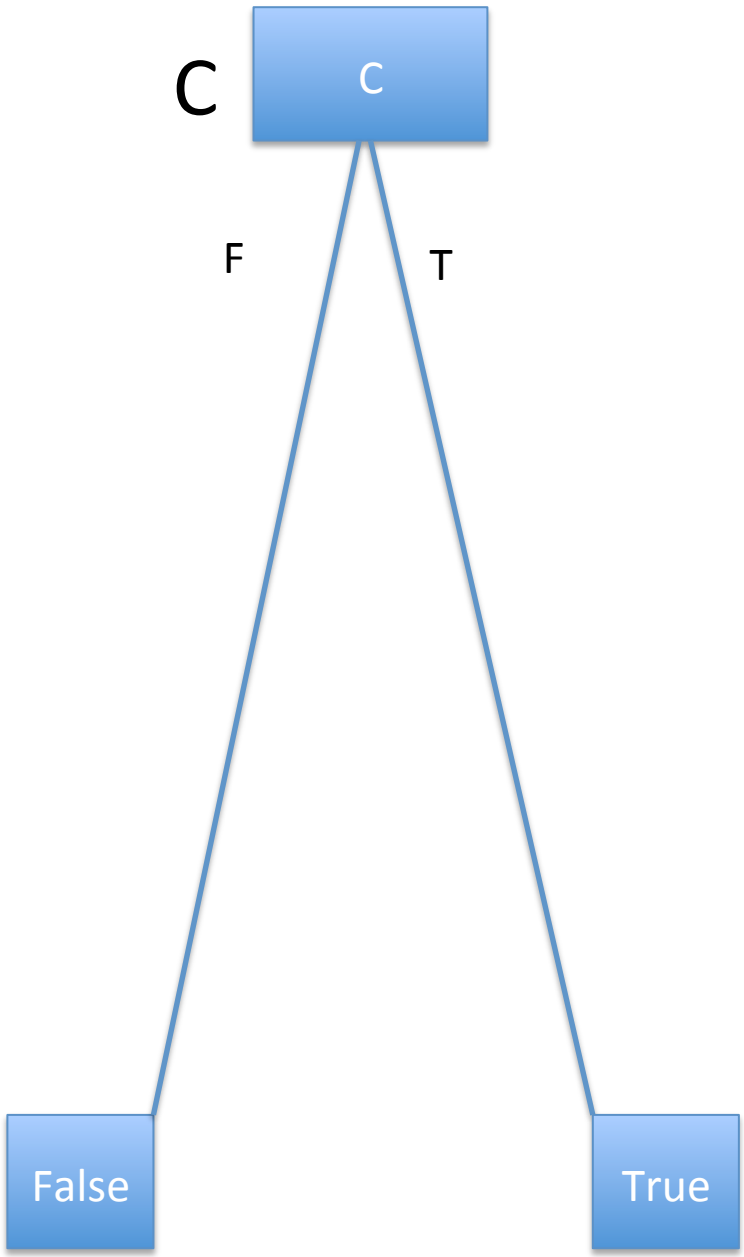


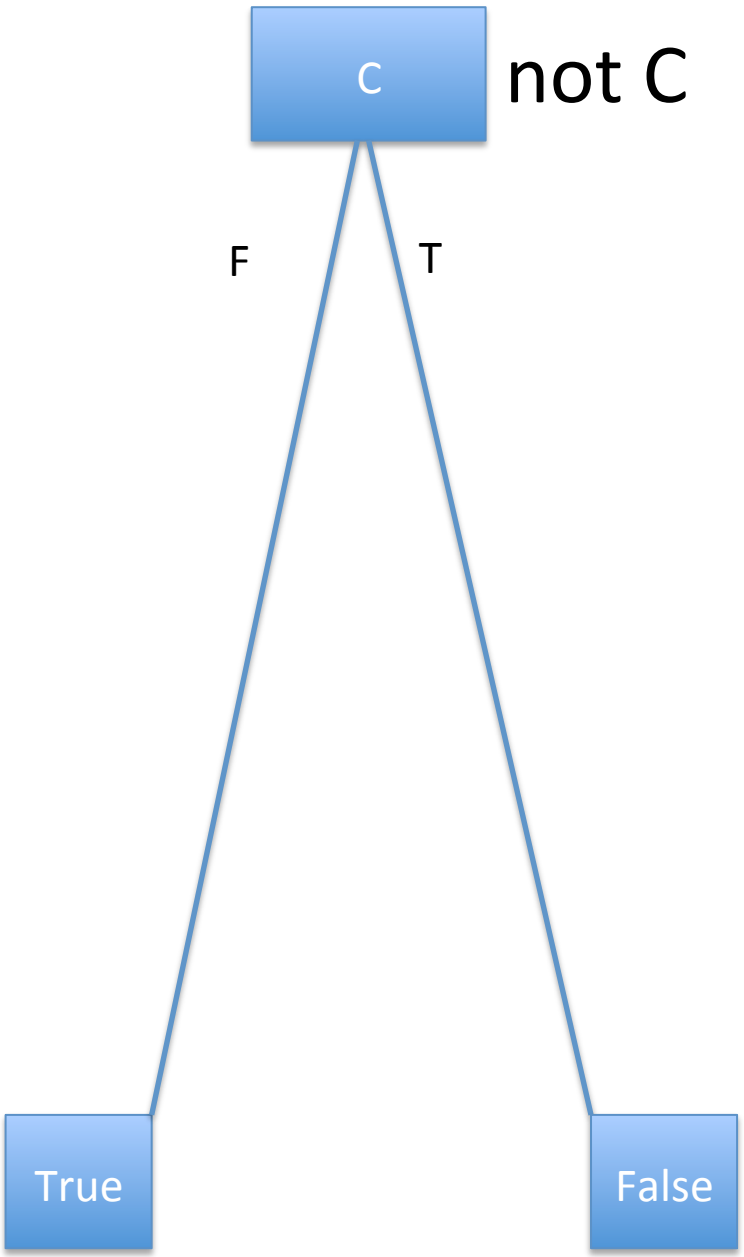
(A ⋀ B) Variables: A < B < C

Continuing to (A ⋀ B) ⋁ (not C)

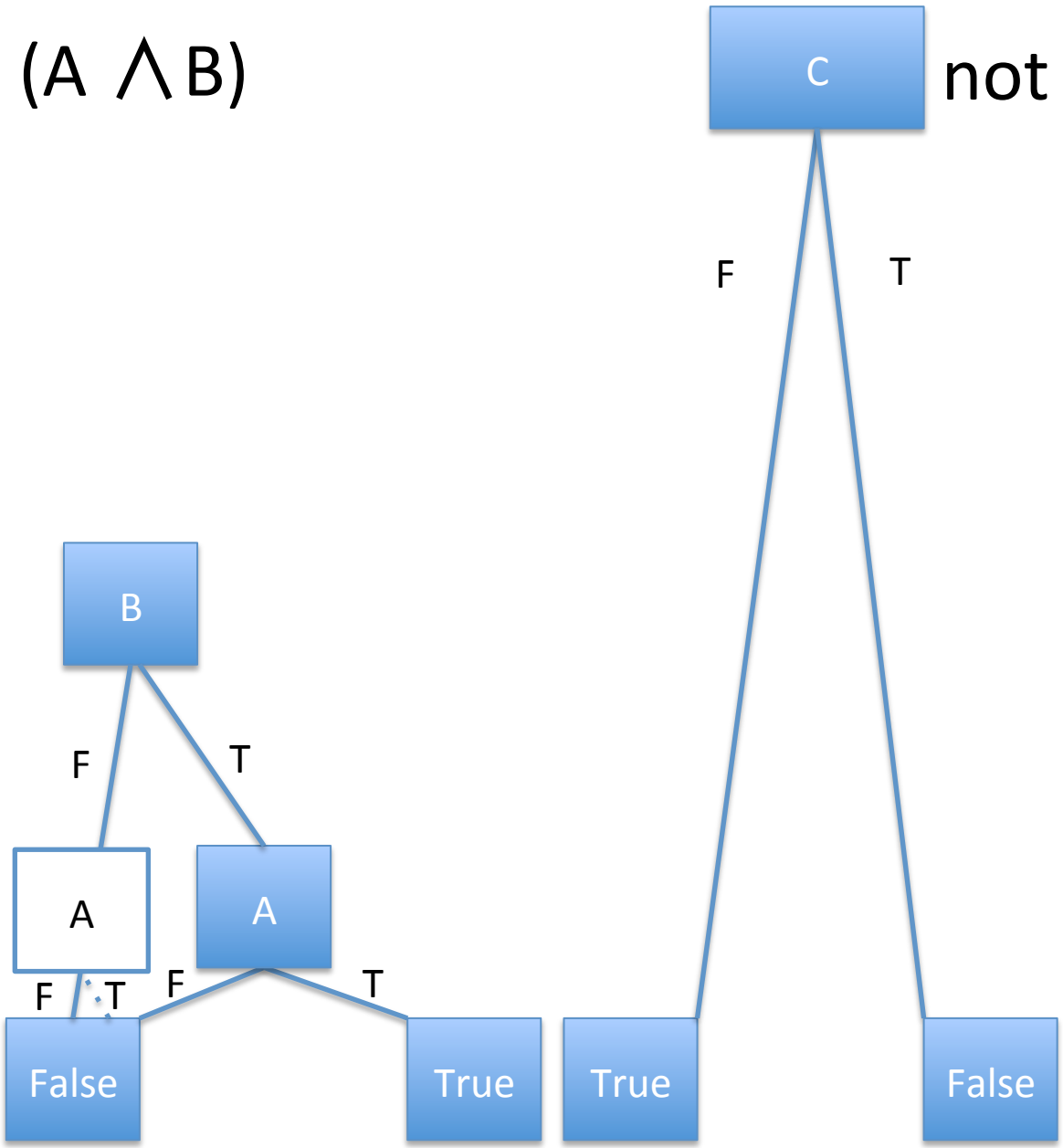


(not C) Variables: C > B > A

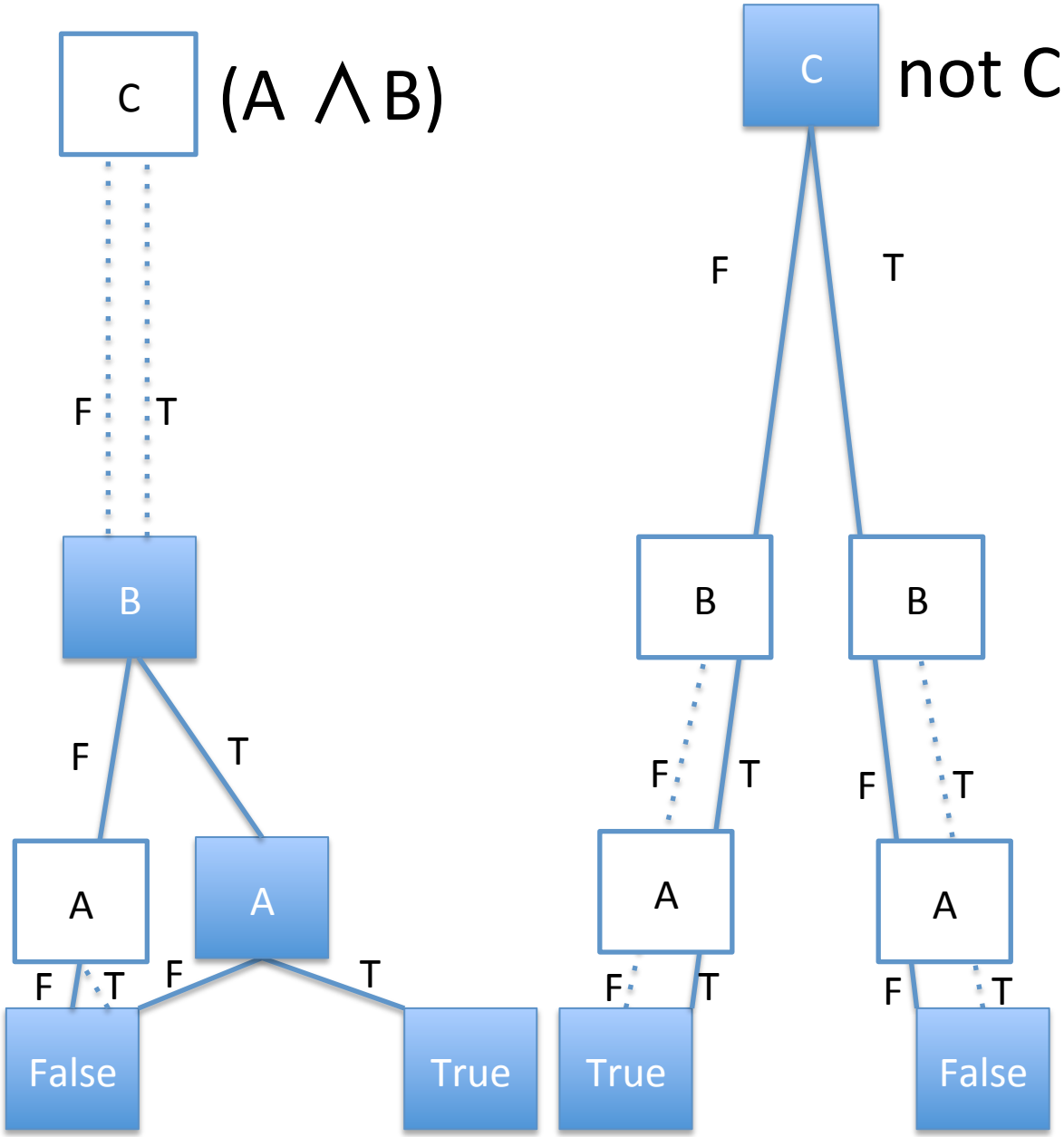




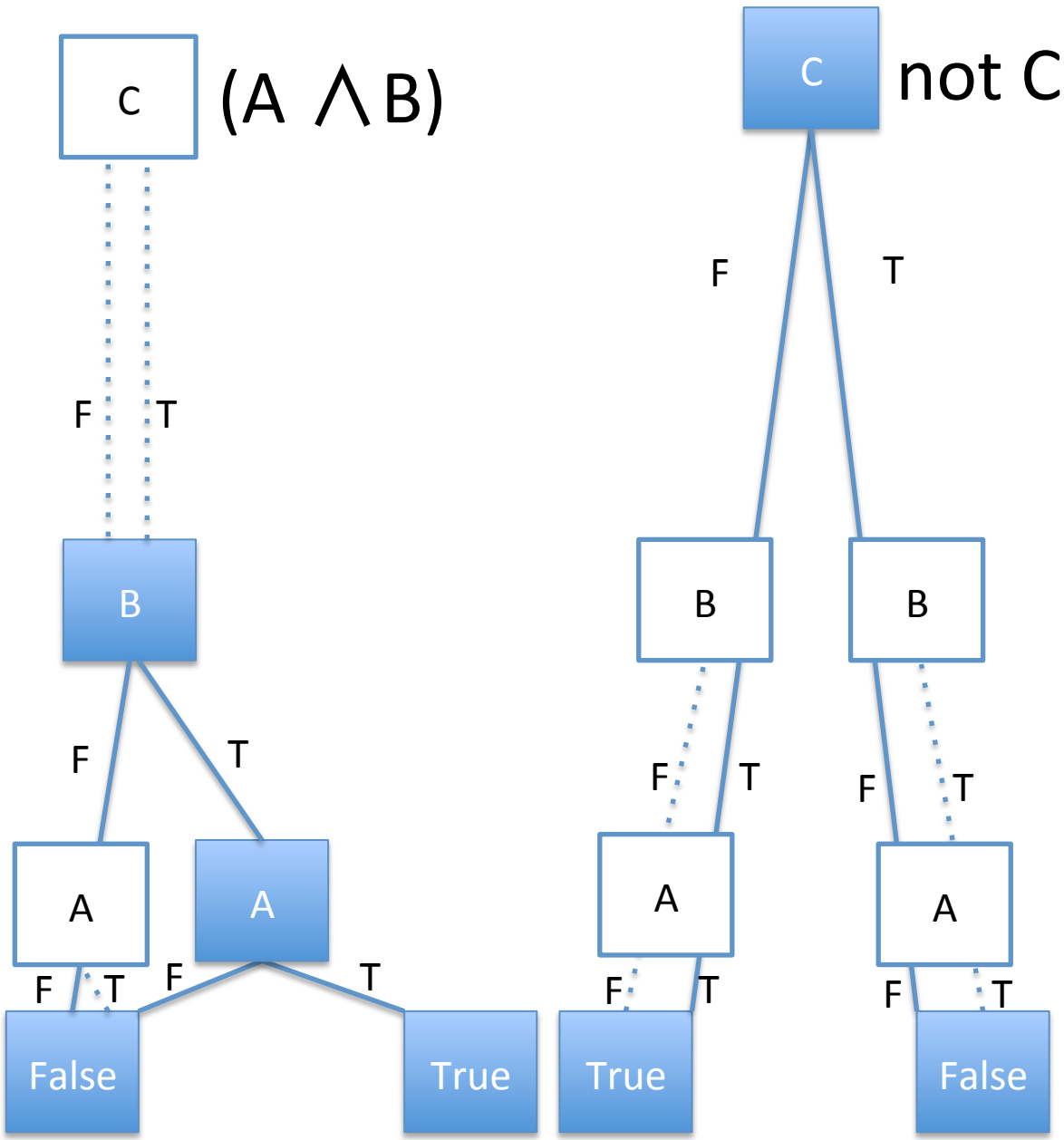
(A ∧ B) ∨ (not C) Variables: C > B > A



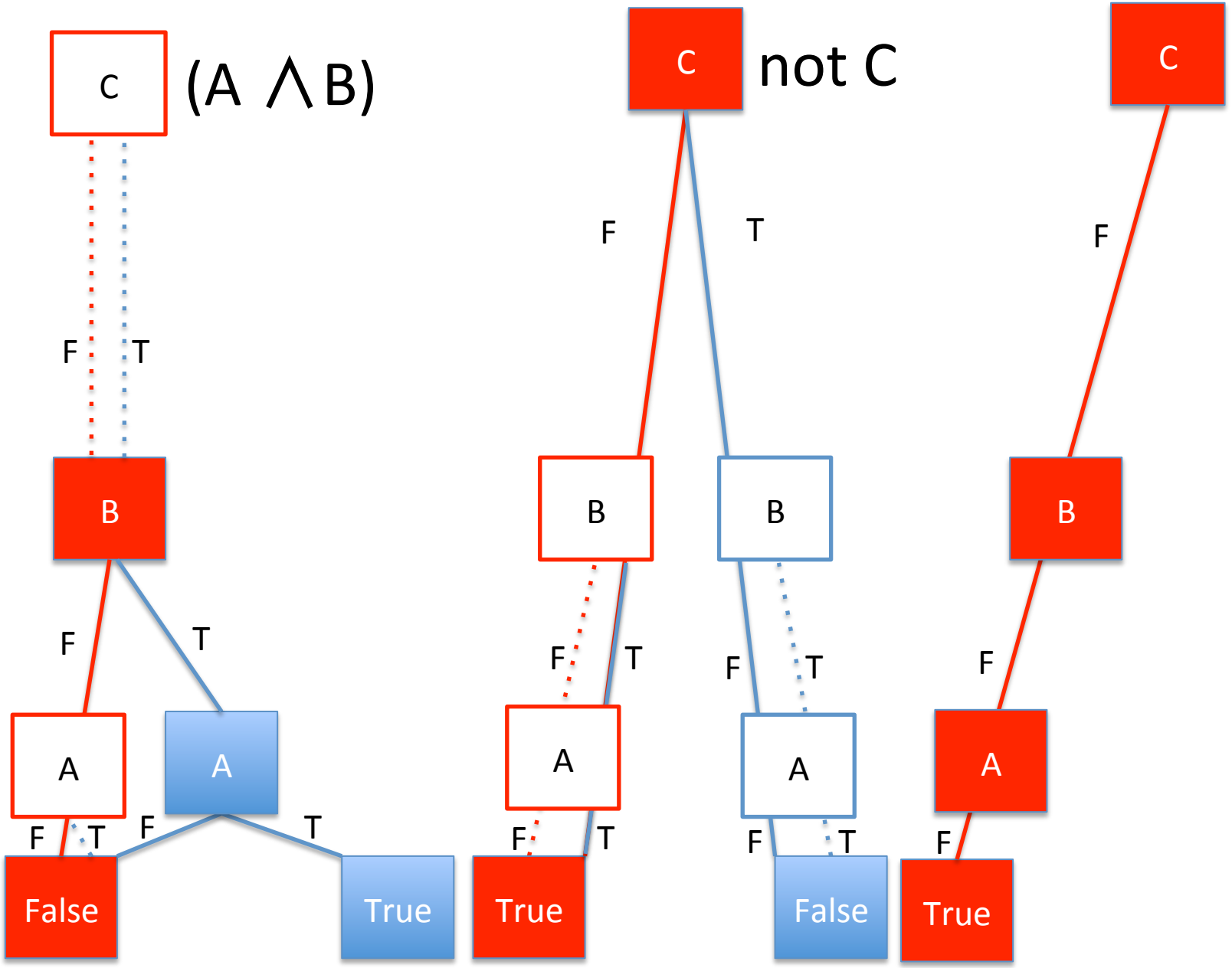
(A ∧ B) ∨ (not C) Variables: C > B > A



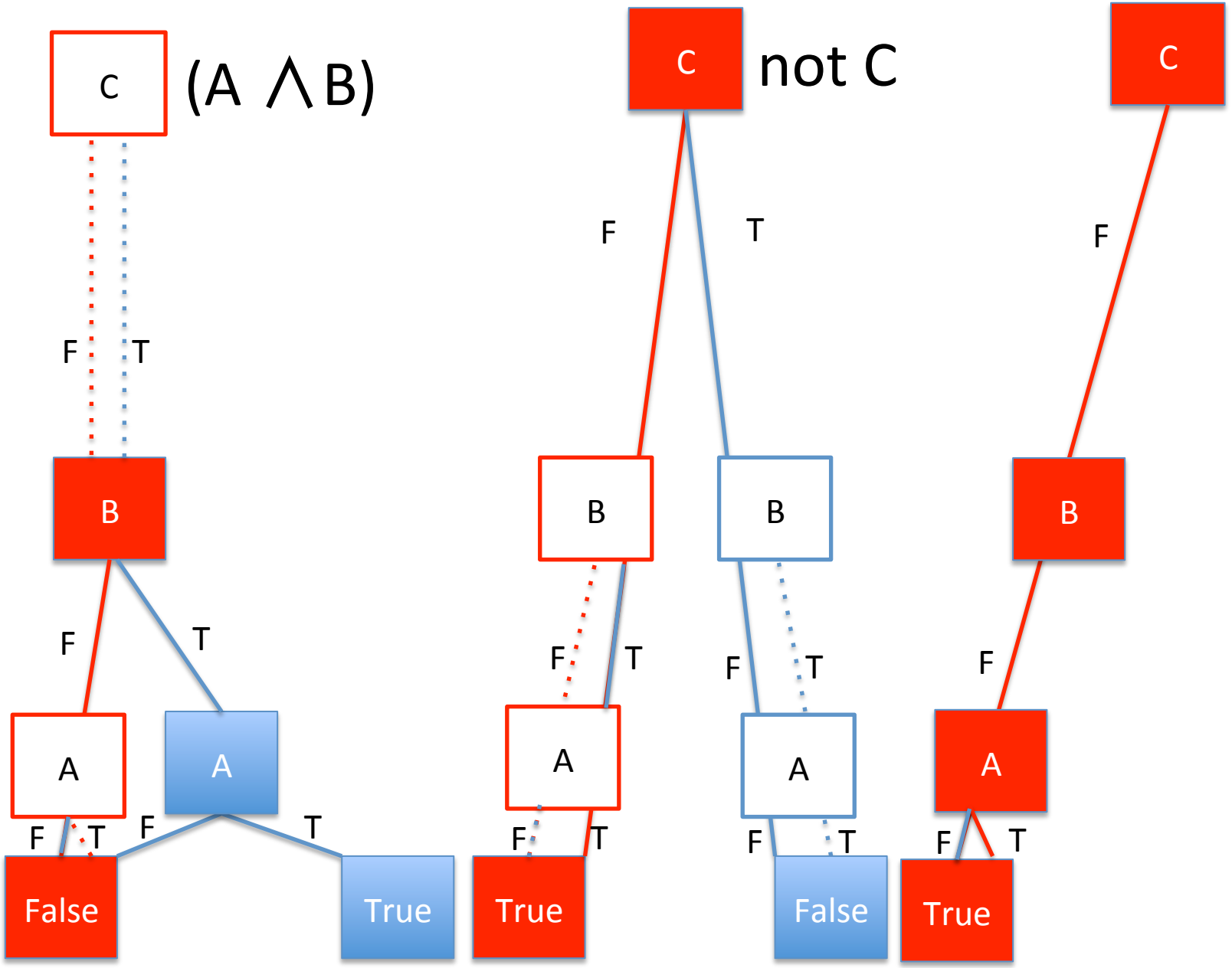
(A ∧ B) ∨ (not C) Variables: C > B > A



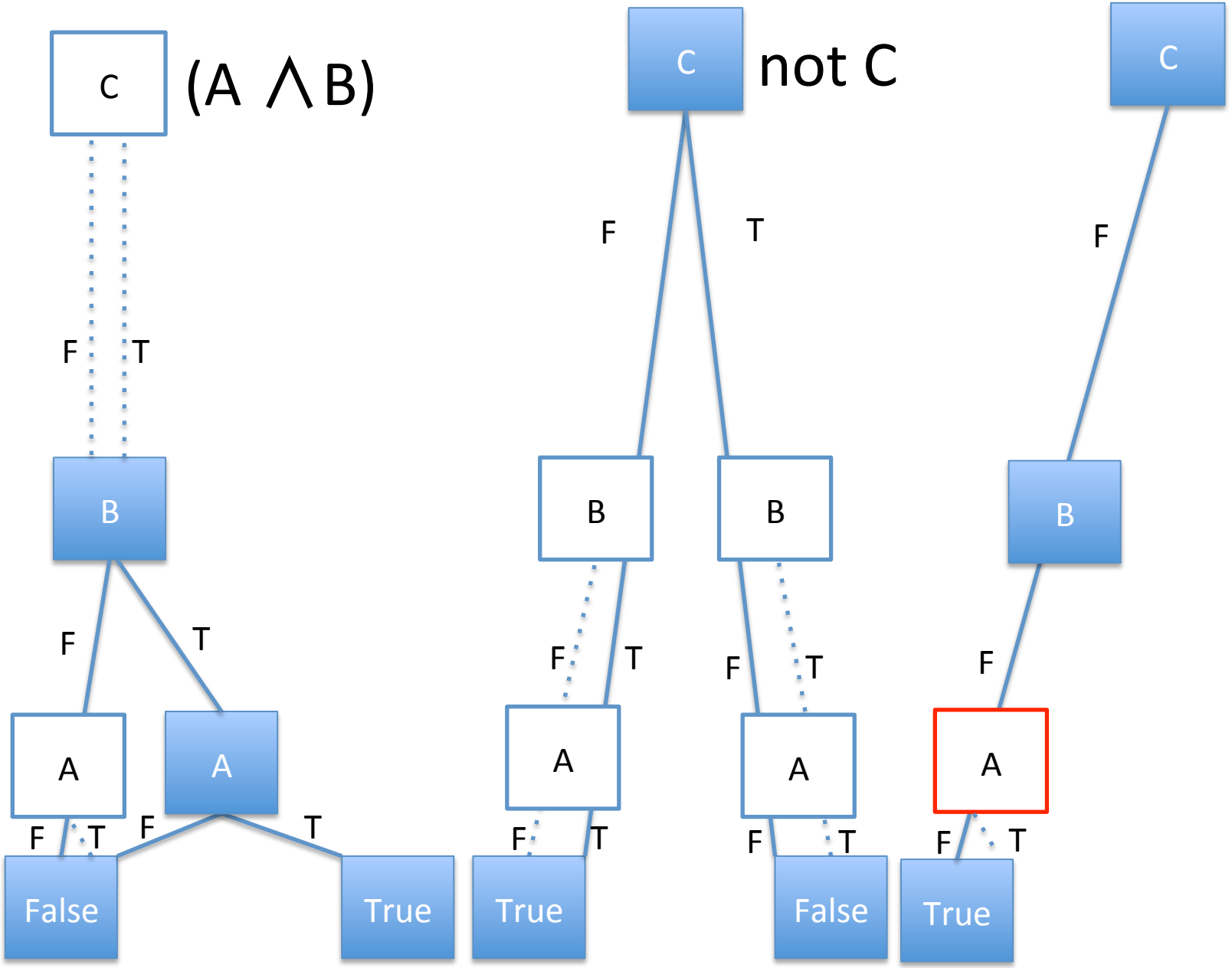
(A ∧ B) ∨ (not C) Variables: C > B > A



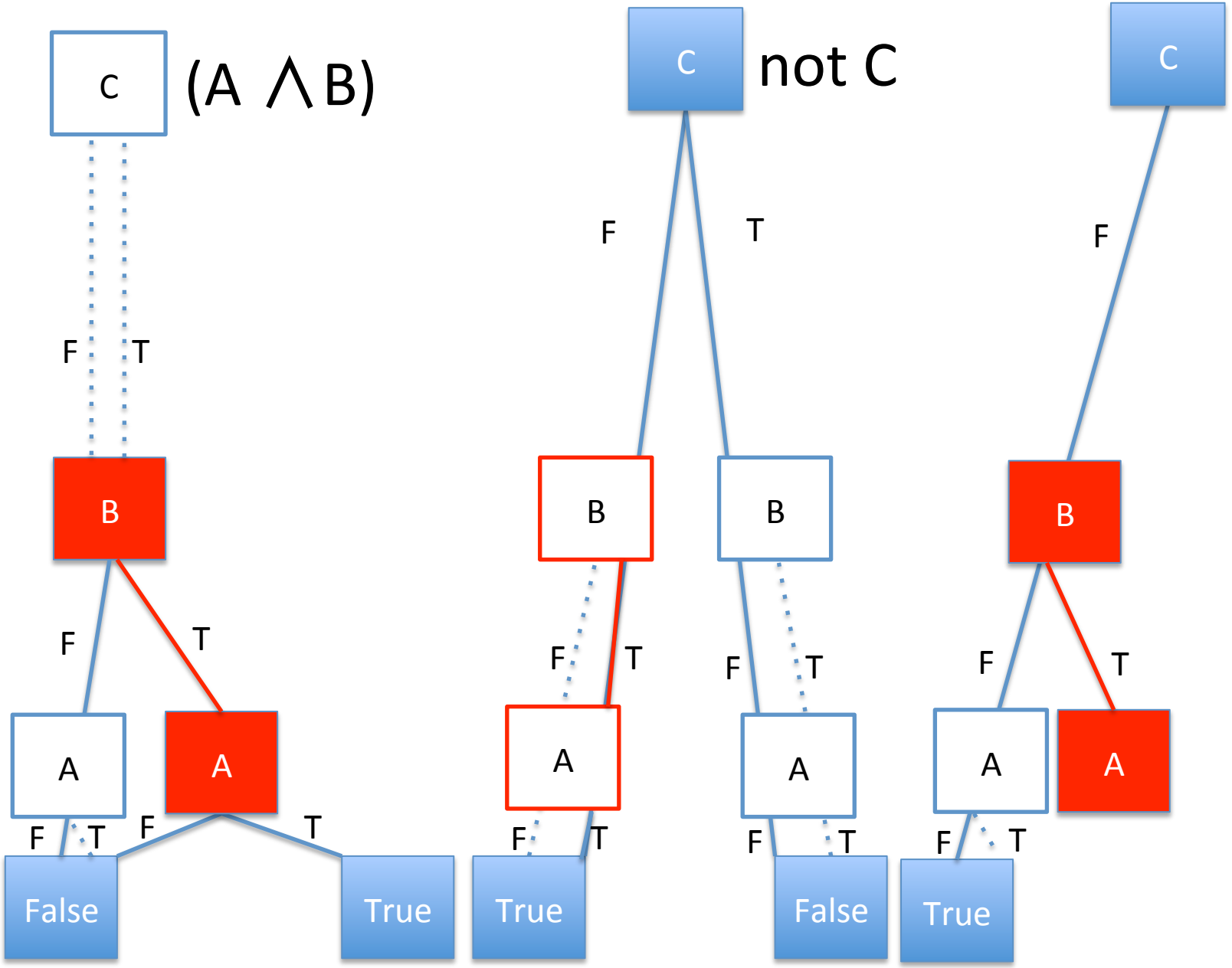
(A ∧ B) ∨ (not C) Variables: C > B > A



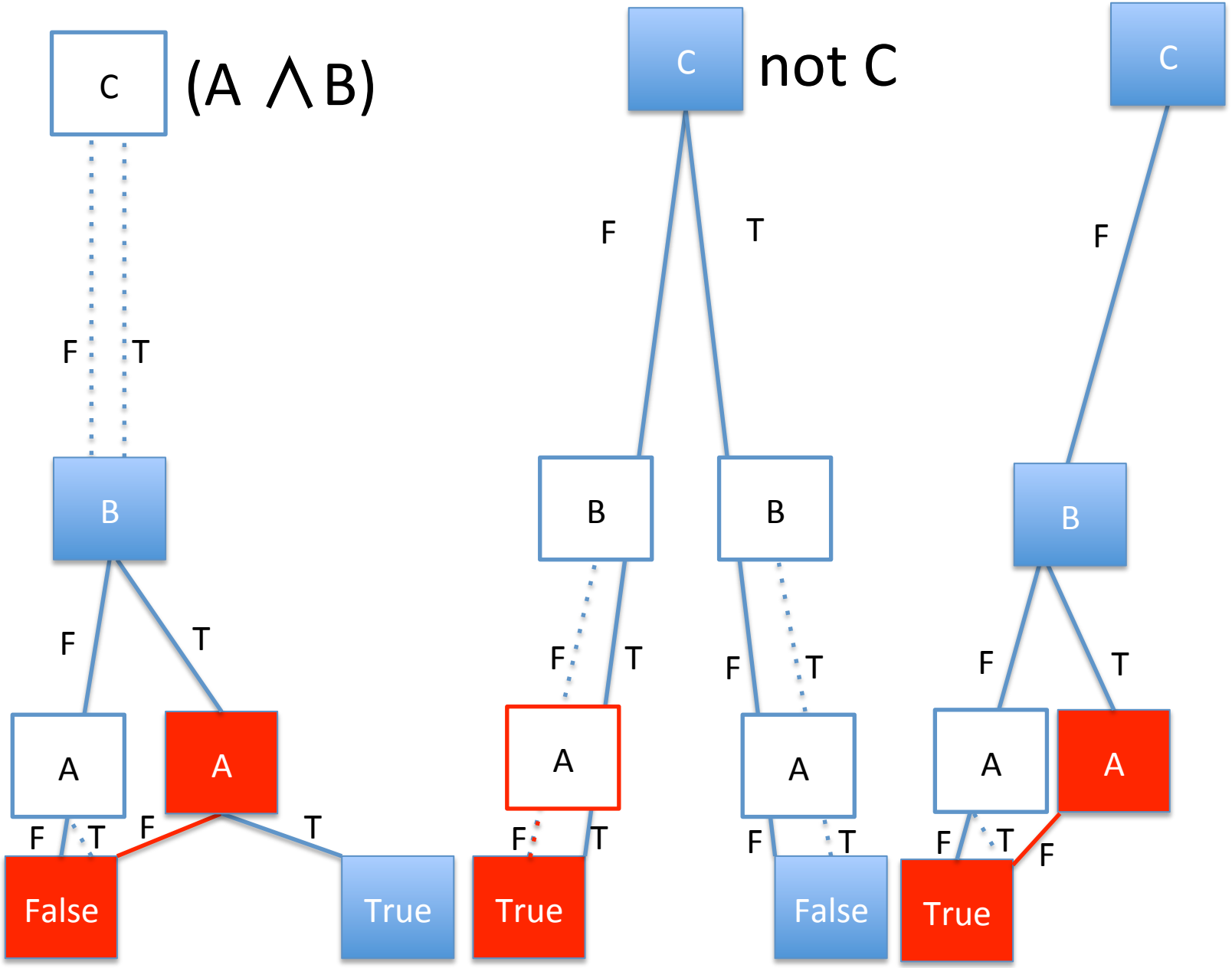
(A ∧ B) ∨ (not C) Variables: C > B > A



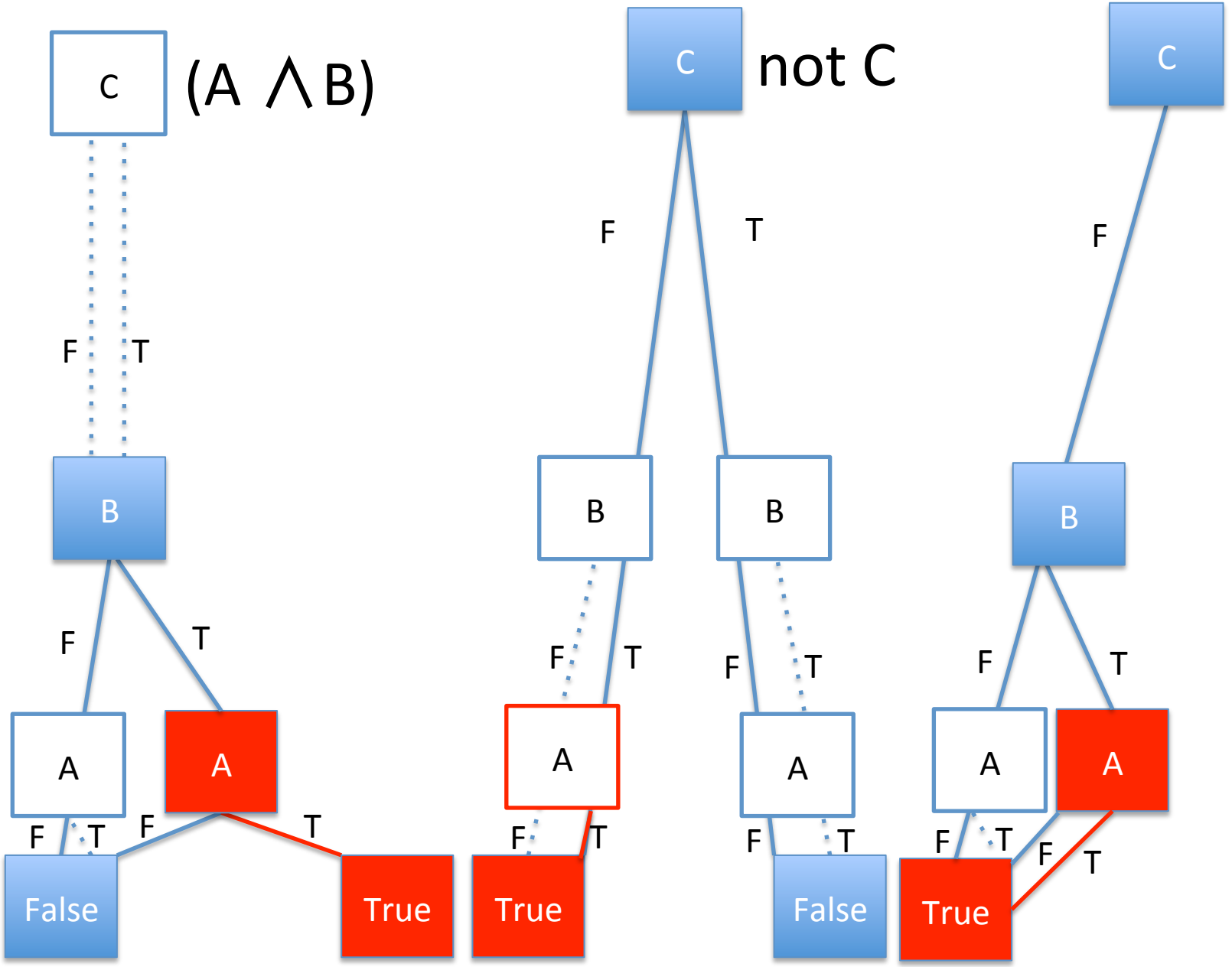
(A ∧ B) ∨ (not C) Variables: C > B > A



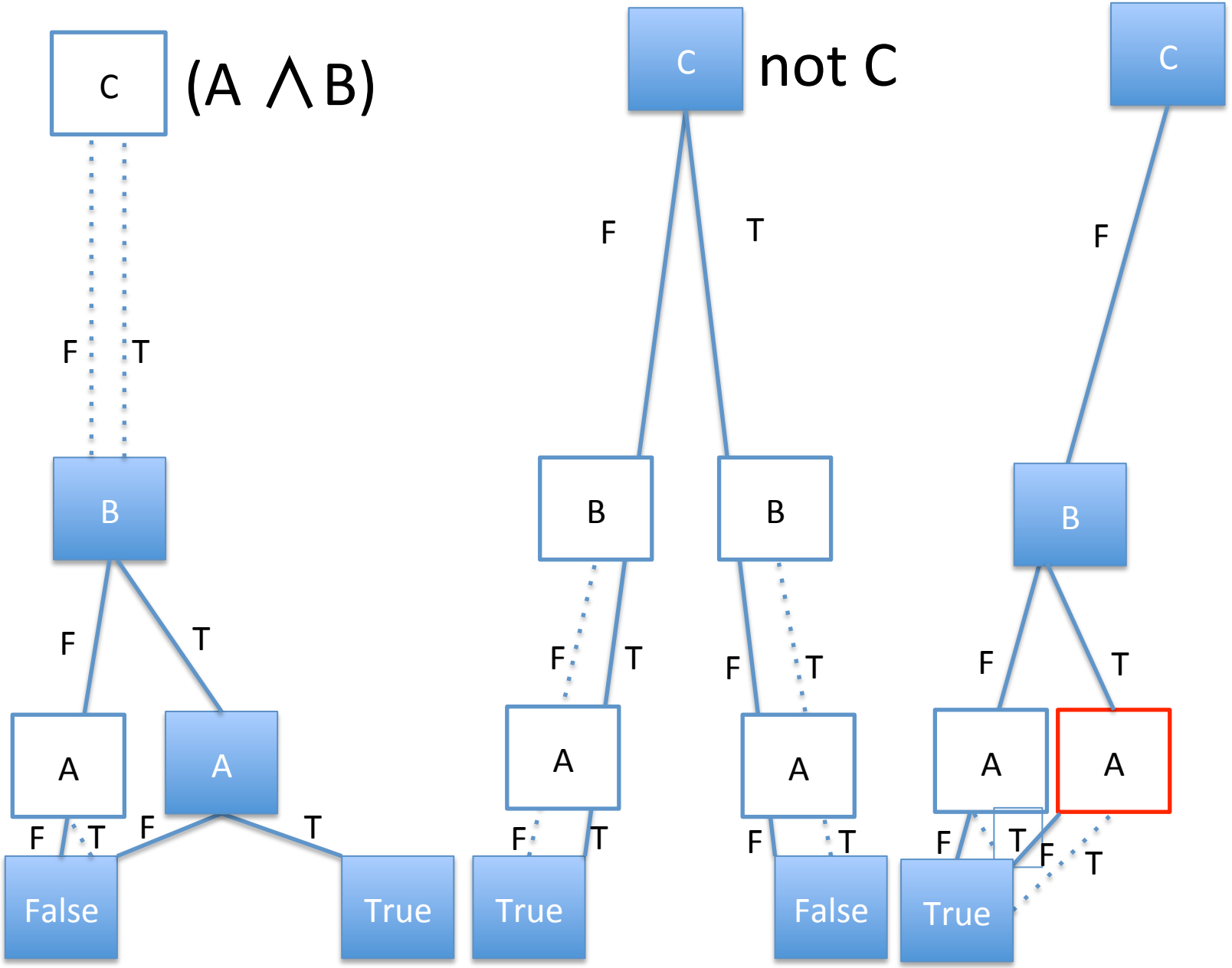
(A ∧ B) ∨ (not C) Variables: C > B > A



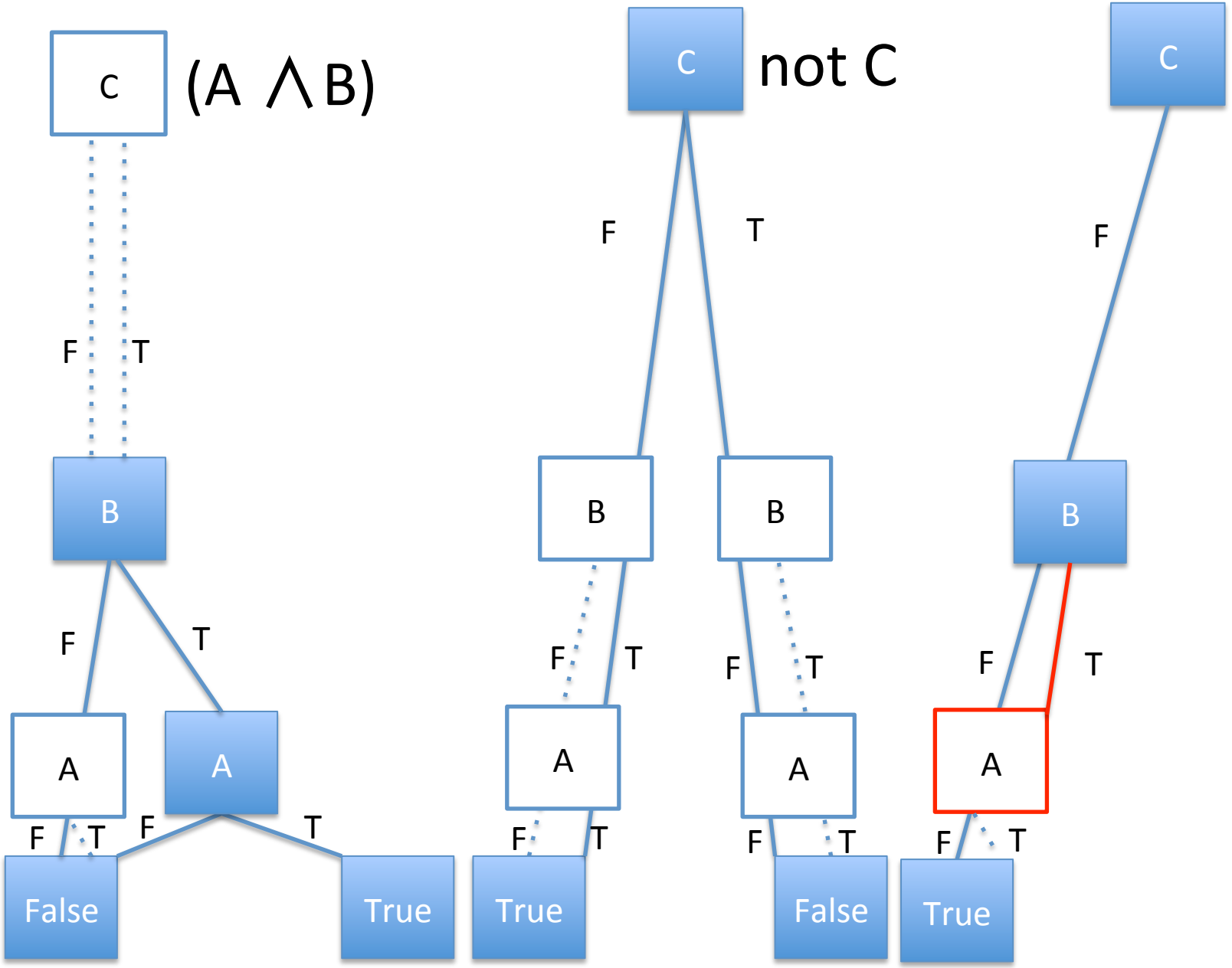
(A ∧ B) ∨ (not C) Variables: C > B > A



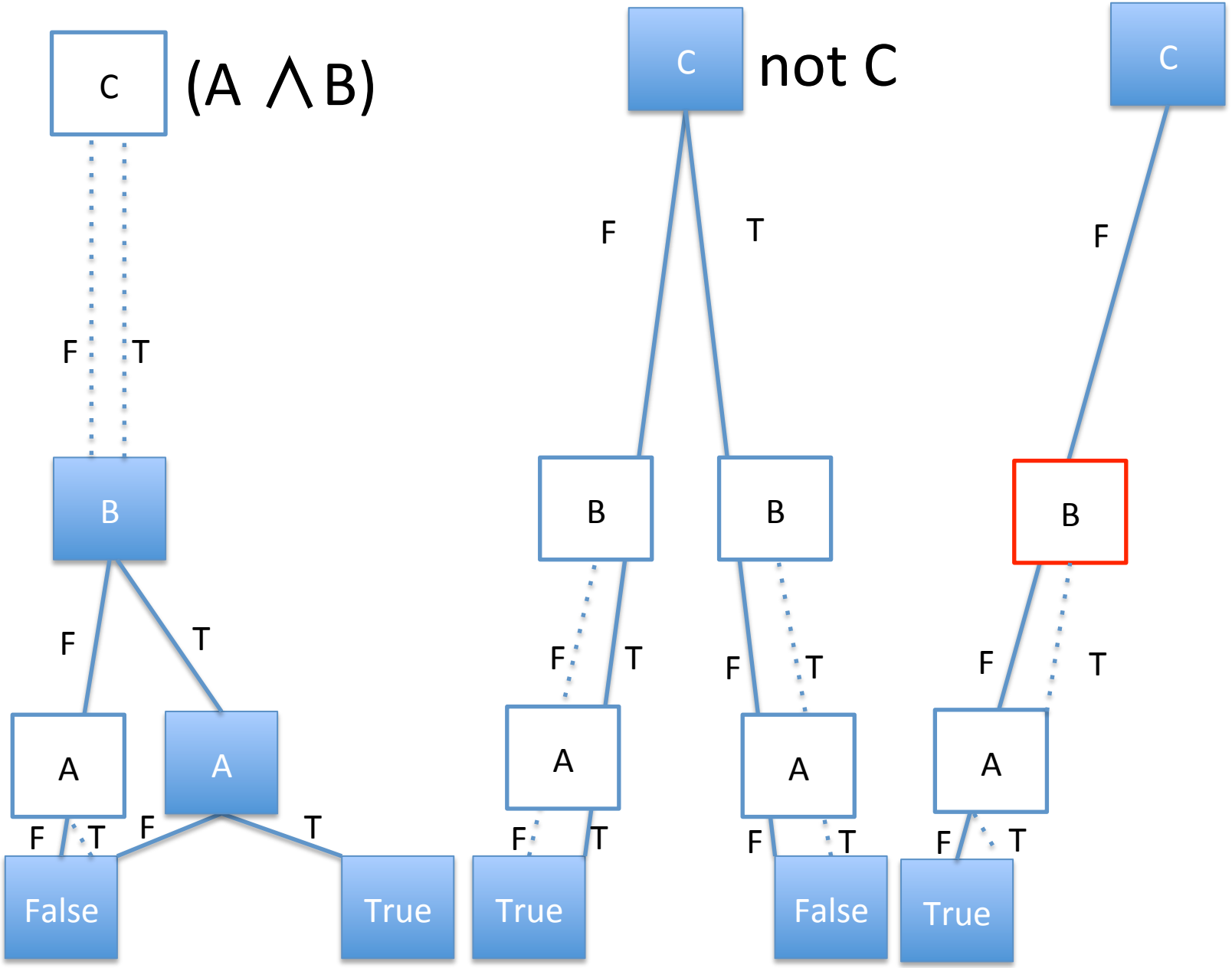
(A ∧ B) ∨ (not C) Variables: C > B > A



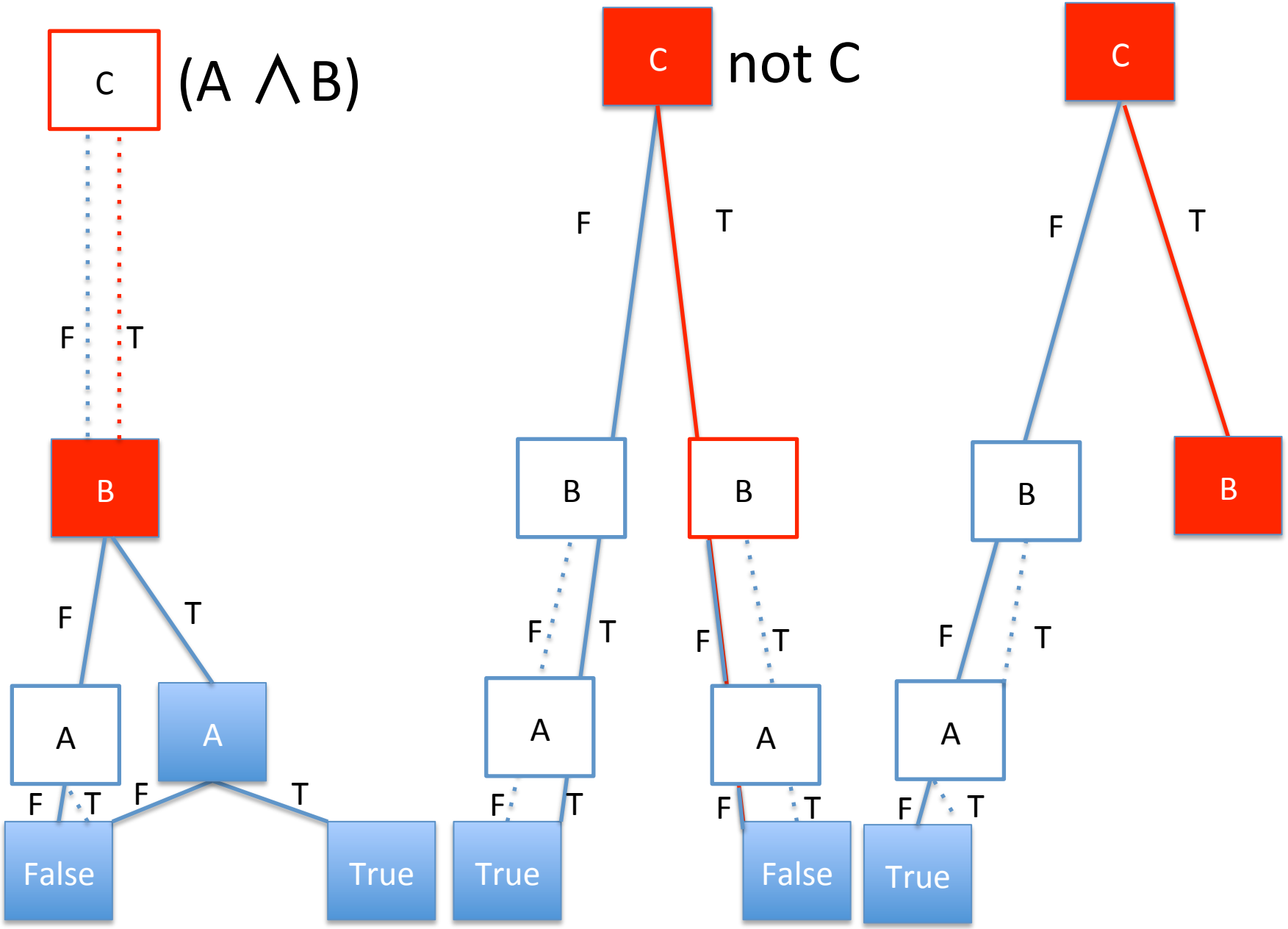
(A ∧ B) ∨ (not C) Variables: C > B > A



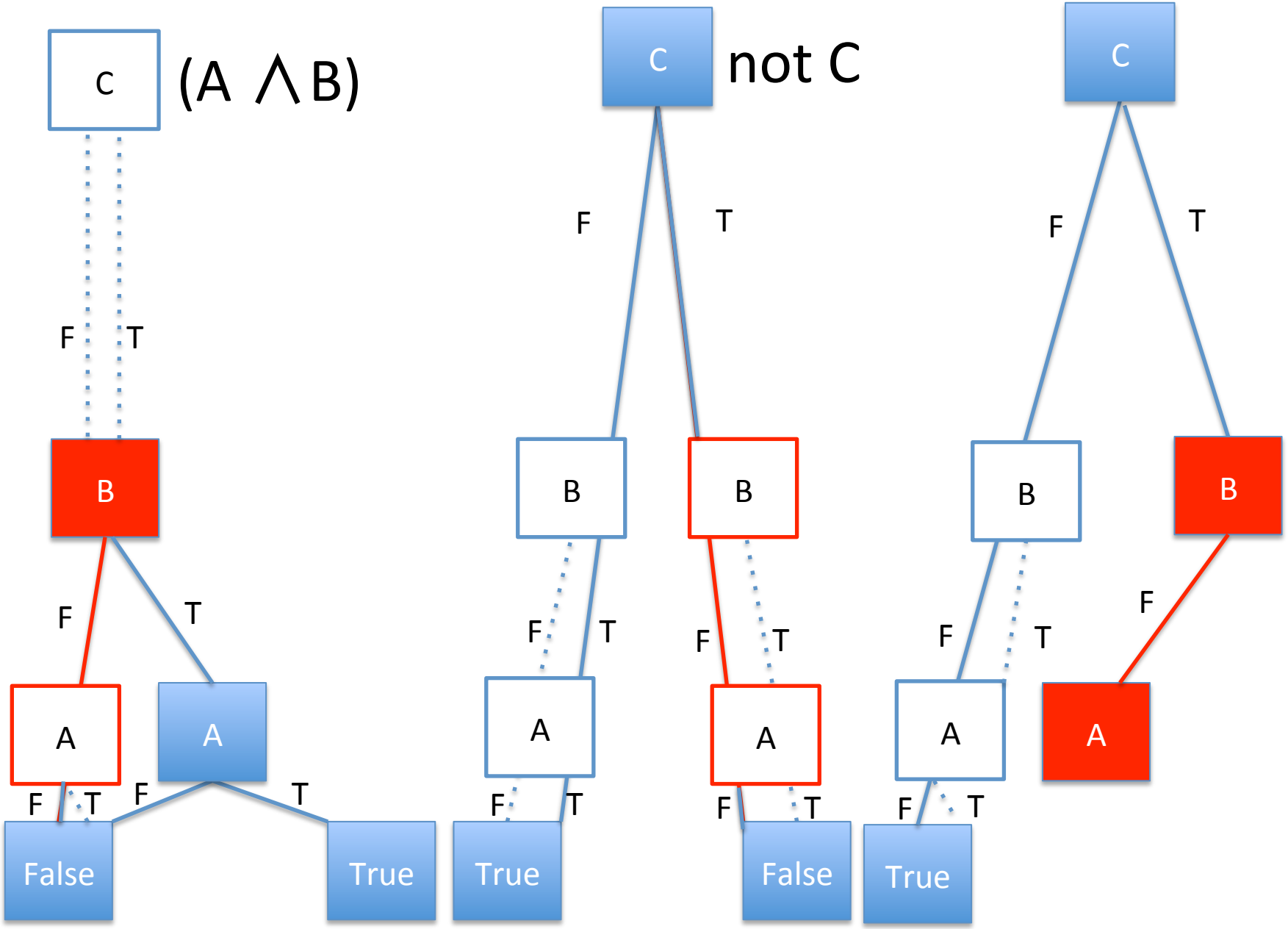
(A ∧ B) ∨ (not C) Variables: C > B > A



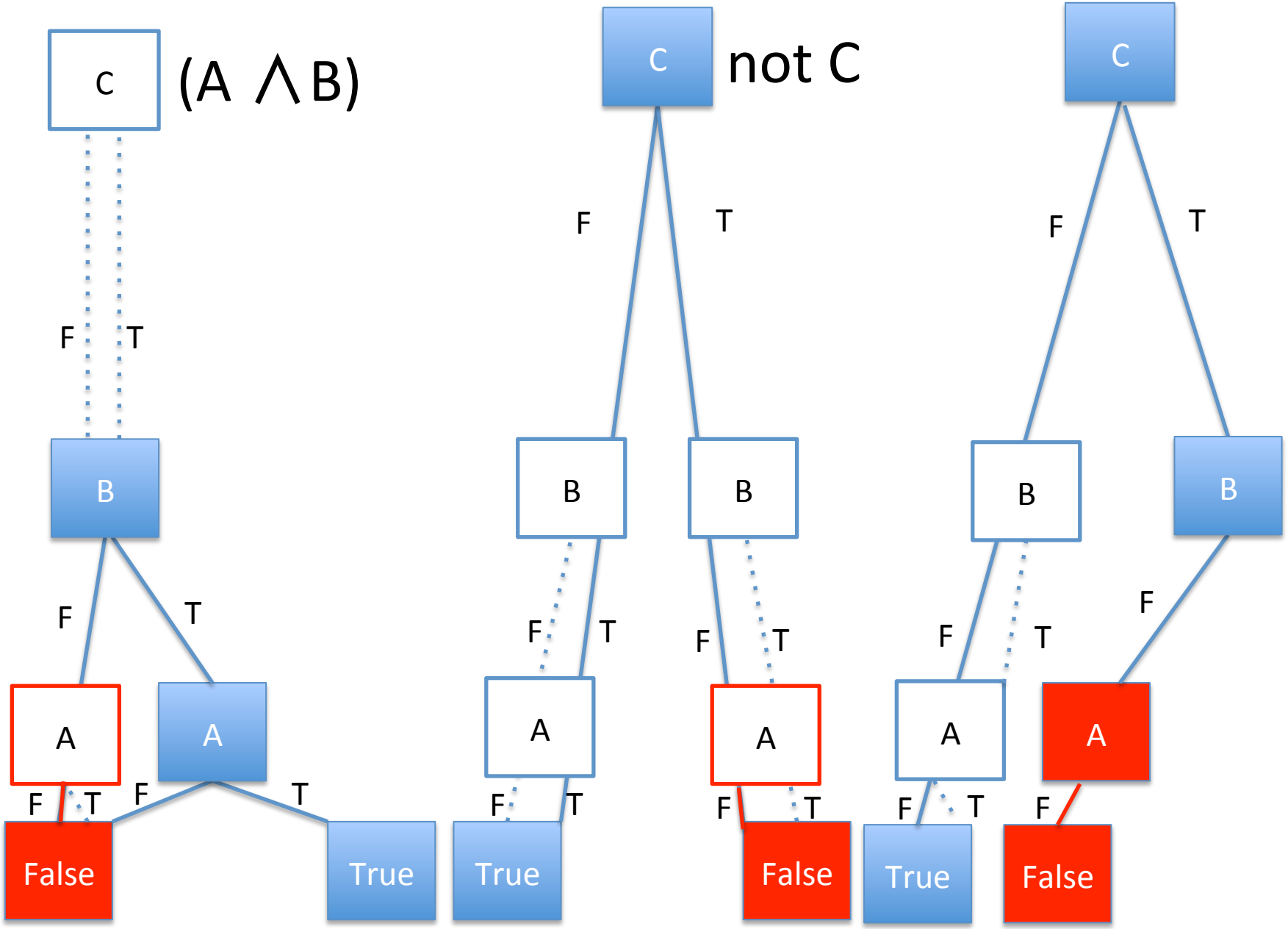
(A ∧ B) ∨ (not C) Variables: C > B > A



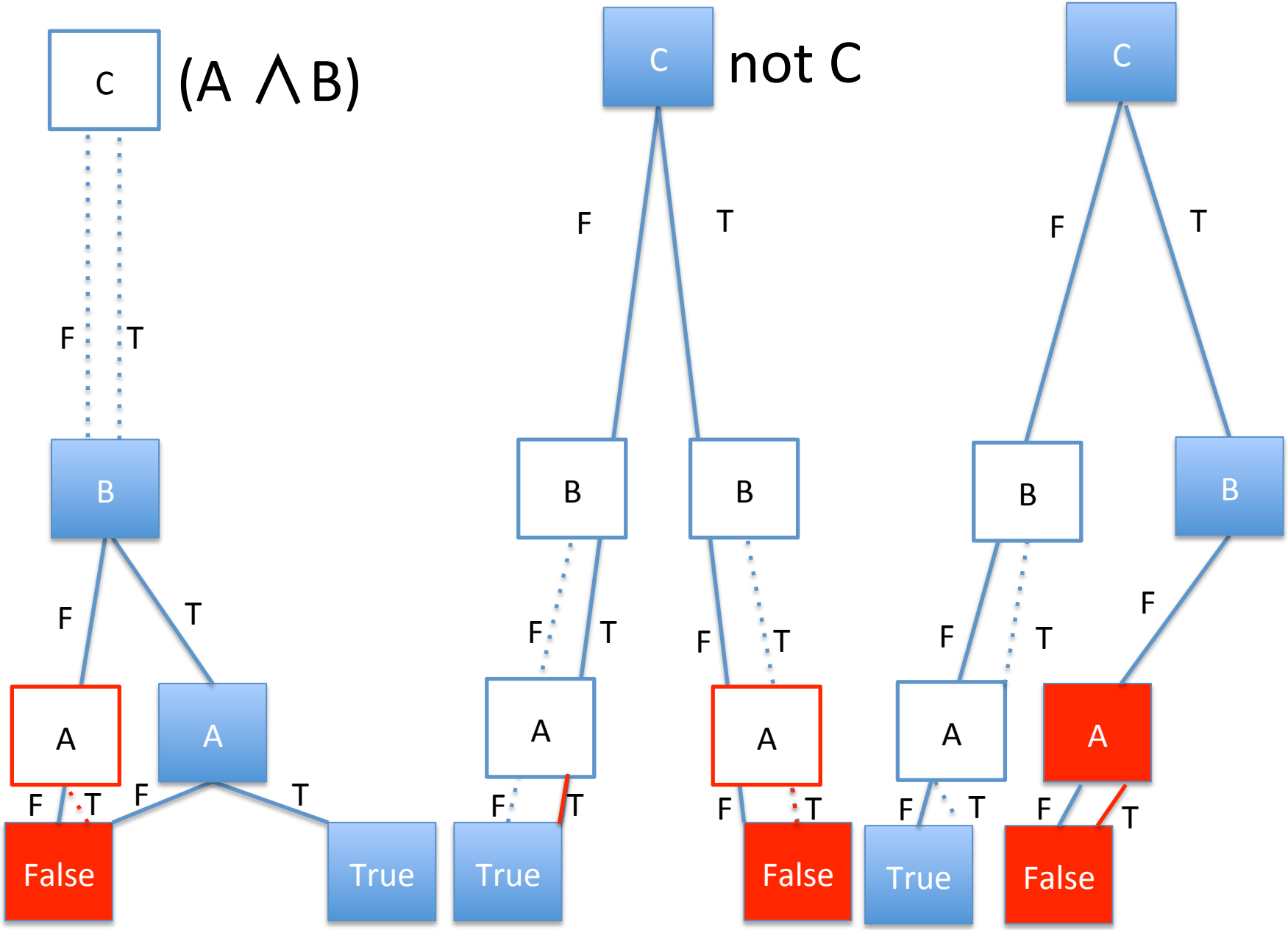
(A ∧ B) ∨ (not C) Variables: C > B > A



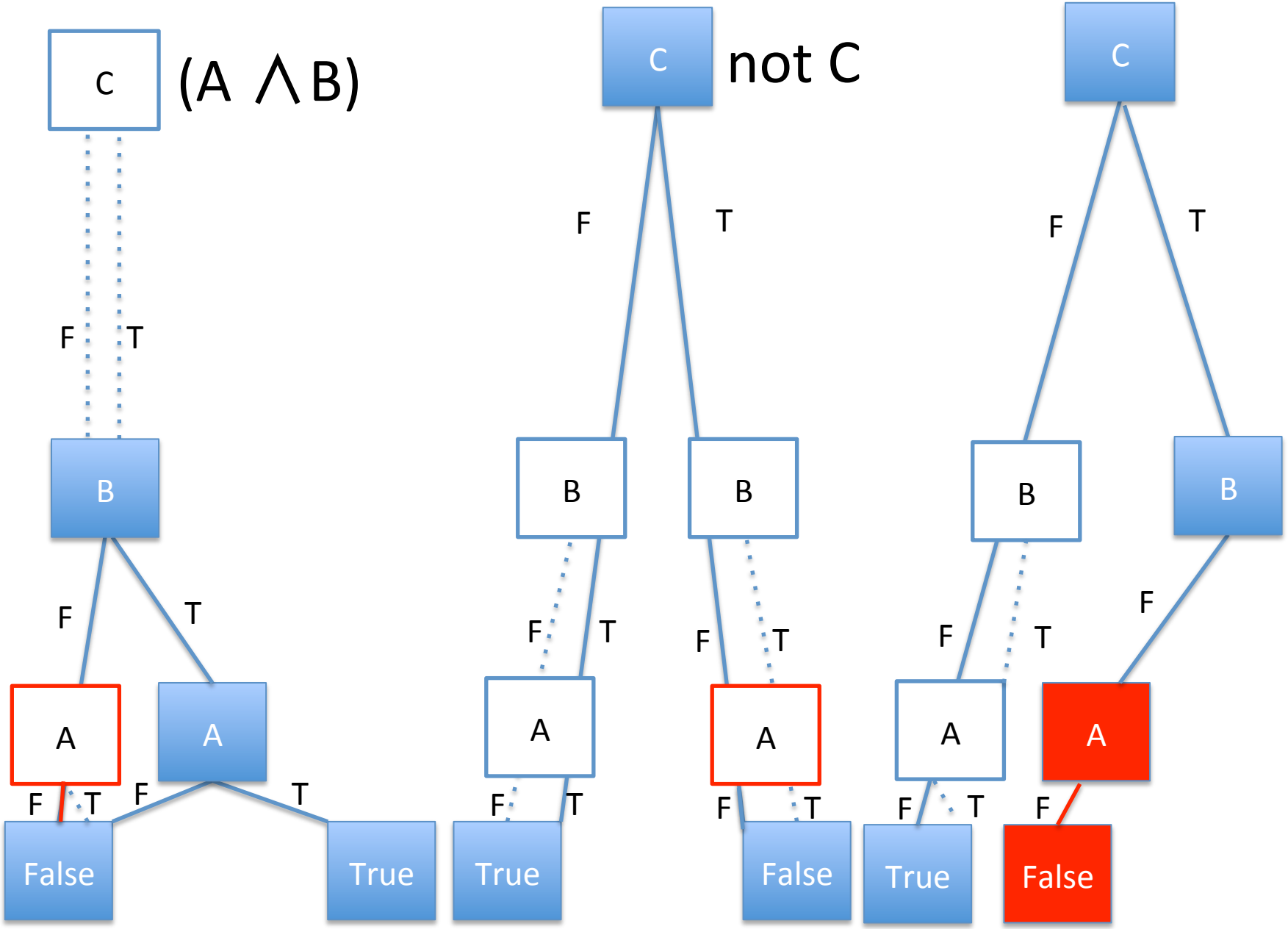
(A ∧ B) ∨ (not C) Variables: C > B > A



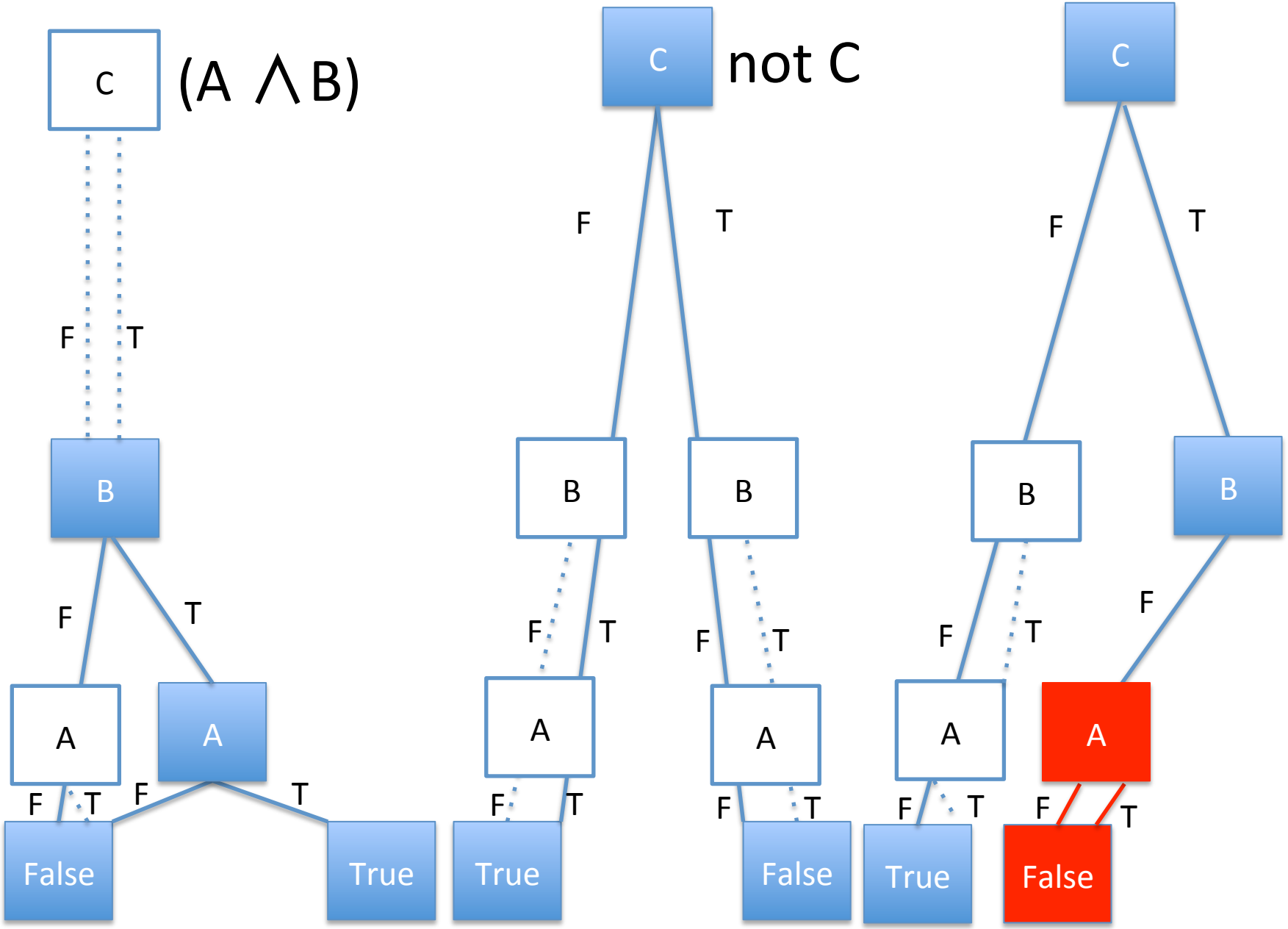
(A ∧ B) ∨ (not C) Variables: C > B > A



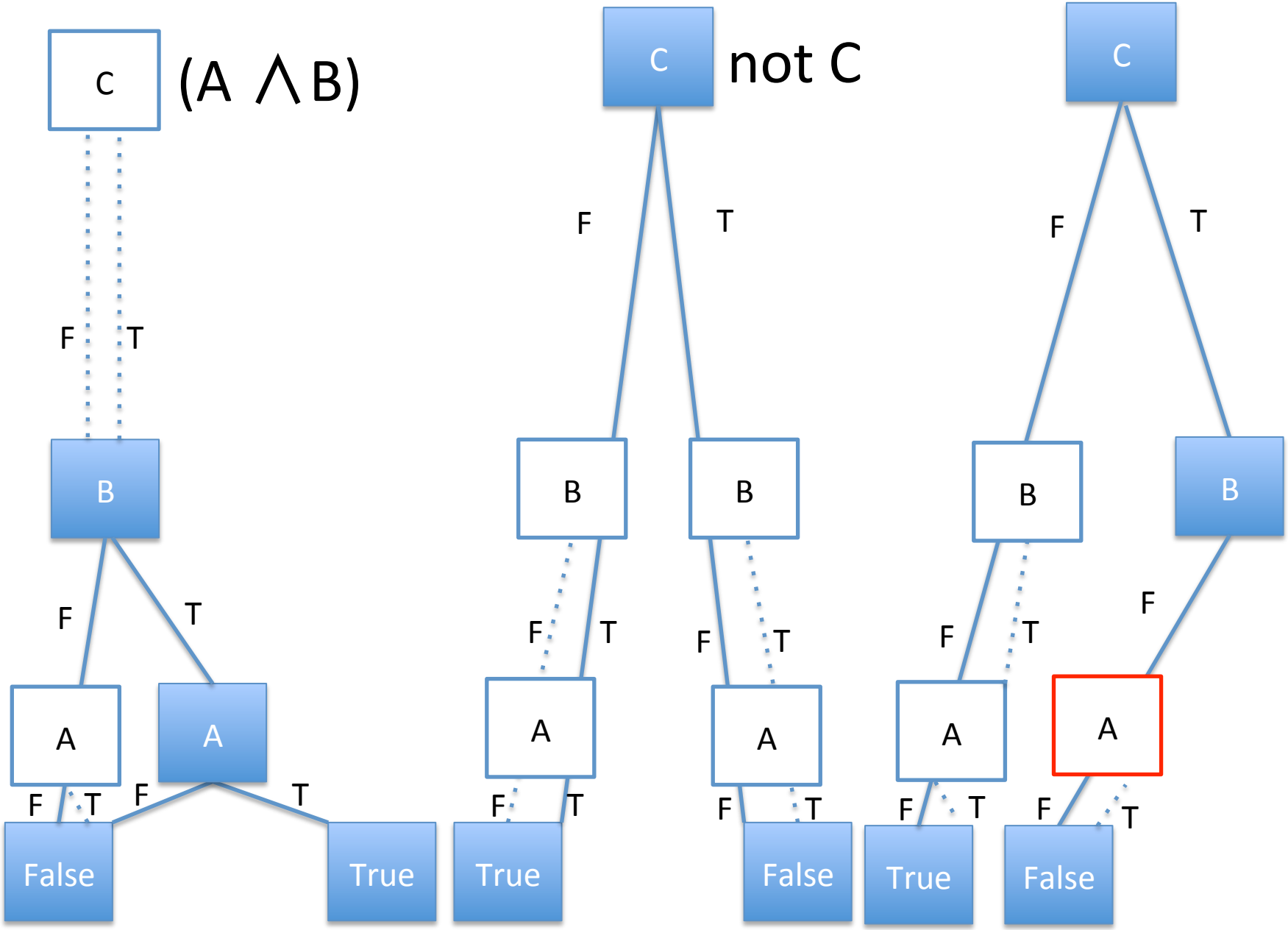
(A ∧ B) ∨ (not C) Variables: C > B > A



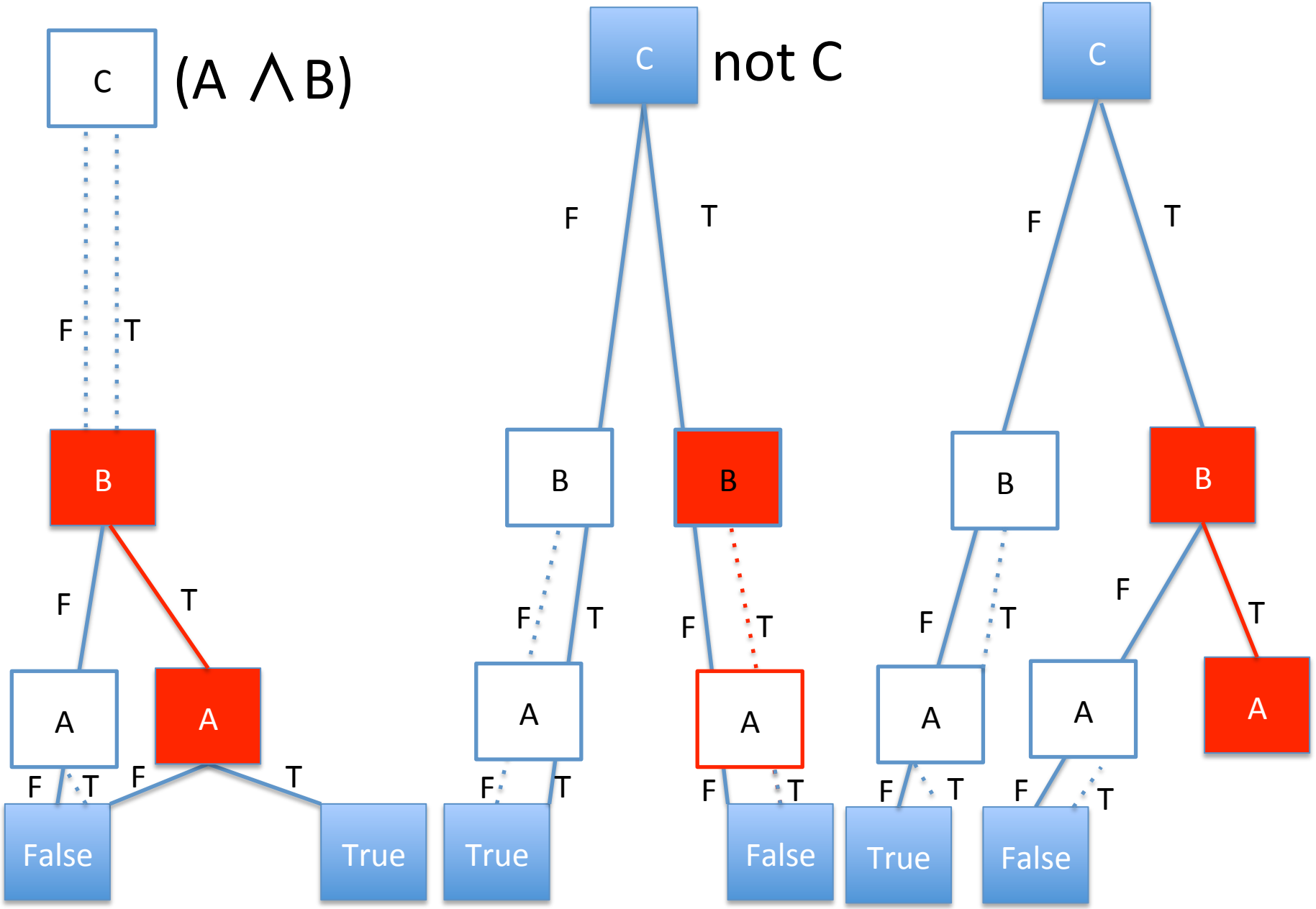
(A ∧ B) ∨ (not C) Variables: C > B > A



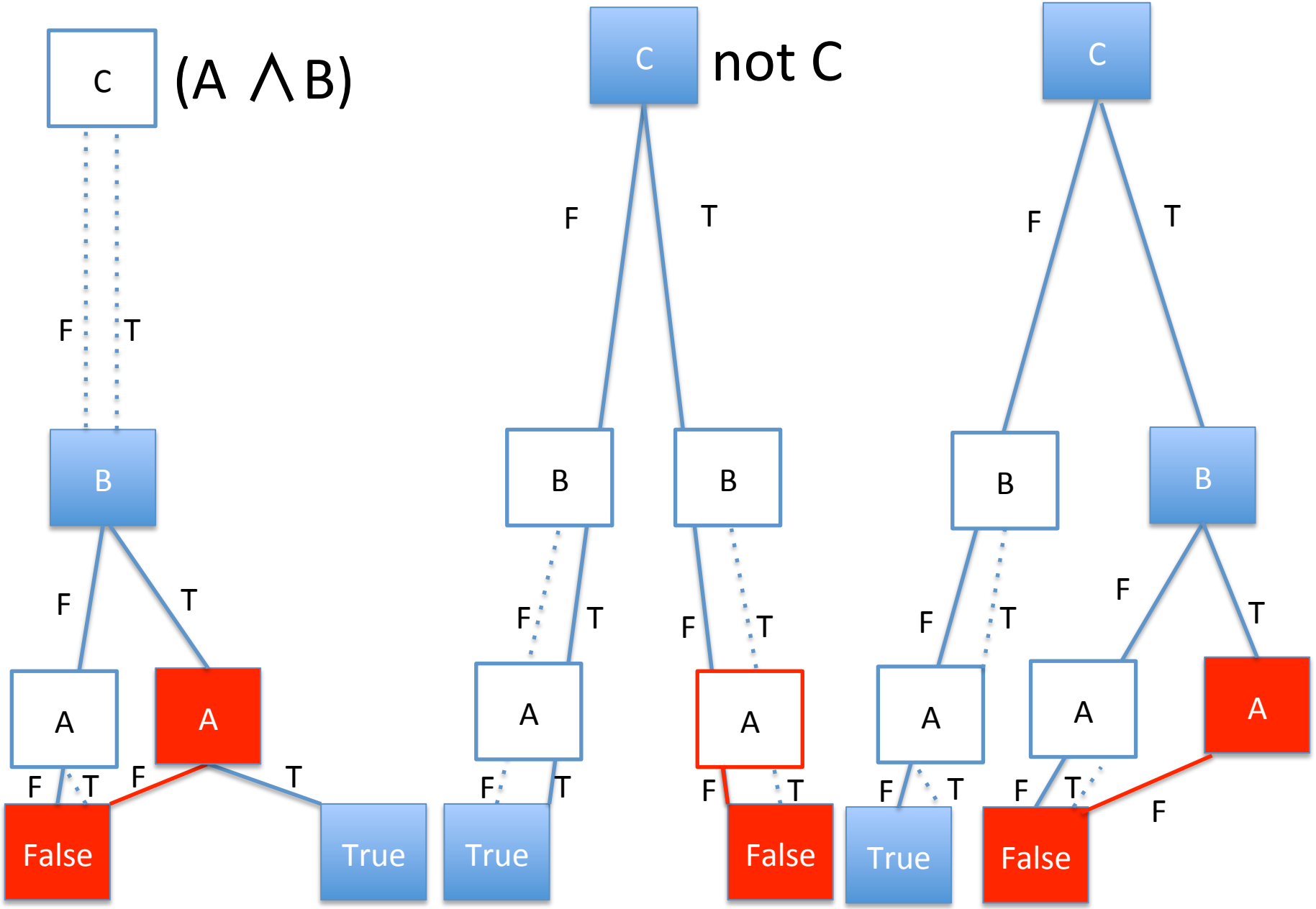
(A ∧ B) ∨ (not C) Variables: C > B > A



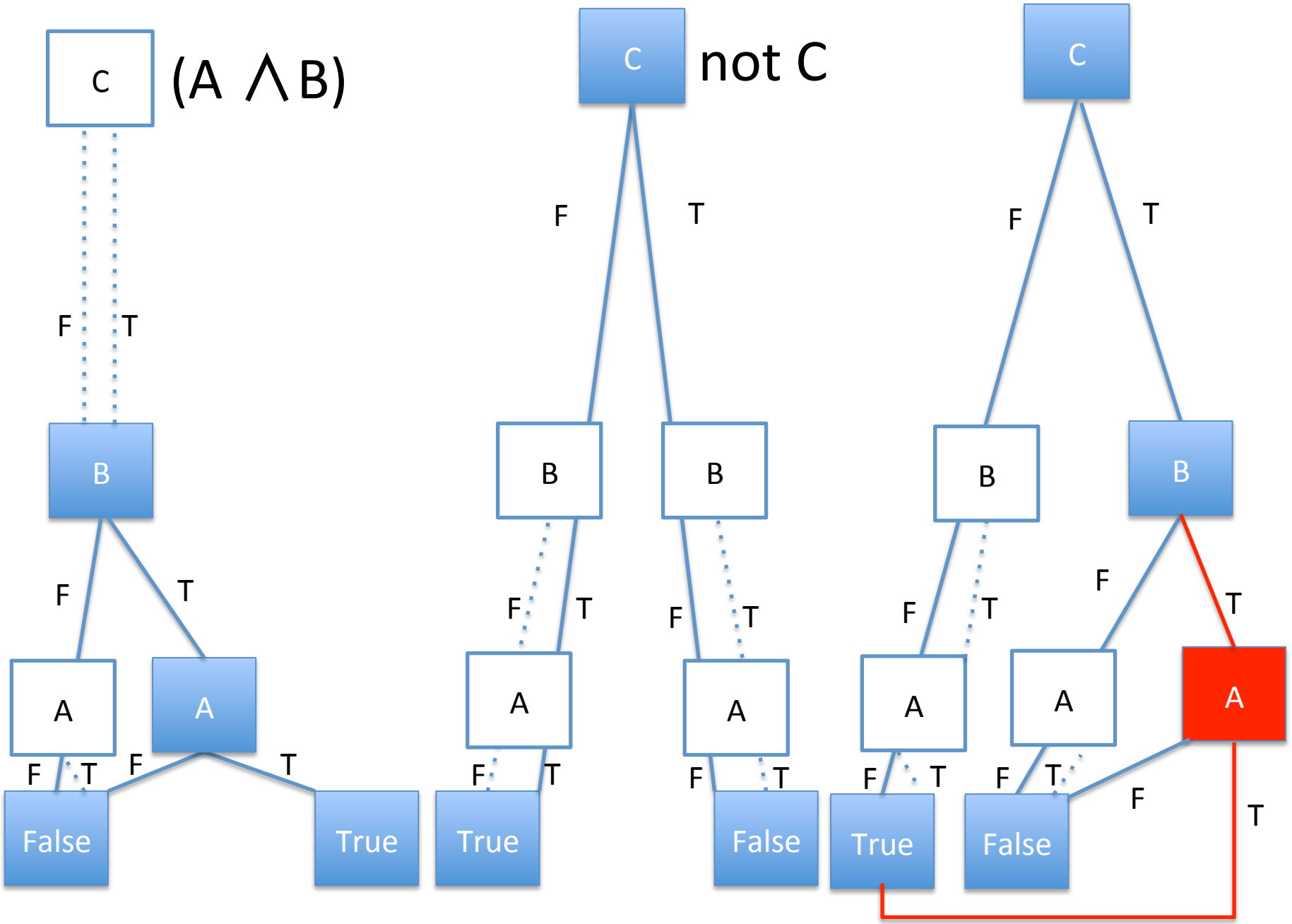
(A ∧ B) ∨ (not C) Variables: C > B > A



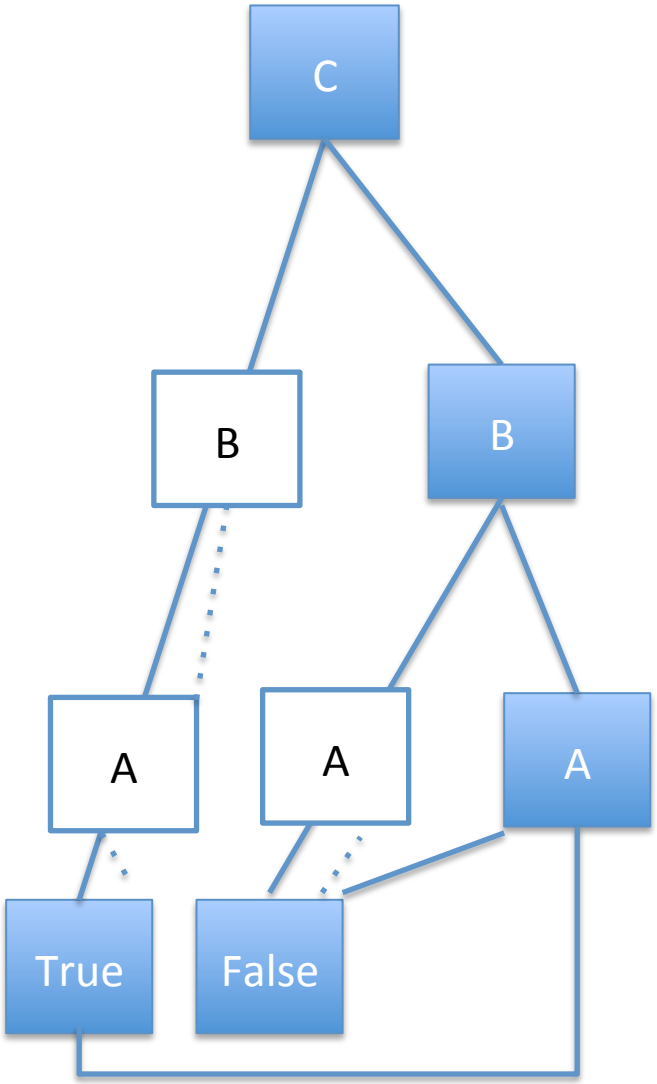
(A ∧ B) ∨ (not C) Variables: C > B > A



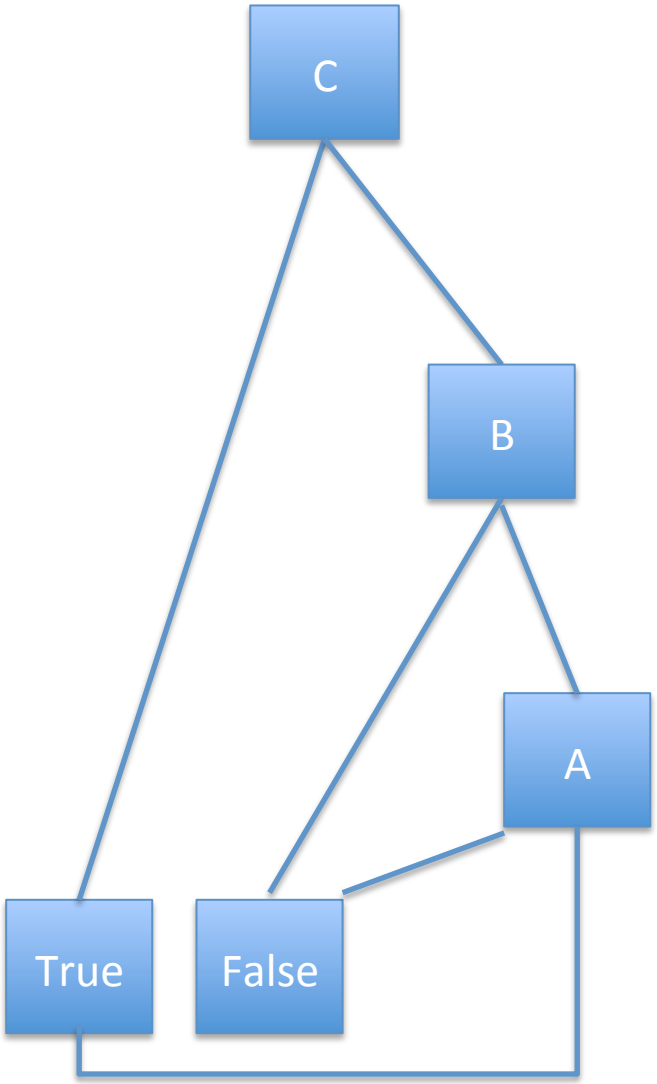
(A ∧ B) ∨ (not C) Variables: C > B > A



(A ∧ B) ∨ (not C) Variables: C > B > A



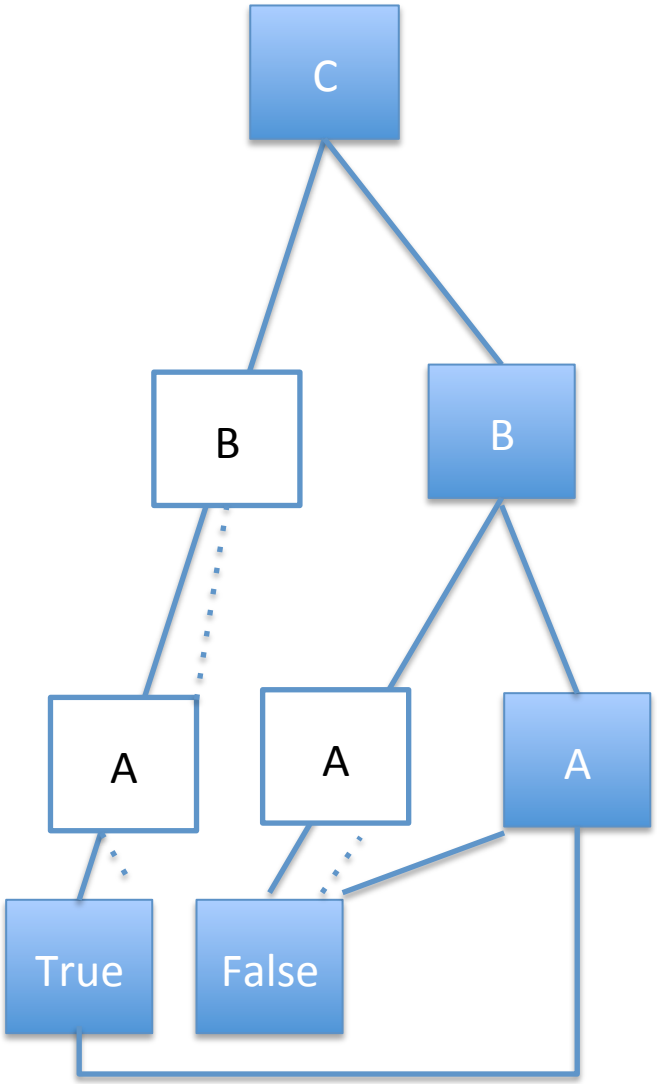
(A ∧ B) ∨ (not C) Variables: C > B > A



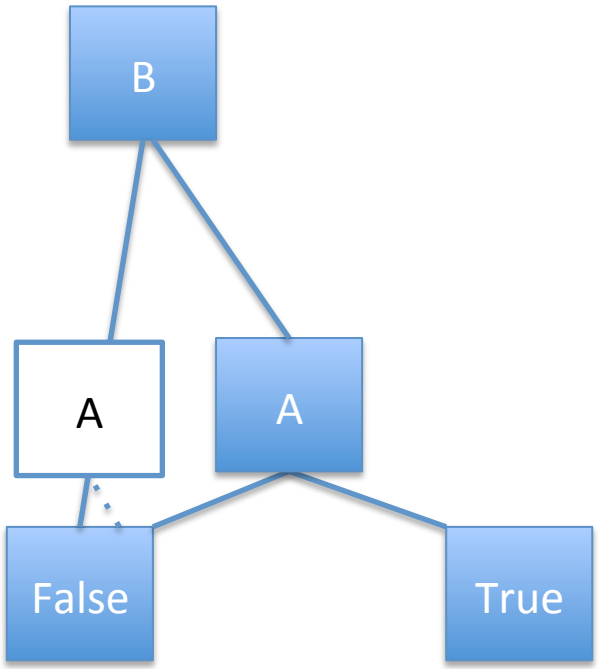
Example

- Find ROBDD for ((A $\wedge$B) $\vee$ (not C)) $\vee$ not (A $\wedge$B)

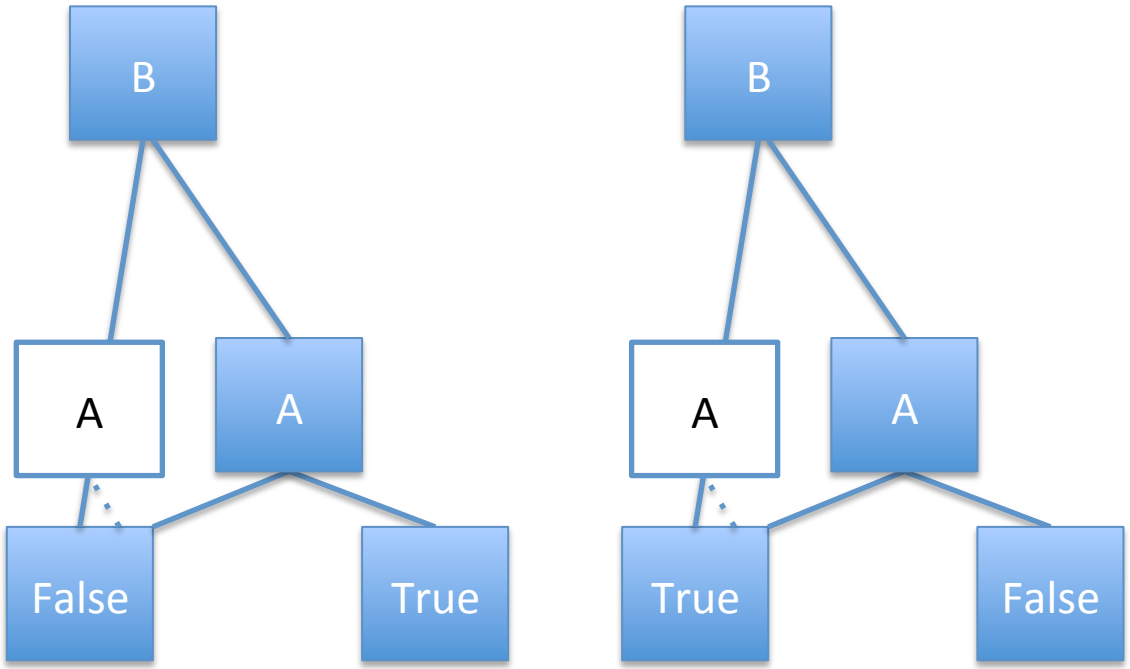
((A ∧ B) ∨ (not C)) Variables: C > B > A



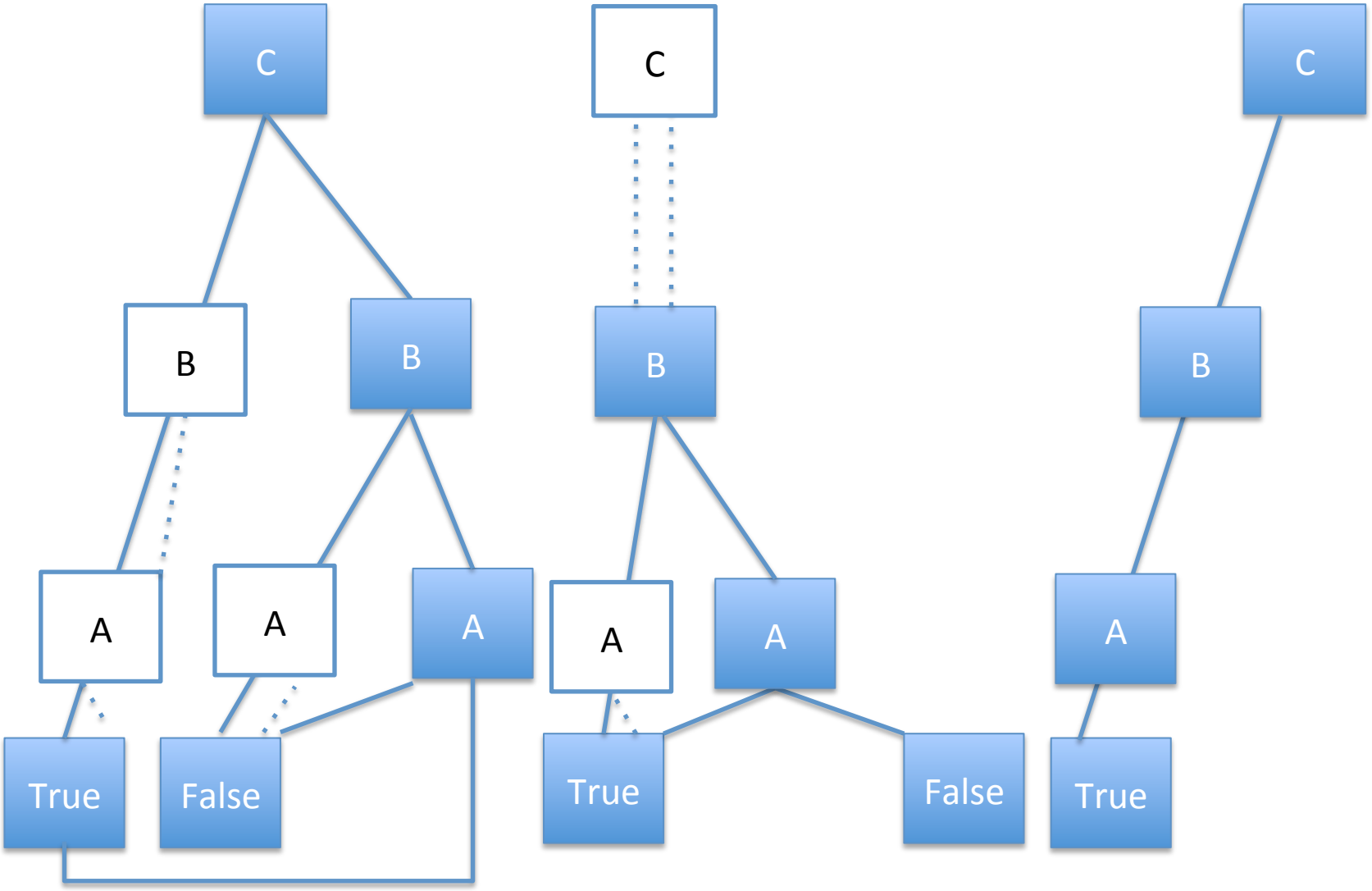
(A ⋀ B) Variables: C > B > A



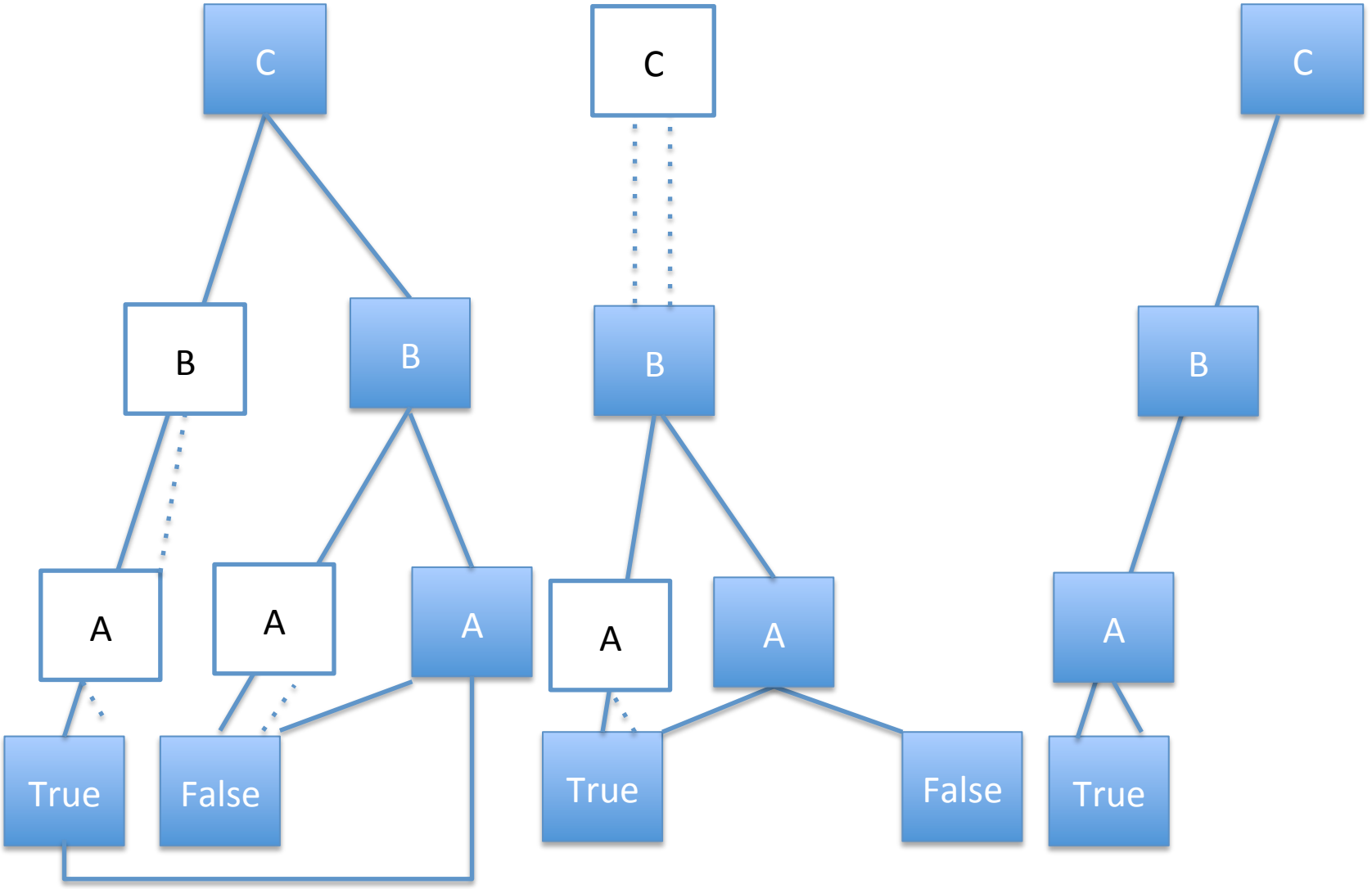
not (A ⋀ B) Variables: C > B > A



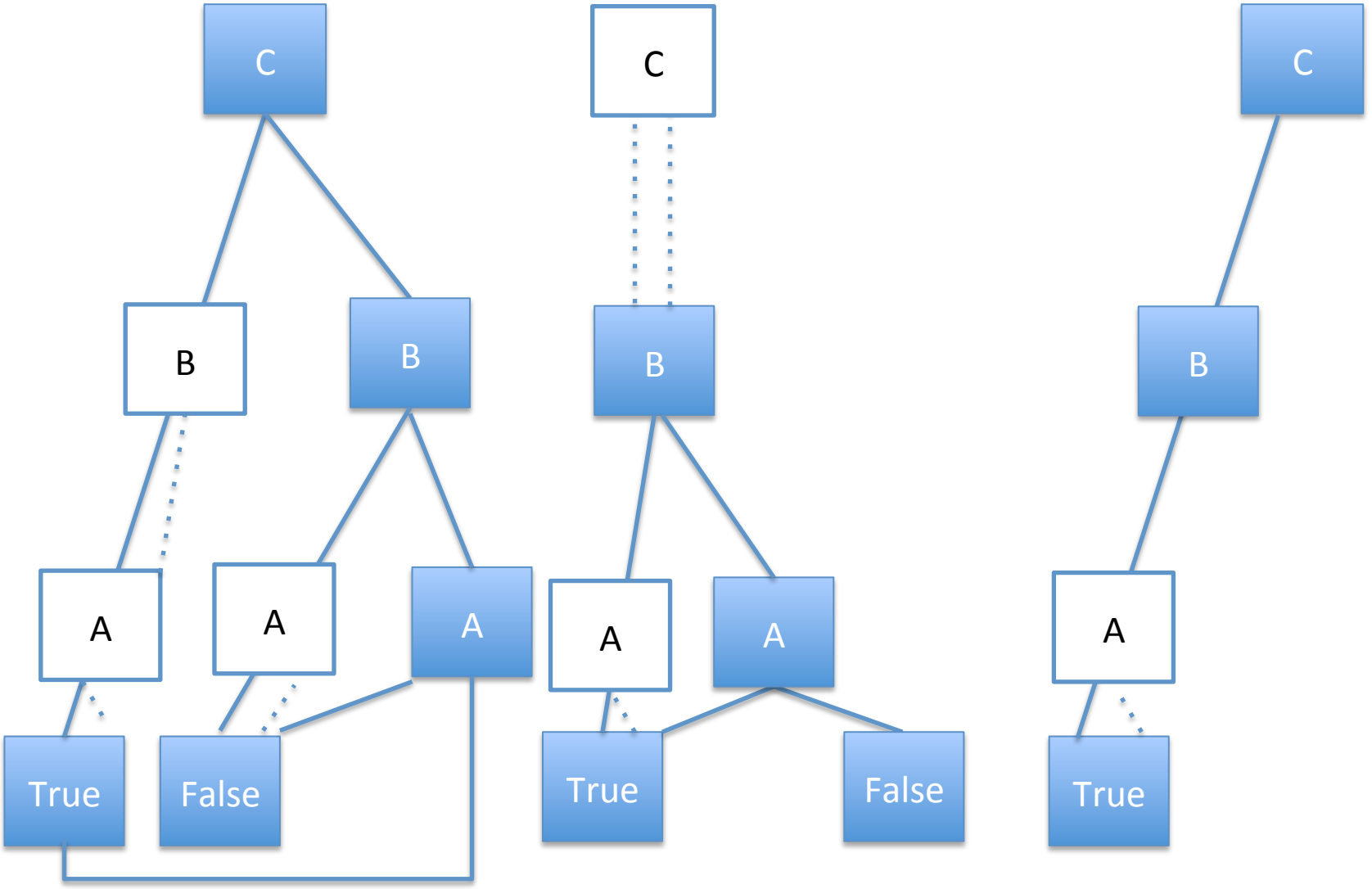
((A ∧ B) ∨ (not C)) ∨ not(A ∧ B) Variables: C > B > A



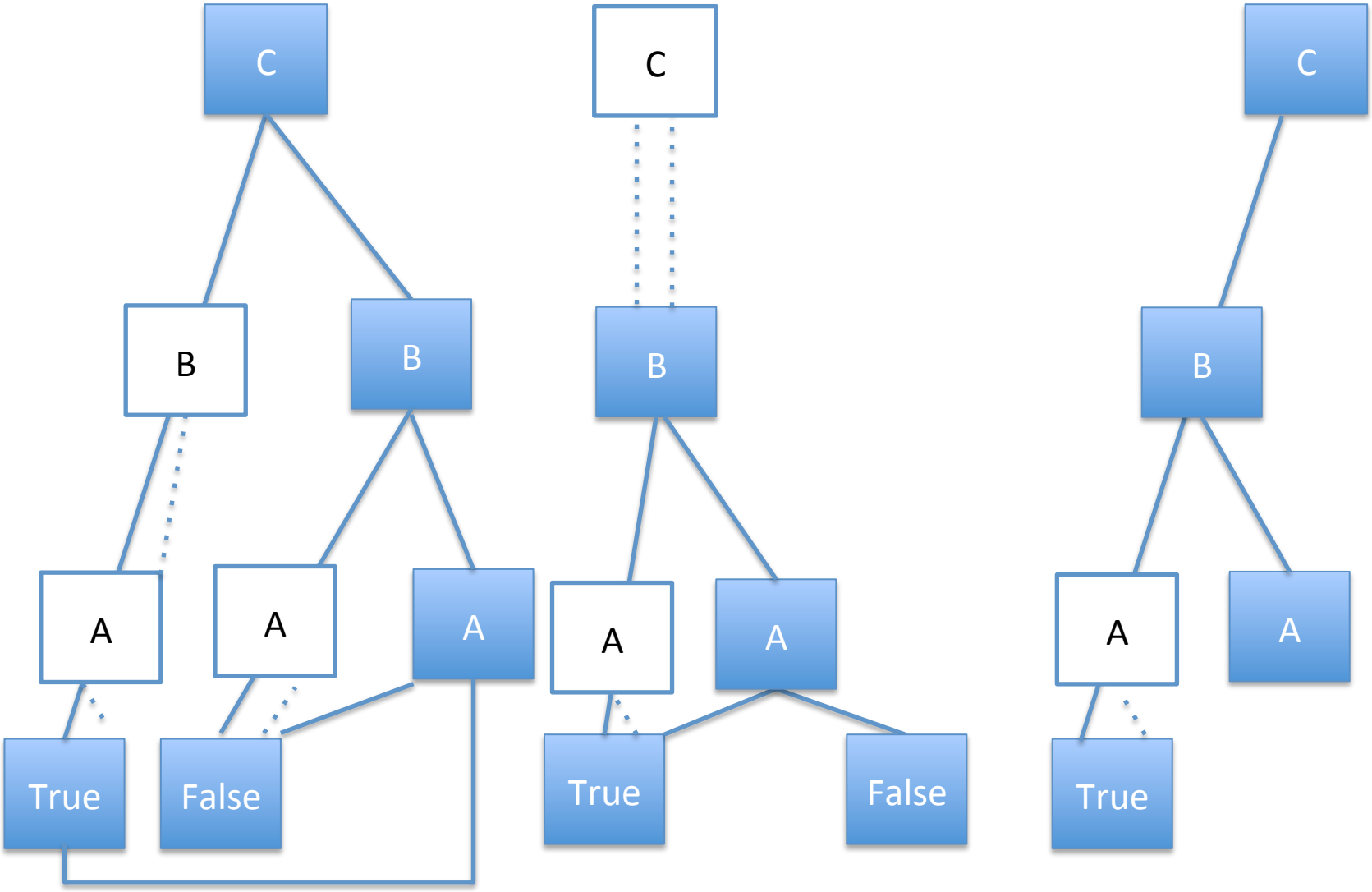
((A ∧ B) ∨ (not C)) ∨ not(A ∧ B) Variables: C > B > A



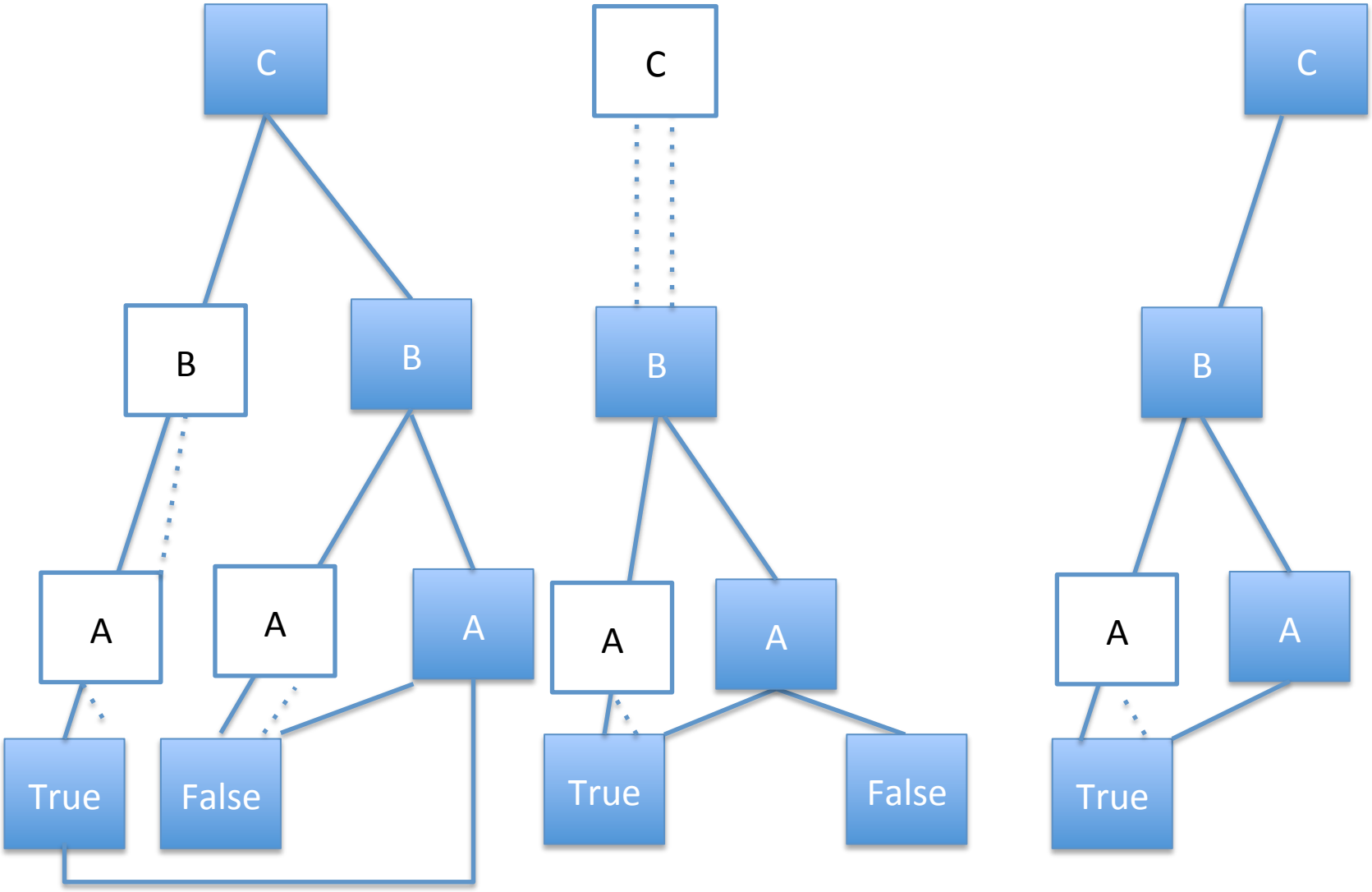
((A ∧ B) ∨ (not C)) ∨ not(A ∧ B) Variables: C > B > A



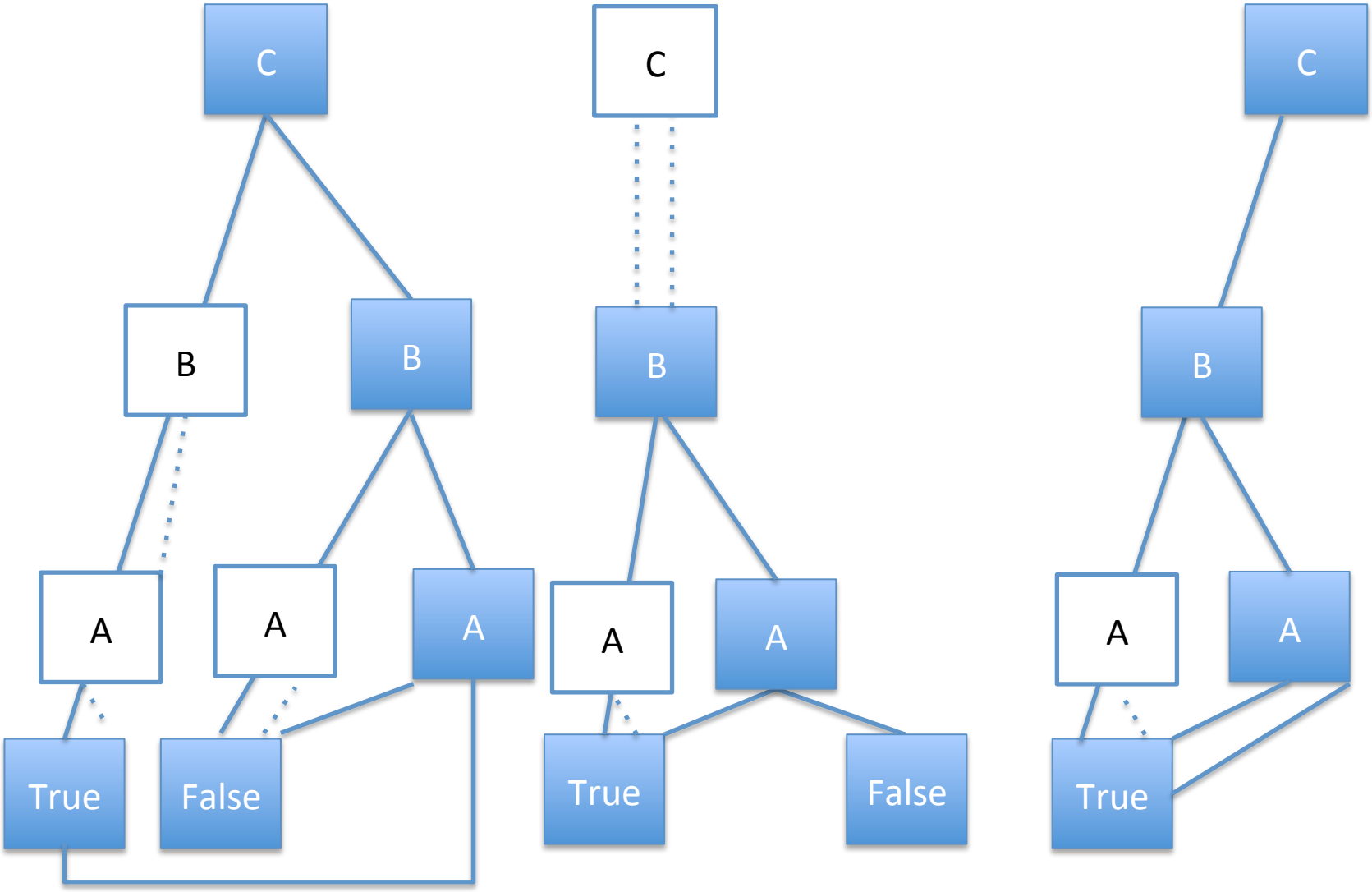
((A ∧ B) ∨ (not C)) ∨ not(A ∧ B) Variables: C > B > A



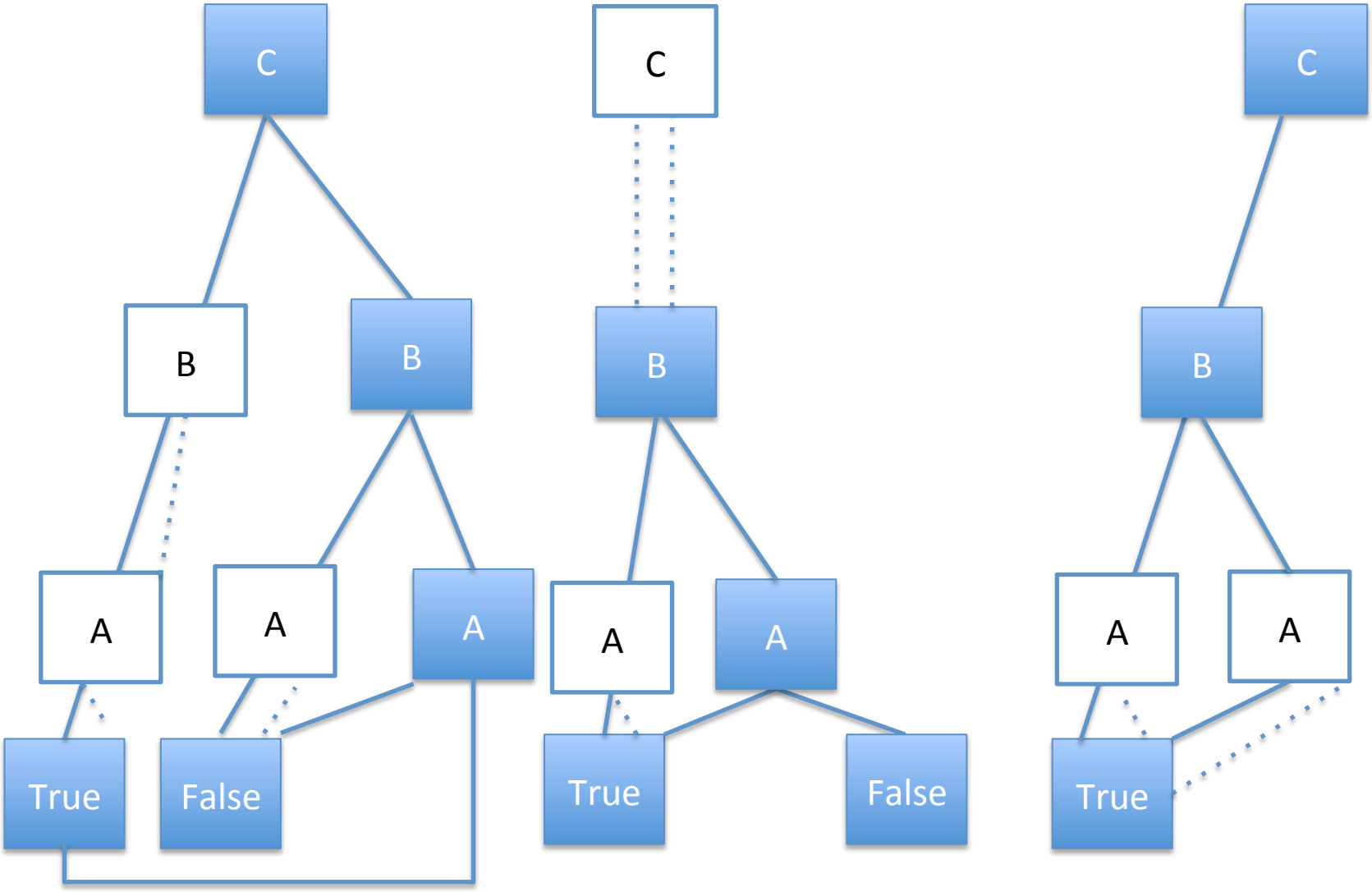
((A ∧ B) ∨ (not C)) ∨ not(A ∧ B) Variables: C > B > A



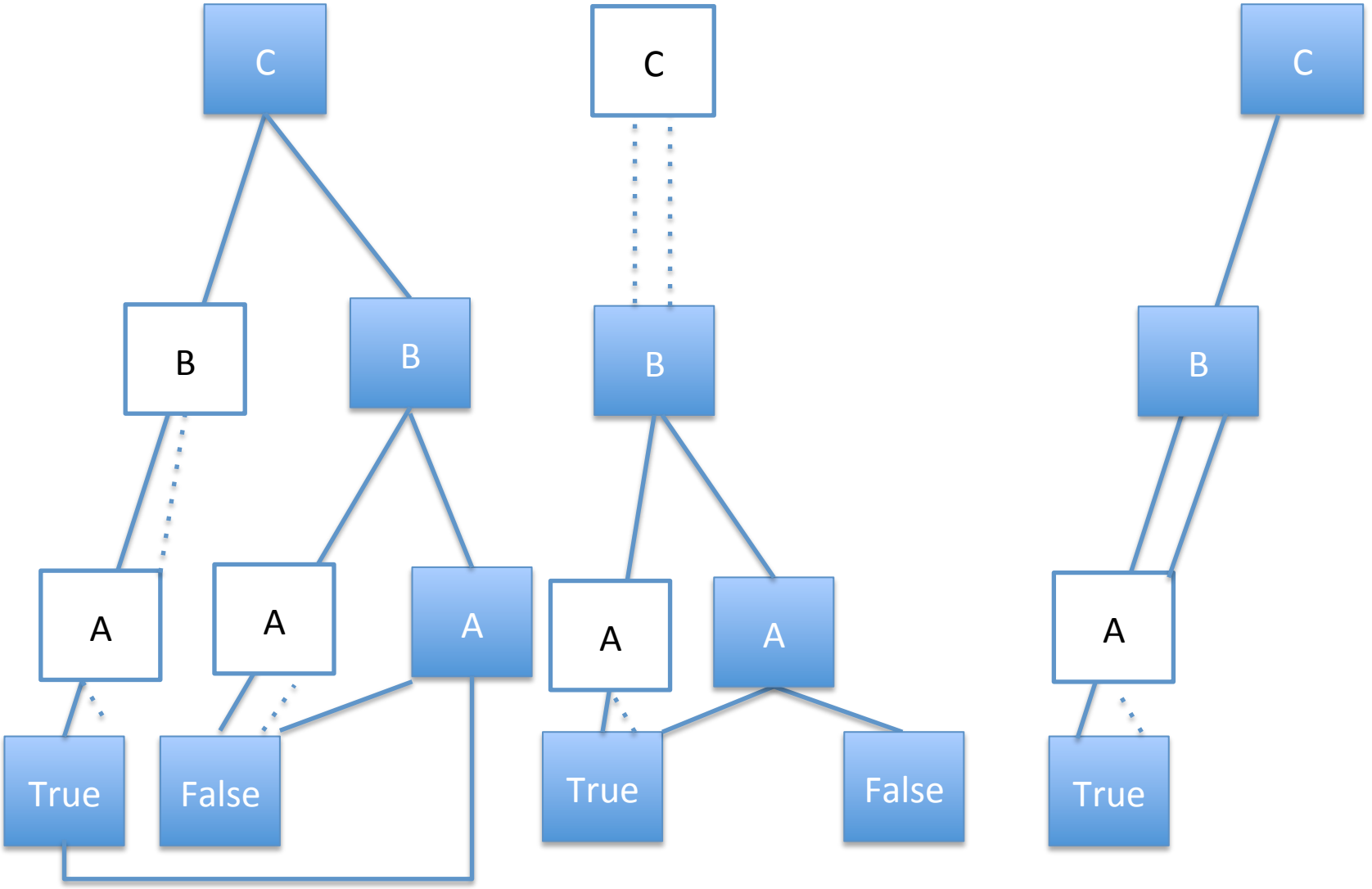
((A ∧ B) ∨ (not C)) ∨ not(A ∧ B) Variables: C > B > A



((A ∧ B) ∨ (not C)) ∨ not(A ∧ B) Variables: C > B > A



((A ∧ B) ∨ (not C)) ∨ not(A ∧ B) Variables: C > B > A



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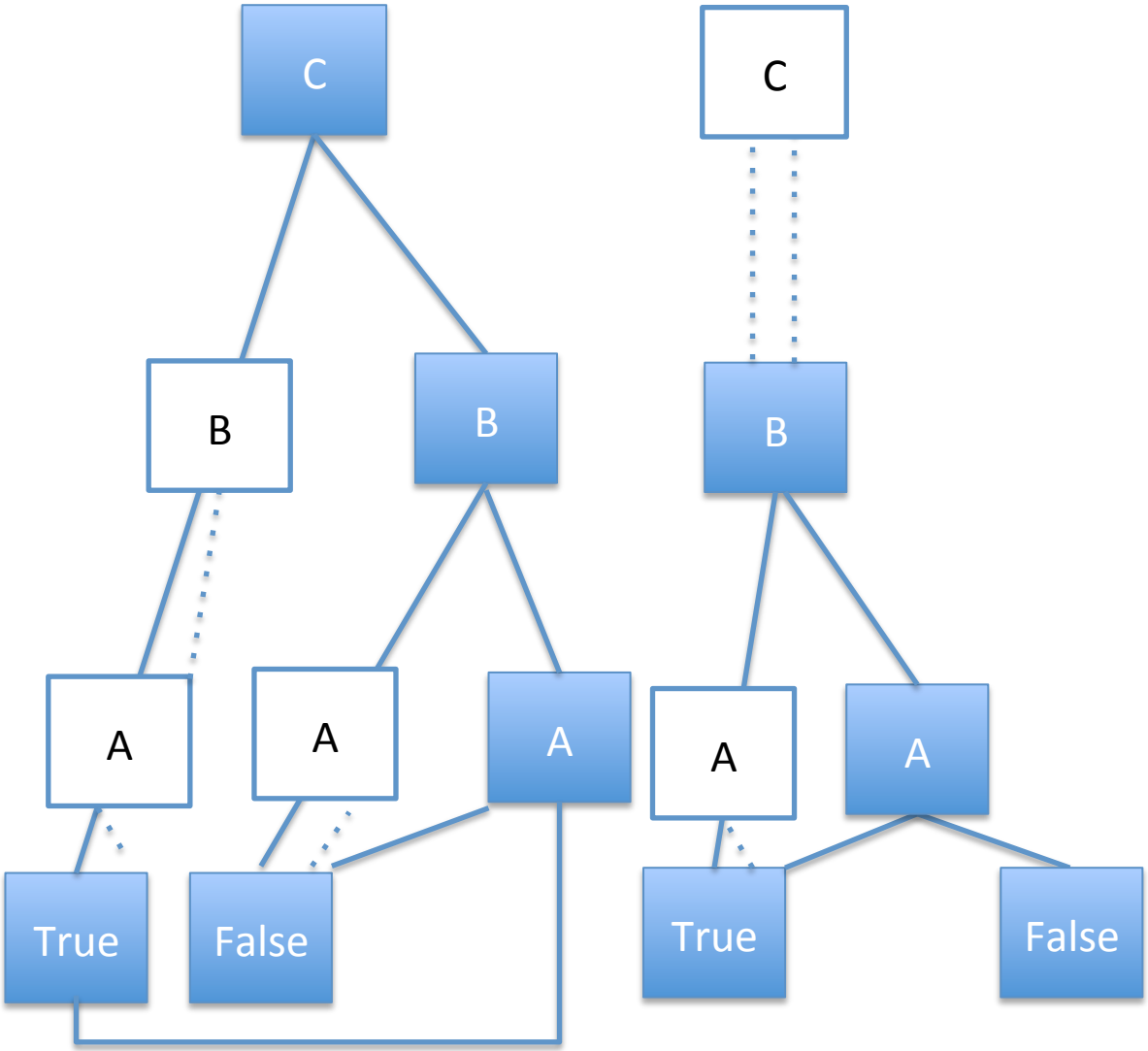

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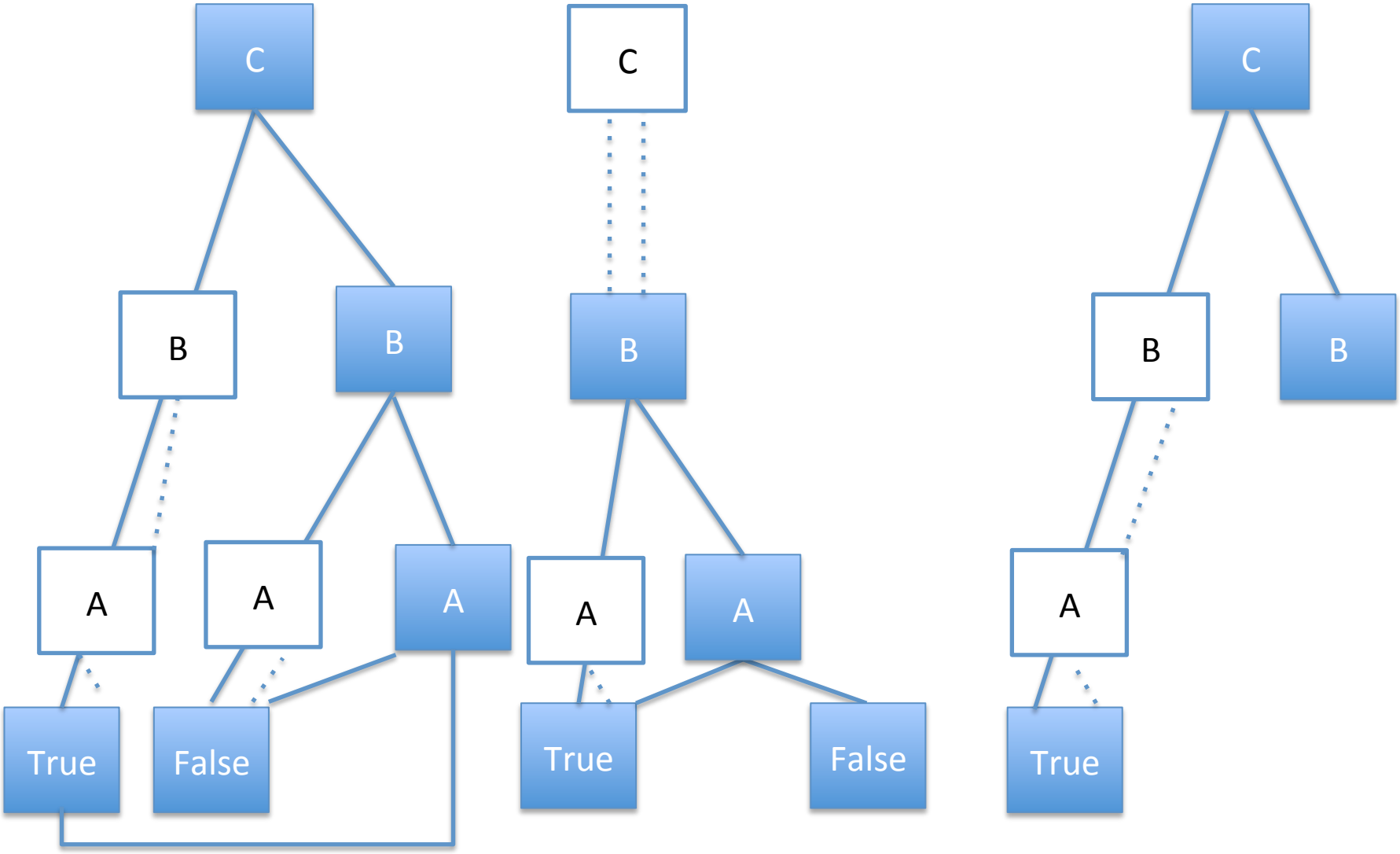
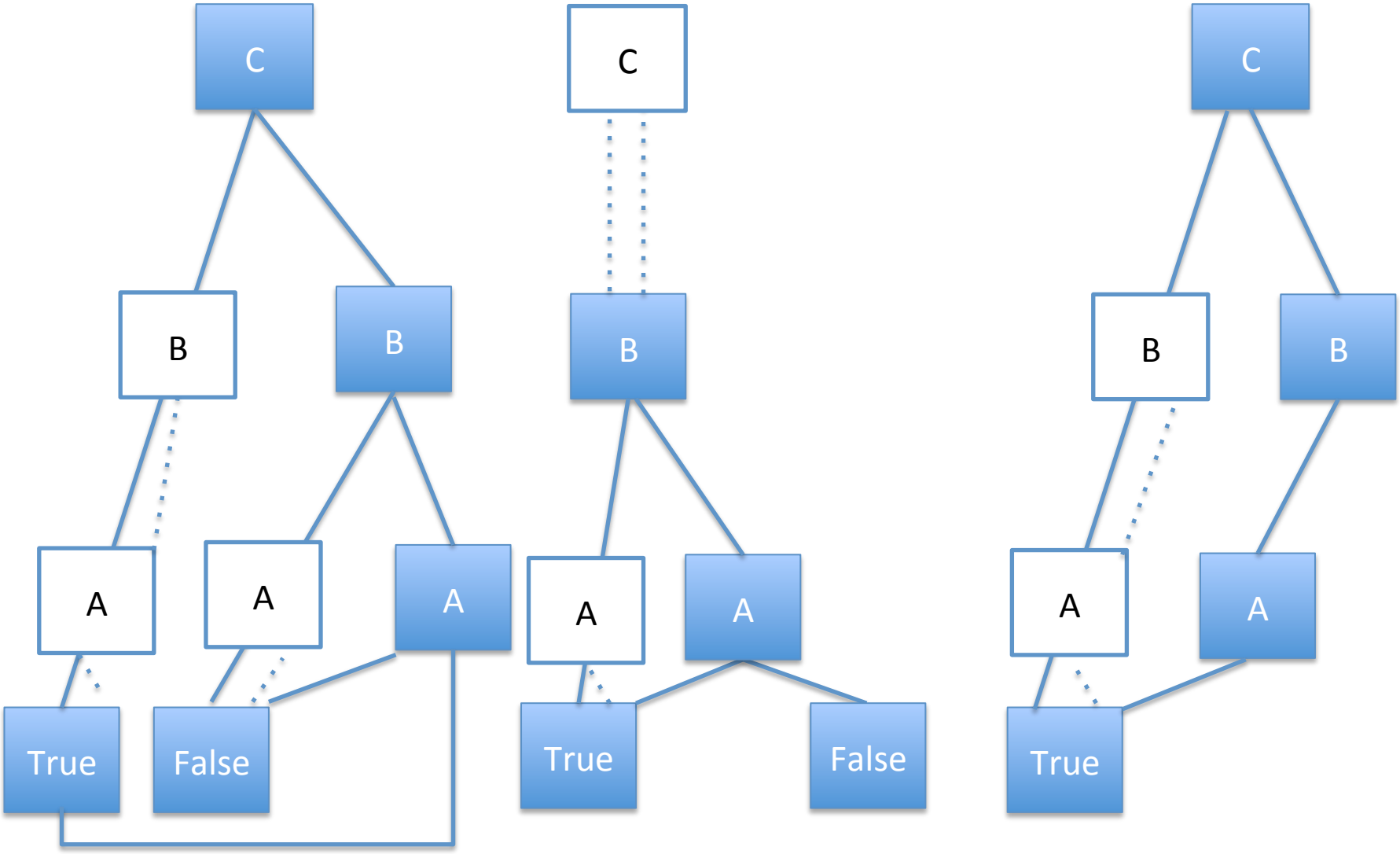


((A ∧ B) ∨ (not C)) ∨ not(A ∧ B) Variables: C > B > A



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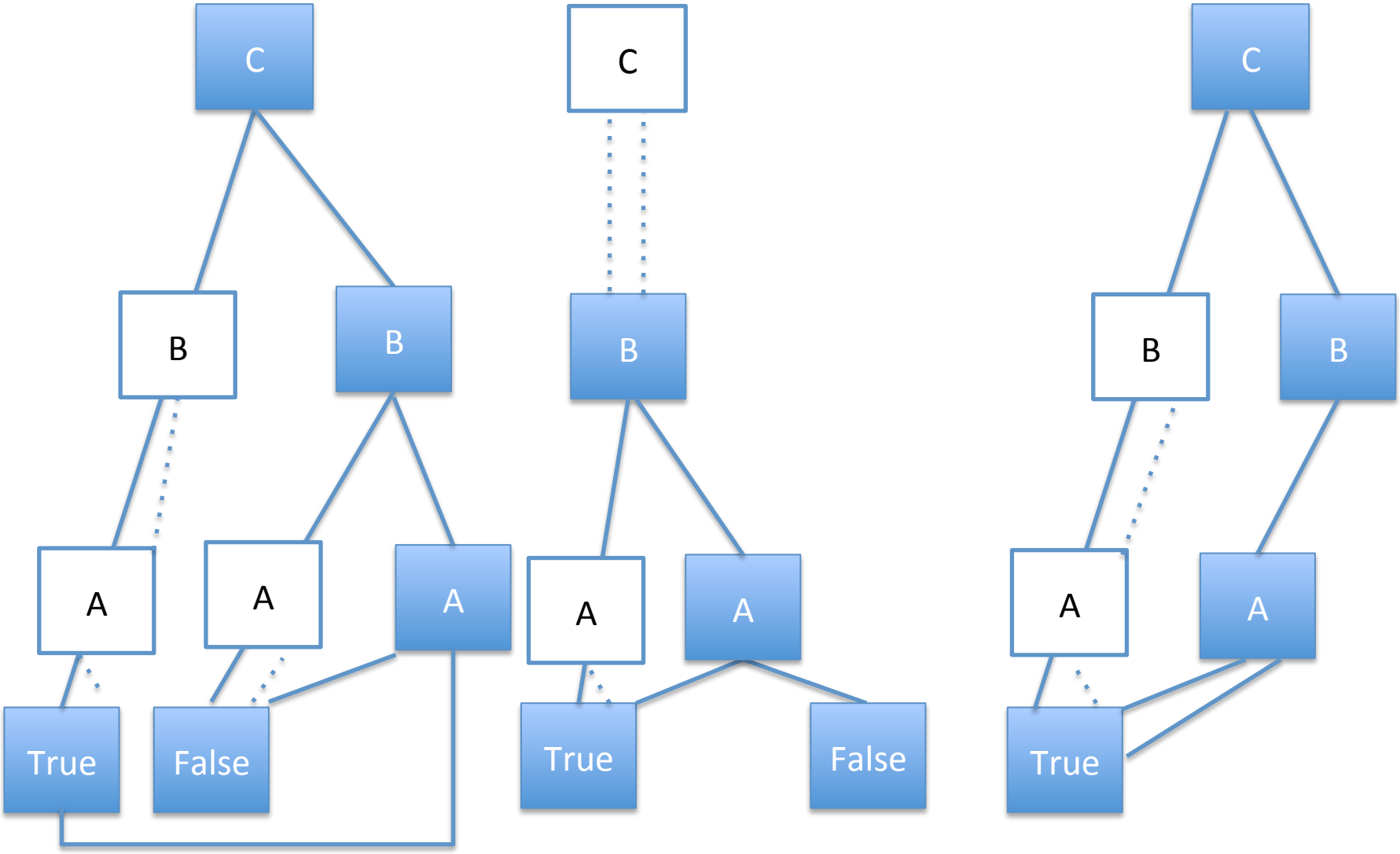

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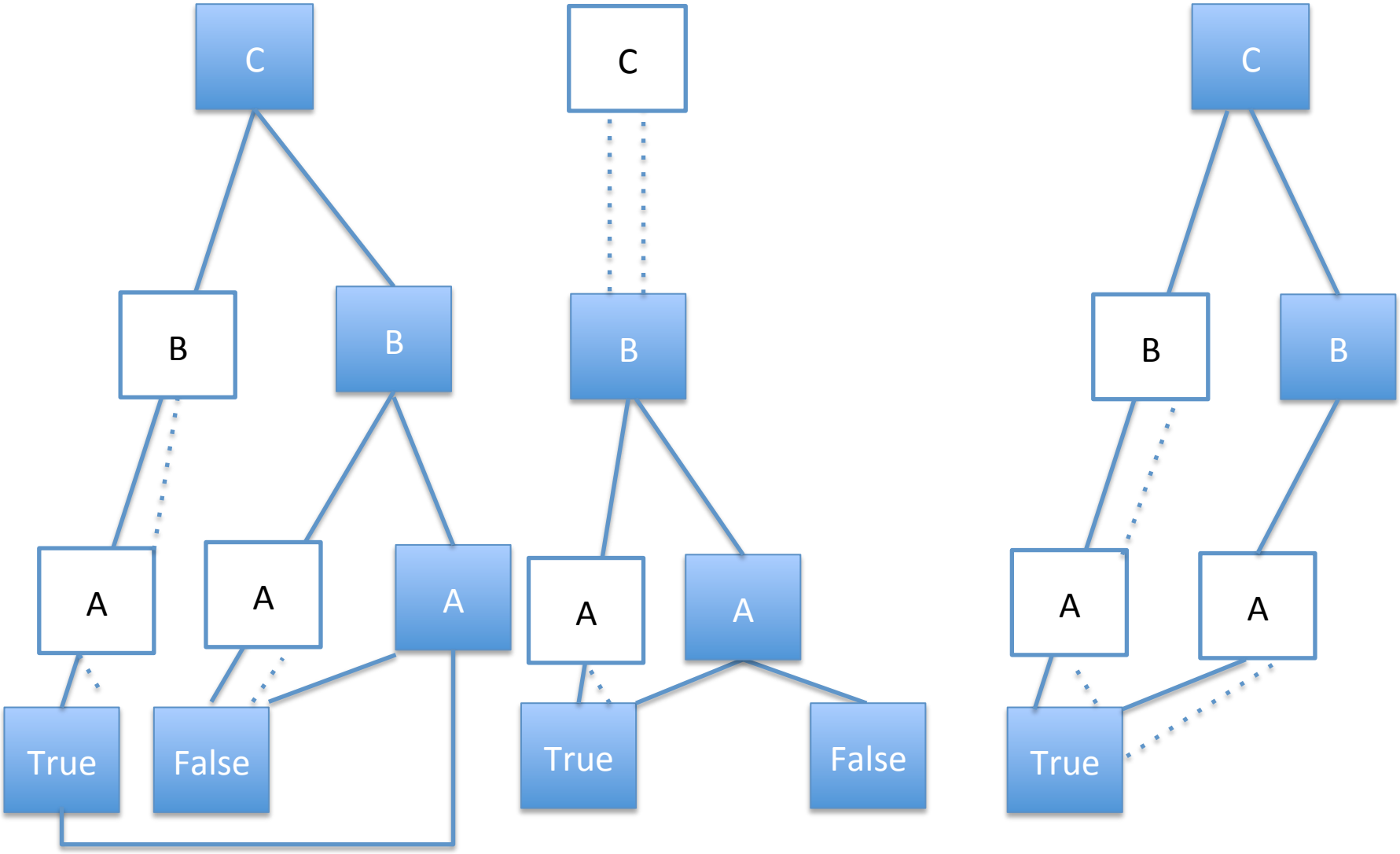

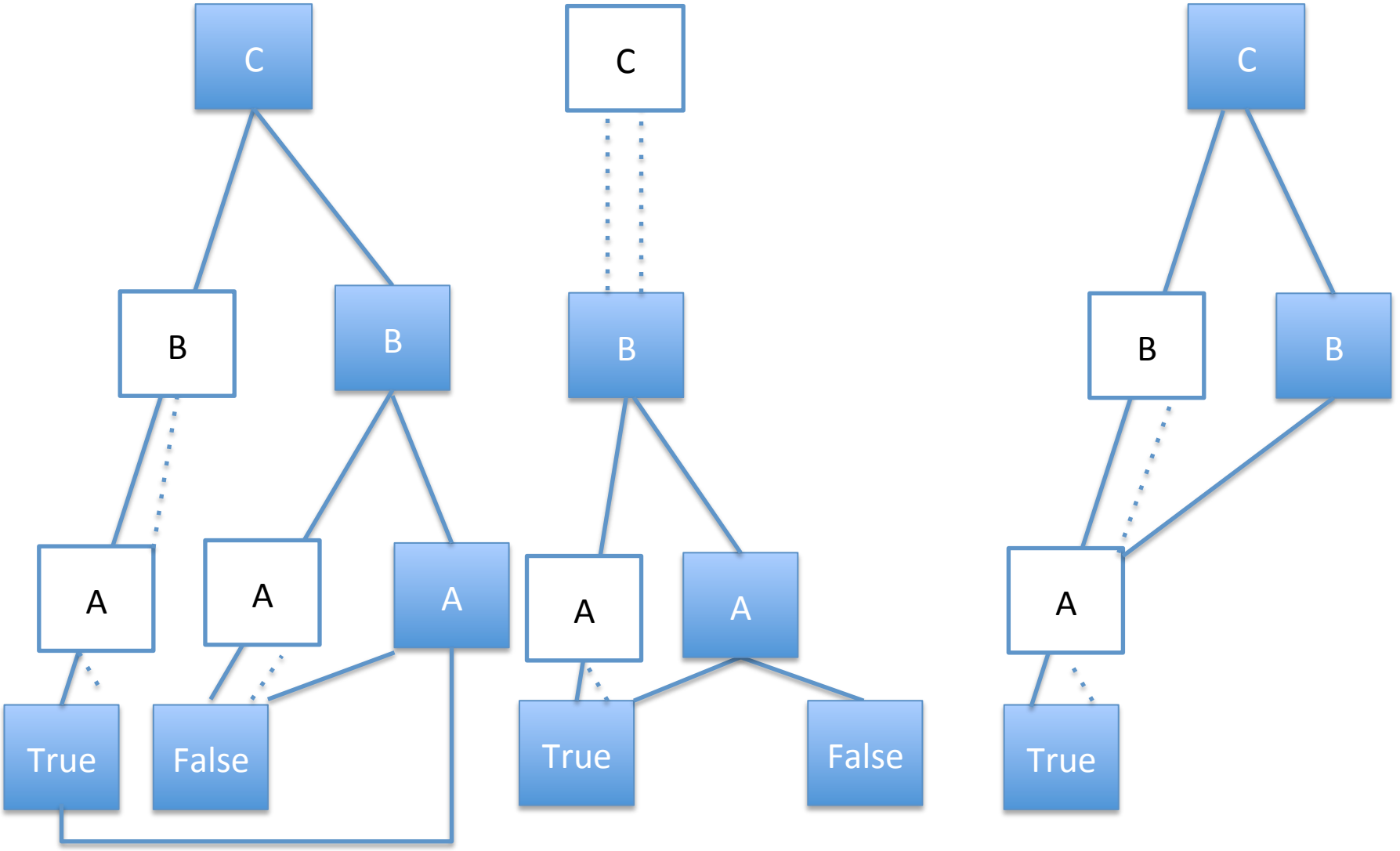
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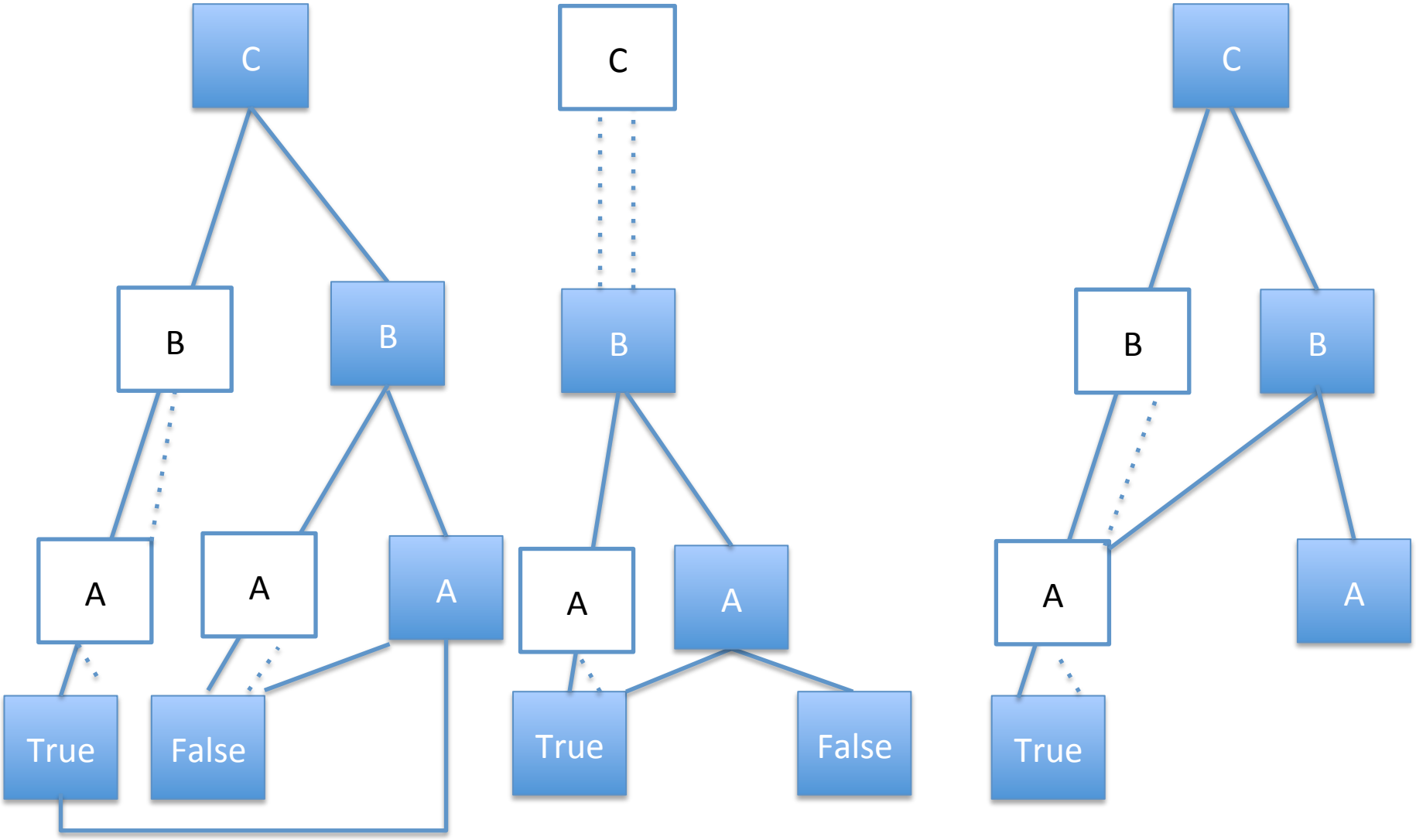
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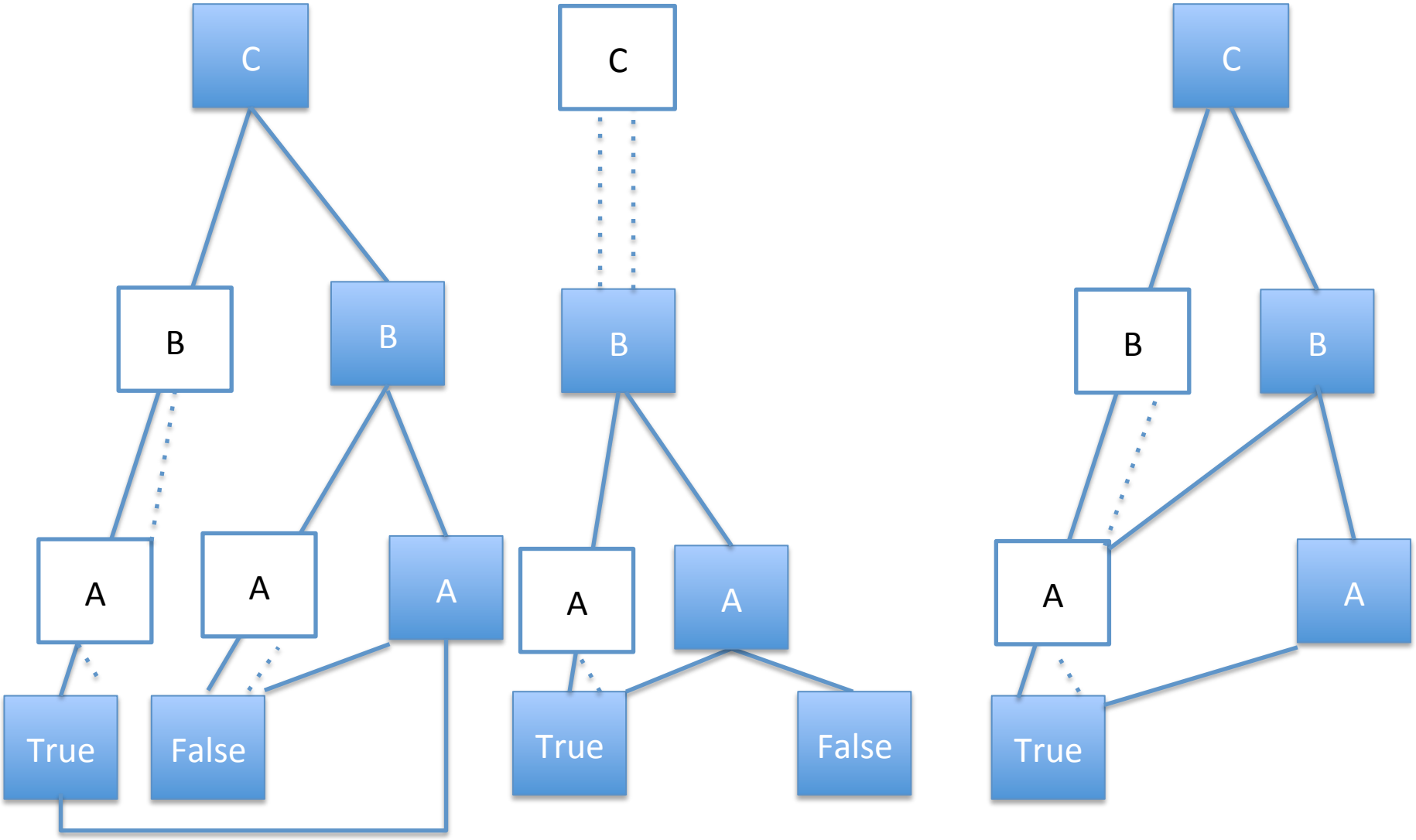
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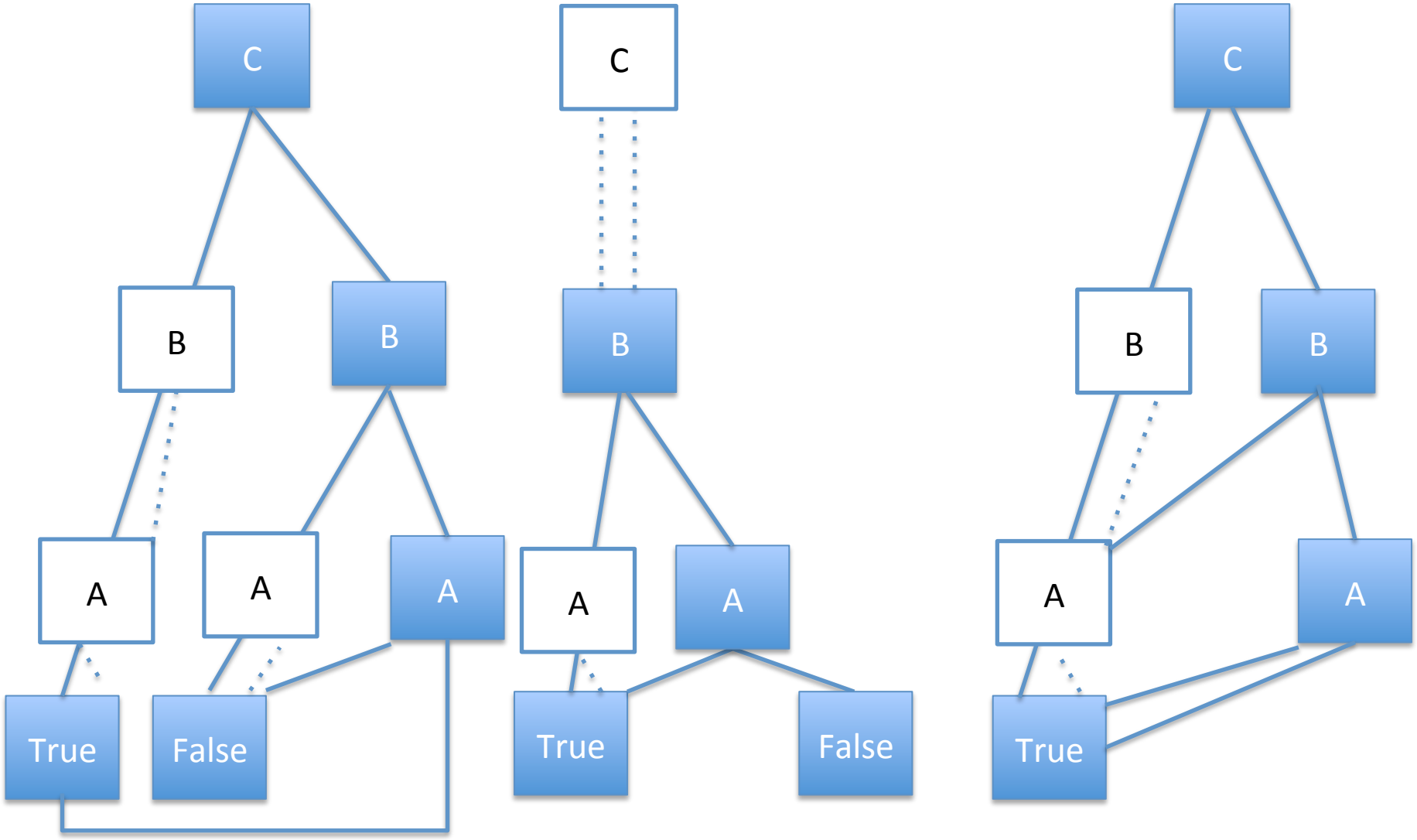
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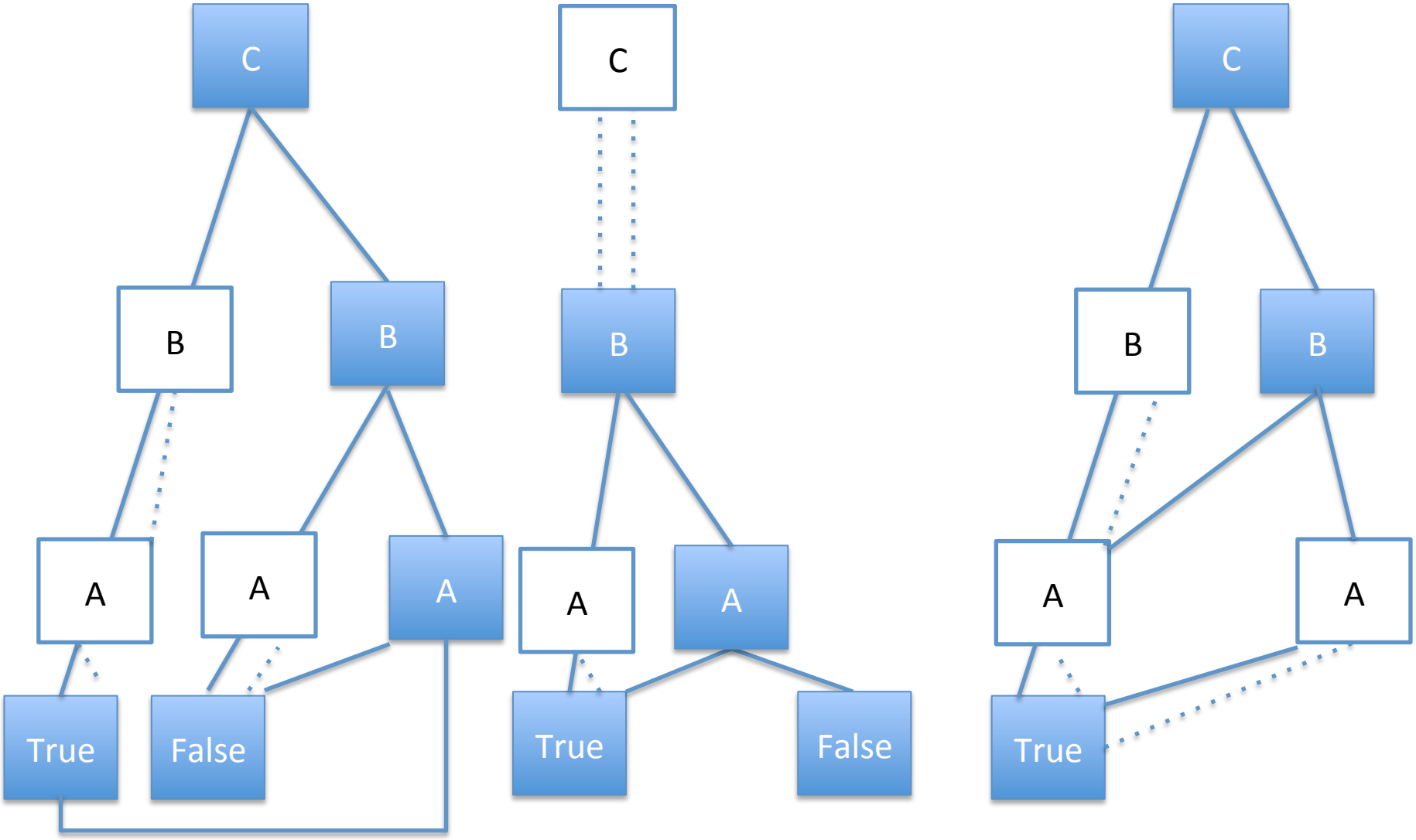
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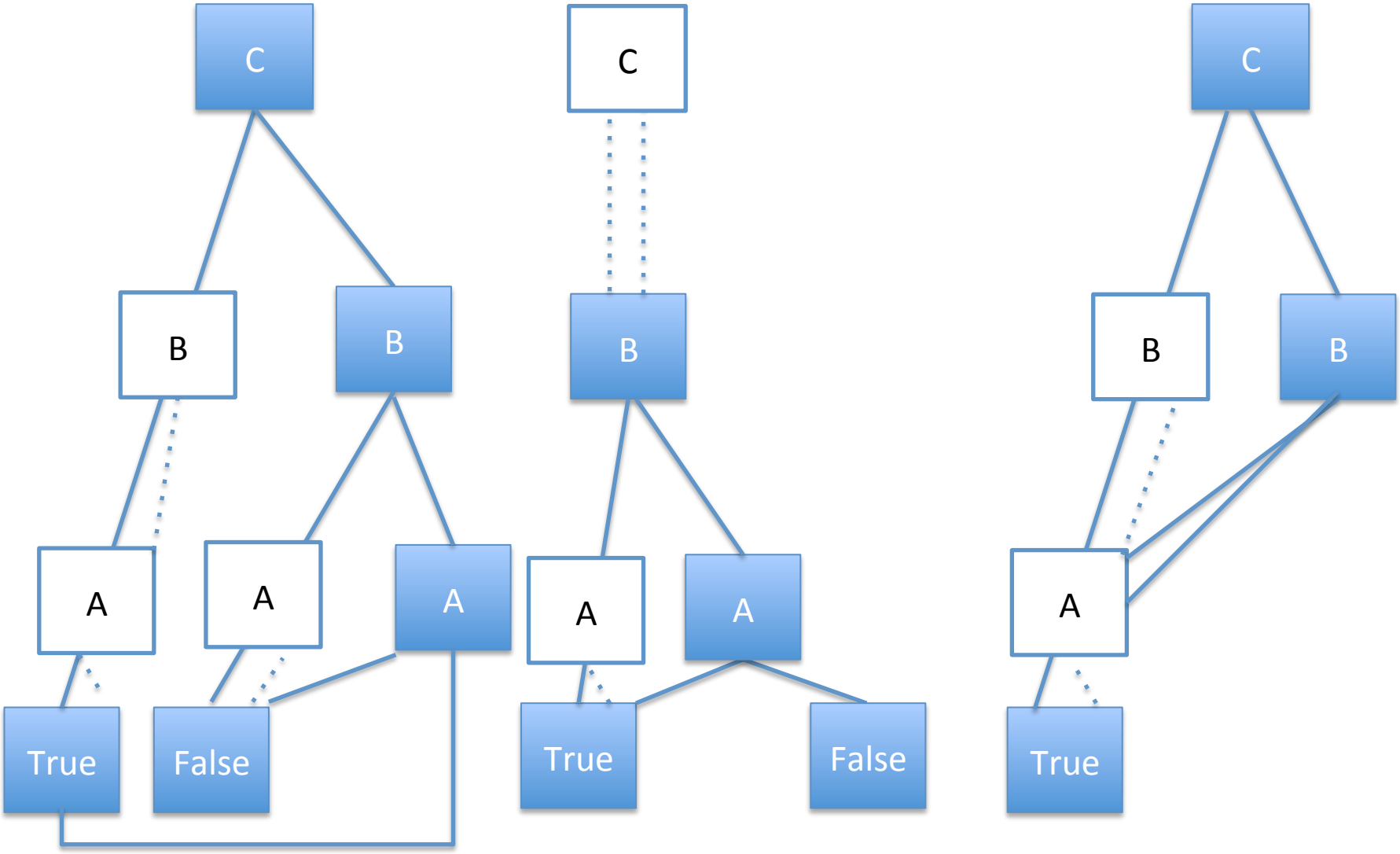
((A ∧ B) ∨ (not C)) ∨ not(A ∧ B) Variables: C > B > A



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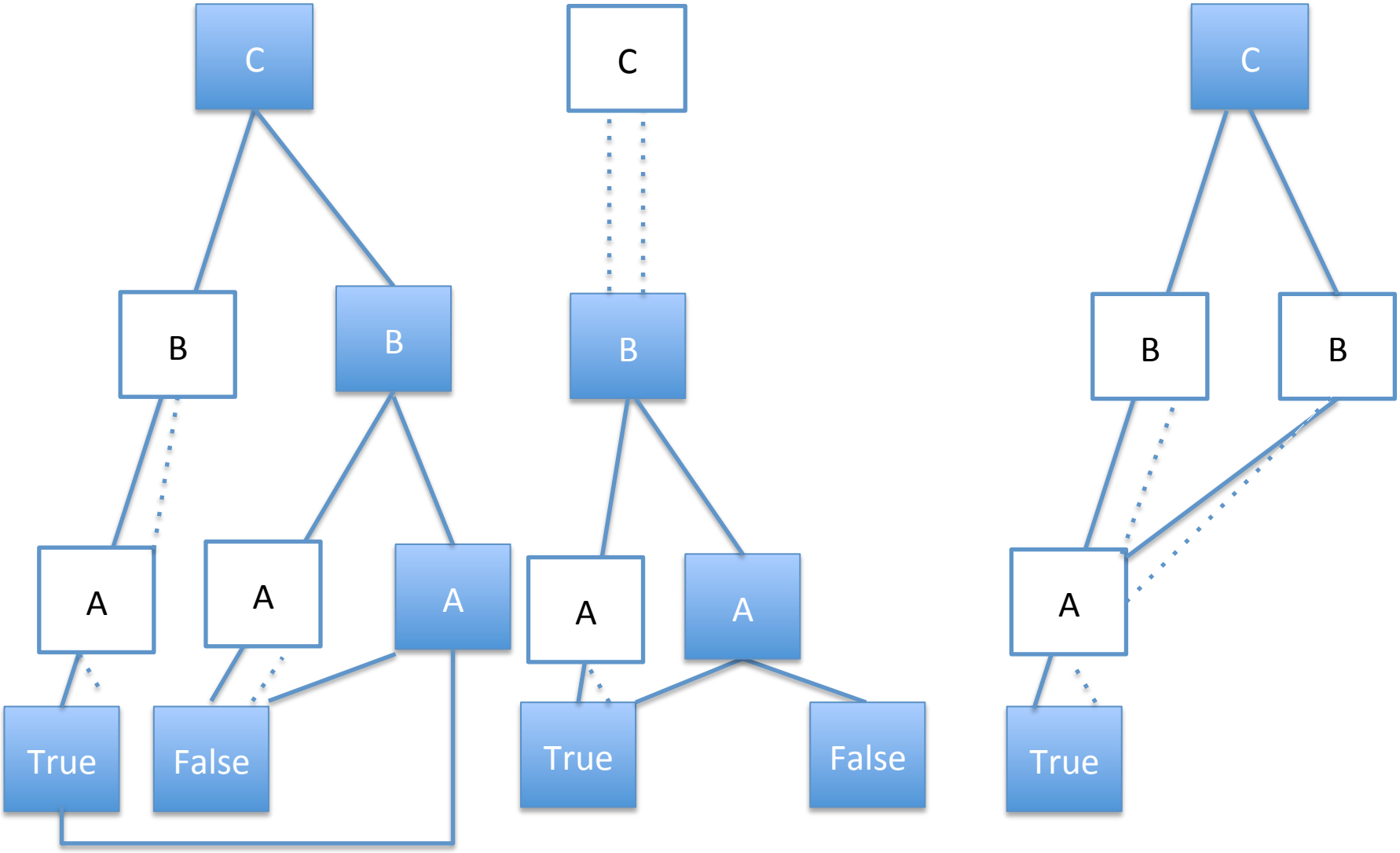

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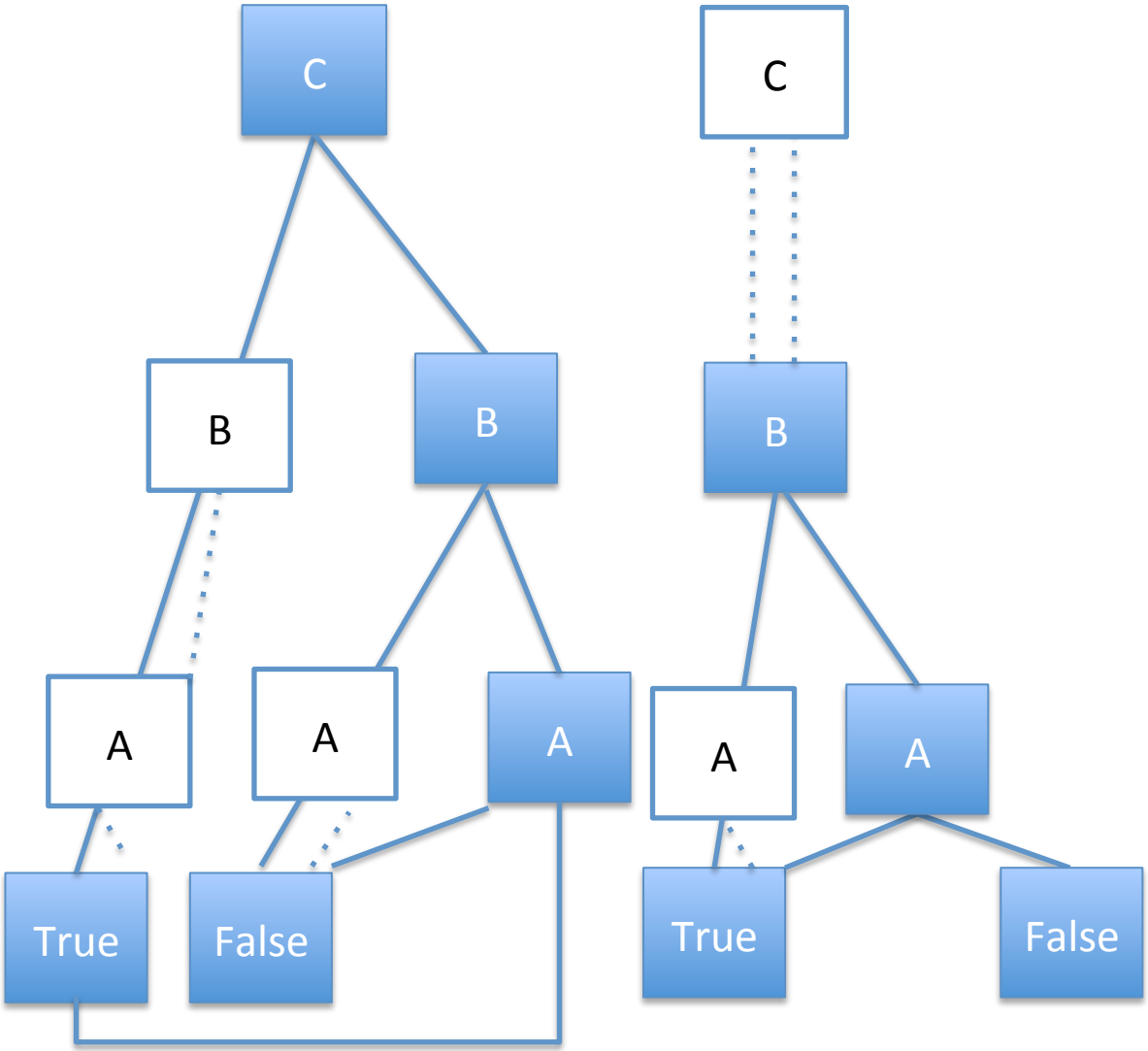

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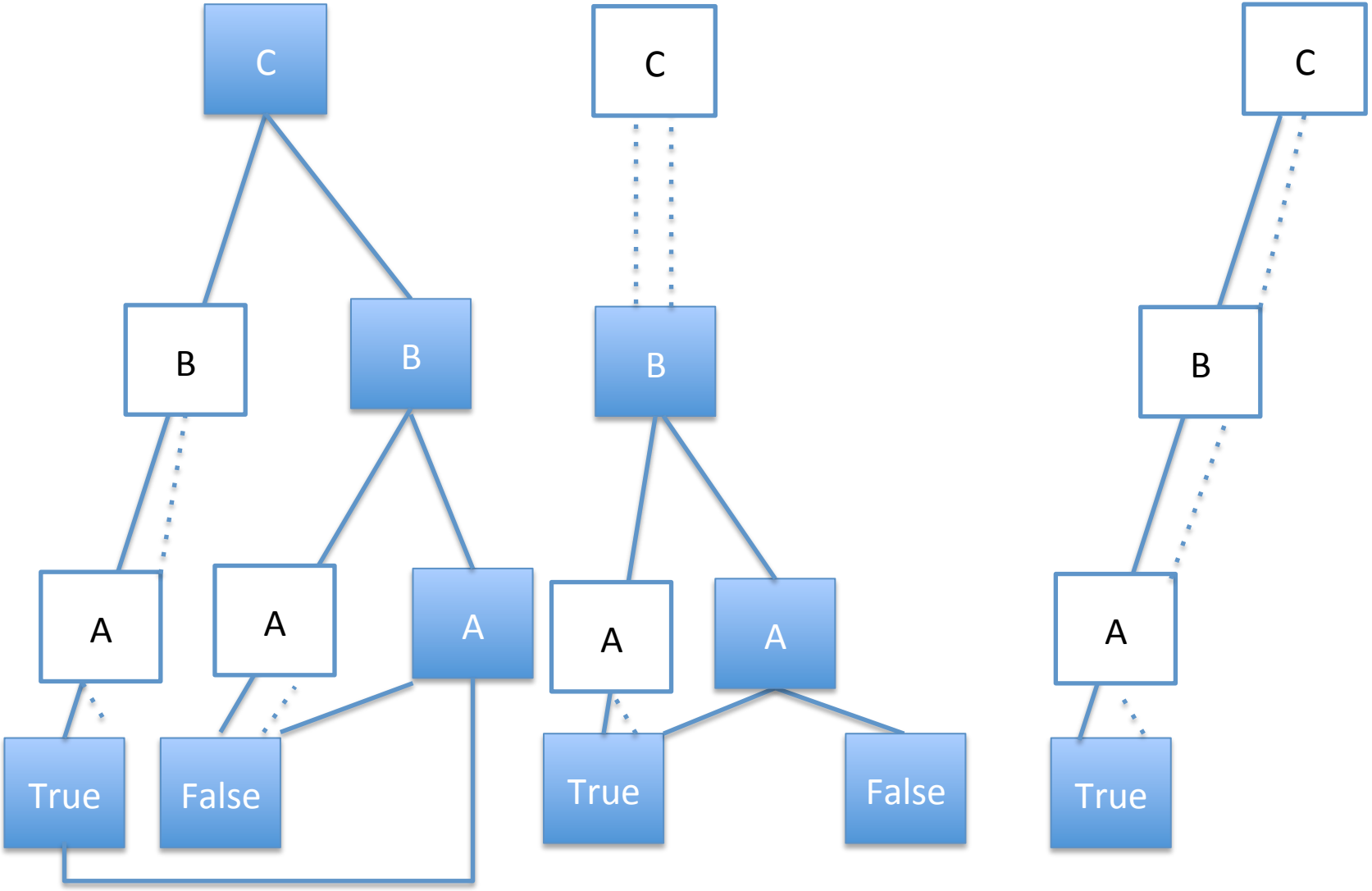

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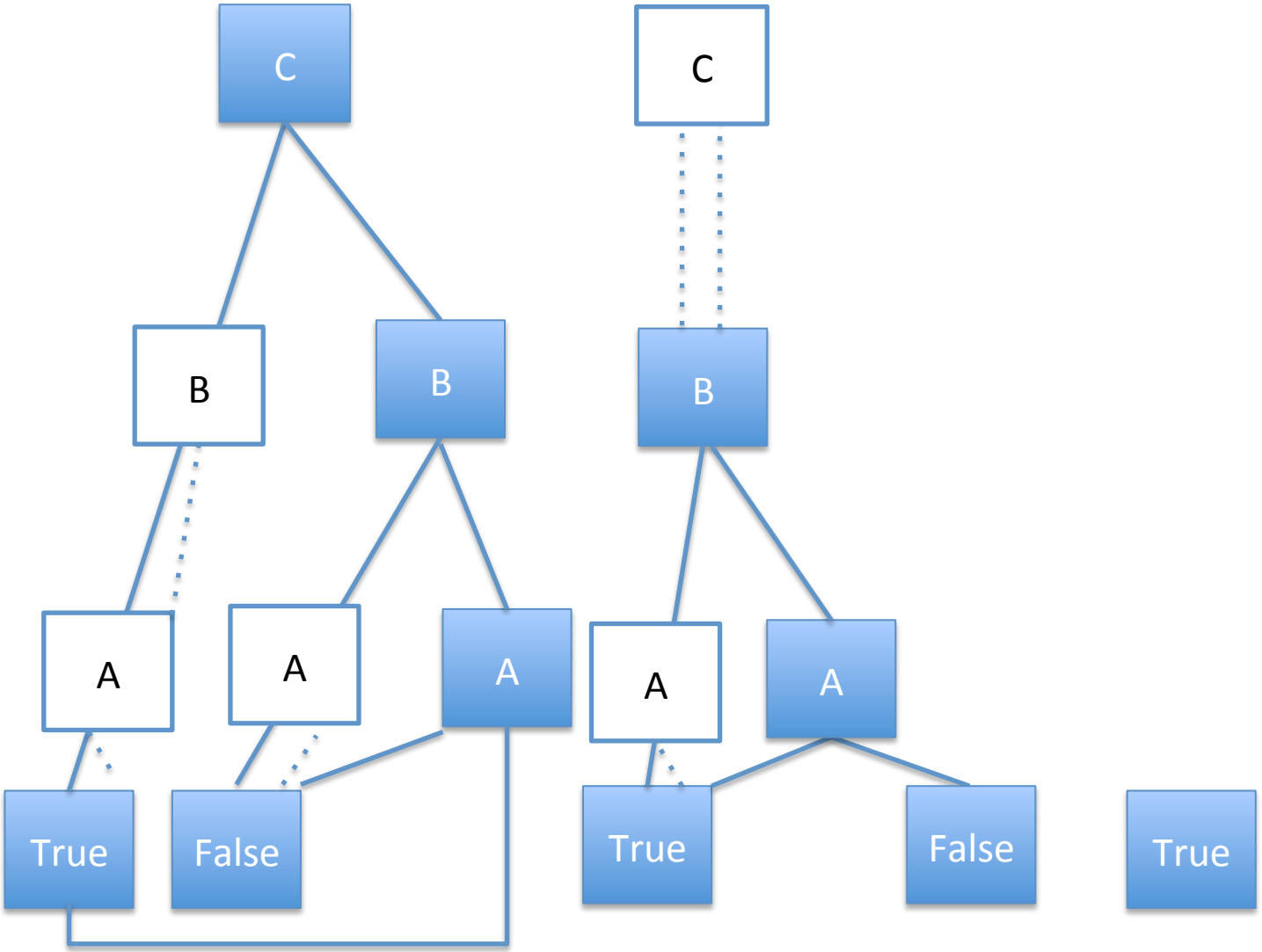

Uses of ROBDDs

- Reduced Order BDD for a proposition is unique for a fixed variable ordering
- Proposition is valid iff its ROBDD is just True
- Proposition is satisfiable its ROBDD is not just False
- Can check if a given valuation satisfies proposition in time linear to number of variables by walking the corresponding branch