HW 2 – Binary Decision Diagrams

CS 477 – Spring 2018

Revision 1.0

Assigned January 31, 2018
Due February 7, 2018, 9:00 pm
Extension 48 hours (20% penalty)

1 Change Log

1.0 Initial Release.

2 Objectives and Background

The purpose of this HW is to test your understanding of

- Binary Decision Diagrams and the Shannon Expansion

Another purpose of HWs is to provide you with experience answering non-programming written questions of the kind you may experience on the midterm and final.

3 Turn-In Procedure

The pdf for this assignment (`hw2.pdf`) should be found in the `assignments/hw2/` subdirectory of your svn directory for this course. Your solution should be put in that same directory. Using your favorite tool(s), you should put your solution in a file named `hw2-submission.pdf`. If you have problems generating a pdf, please seek help from the course staff. Your answers to the following questions are to be submitted electronically from within `assignments/hw2/` subdirectory by committing the file as follows:

```
svn add hw2-submission.pdf
svn commit -m "Turning in hw2"
```

4 Problem

For each of the following propositions,

a. (4 pts each) give the Shannon expansion (put it in if_then_else_ form), where you use the alphabetical ordering from least to greatest in generating the conditionals by the algorithm shown in class,

b. (5 pts each) give the reduced ordered binary decision diagram (ROBDD), with the variables order smallest to largest alphabetically,

c. (5 pts each) give the reduced ordered binary decision diagram (ROBDD), with the variables order reverse alphabetically,

d. (3pts each) say whether it is satisfiable, and if it is, give a valuation satisfying it.

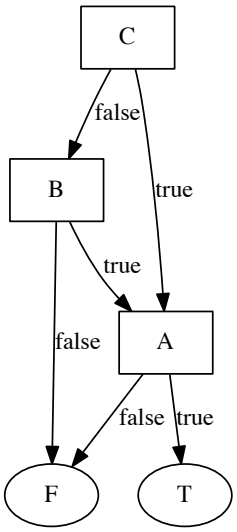
Since many of you will want to know, I used dot/graphiz to generate the pdfs for the directed graphs you need in b and c. You should feel free to draw them by hand.

1. $(A \wedge B) \vee (A \wedge C)$

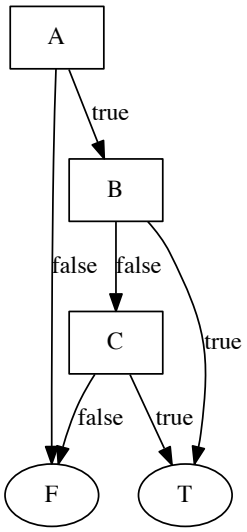
Solution:

a.
```
if C   then if B   then if A   then True
                               else False
                   else if A   then True
                               else False
       else if B   then if A   then True
                               else False
                   else if A   then False
                               else False
```

b. $C > B > A$



c. $A > B > C$



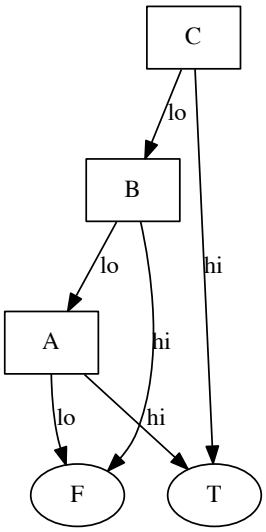
d. I only asked for one valuation, but I'm giving a description of all here.
[$\{A \mapsto \mathsf{True}; B \mapsto \mathsf{True}; C \mapsto \mathsf{True}\}$;
$\{A \mapsto \mathsf{True}; B \mapsto \mathsf{True}; C \mapsto \mathsf{False}\}$;
$\{A \mapsto \mathsf{True}; B \mapsto \mathsf{False}; C \mapsto \mathsf{True}\}$]

2. $(A \Rightarrow (B \Rightarrow C)) \wedge ((A \Rightarrow B) \Rightarrow C)$

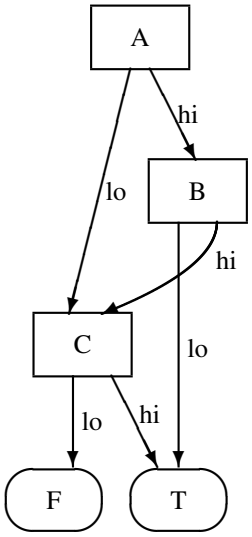
Solution:

a.
```
if C   then if B   then if A   then True
                               else True
                   else if A   then True
                               else True
       else if B   then if A   then False
                               else False
                   else if A   then True
                               else False
```

b. $C > B > A$



c. $A > B > C$



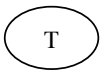
d. [$\{A \mapsto \mathsf{True}; B \mapsto \mathsf{True}; C \mapsto \mathsf{True}\}$;
$\{A \mapsto \mathsf{True}; B \mapsto \mathsf{False}; C \mapsto \mathsf{True}\}$;
$\{A \mapsto \mathsf{True}; B \mapsto \mathsf{False}; C \mapsto \mathsf{False}\}$;
$\{A \mapsto \mathsf{False}; B \mapsto \mathsf{True}; C \mapsto \mathsf{True}\}$;
$\{A \mapsto \mathsf{False}; B \mapsto \mathsf{False}; C \mapsto \mathsf{True}\}$]

3. $(A \Rightarrow B) \Rightarrow ((A \wedge C) \Rightarrow (B \wedge C))$

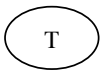
Solution:

a.
```
if C  then if B  then if A  then True
                            else True
                 else if A  then True
                            else True
      else if B  then if A  then True
                            else True
                 else if A  then True
                            else True
```

b. $C > B > A$



c. $A > B > C$



d.

$$
\begin{array}{ll}
[& \{A \mapsto \mathsf{True}; B \mapsto \mathsf{True}; C \mapsto \mathsf{True}\}; \\
& \{A \mapsto \mathsf{True}; B \mapsto \mathsf{True}; C \mapsto \mathsf{False}\}; \\
& \{A \mapsto \mathsf{True}; B \mapsto \mathsf{False}; C \mapsto \mathsf{True}\}; \\
& \{A \mapsto \mathsf{True}; B \mapsto \mathsf{False}; C \mapsto \mathsf{False}\}; \\
& \{A \mapsto \mathsf{False}; B \mapsto \mathsf{True}; C \mapsto \mathsf{True}\}; \\
& \{A \mapsto \mathsf{False}; B \mapsto \mathsf{True}; C \mapsto \mathsf{False}\}; \\
& \{A \mapsto \mathsf{False}; B \mapsto \mathsf{False}; C \mapsto \mathsf{True}\}; \\
& \{A \mapsto \mathsf{False}; B \mapsto \mathsf{False}; C \mapsto \mathsf{False}\} \ ]
\end{array}
$$

That is, every assignment models $(A \Rightarrow B) \Rightarrow ((A \wedge C) \Rightarrow (B \wedge C))$; it is a tautology.