

CS477 Formal Software Development Methods

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How to build a Reduced Ordered BDD

- Done by recursion on the structure of the formula
- Order propositional variables
- For each variable, create a three node tree corresponding to (if Var then true else false)
- For formula not P, build ROBDD from ROBDD for P by flipping values in the leaves

How to build a Reduced Ordered BDD

- For $P \text{ <op> } Q$
 - Fill in each branch of BDDs for P and Q with all nodes appearing in either BDD, not already on branch
 - For each ordered sequence of values (done in order of variables, greatest to least, false before true) build the branch in partial BDD for $P \text{ <op> } Q$
 - Value in end leaf is $v1 \text{ <op> } v2$ where $v1$ is value at end of branch in BDD for P , $v2$ for branch in Q

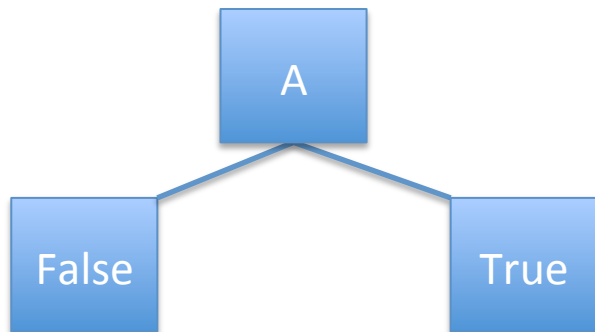
How to build a Reduced Ordered BDD

- When you add a new node
 - If the outedges point to same node, move all edges pointing to it to point to what it points to, and remove / ghost it.
 - If there exists another node with the same label whose edges point the same nodes, remove the new node and move all edges pointing to it to the other node

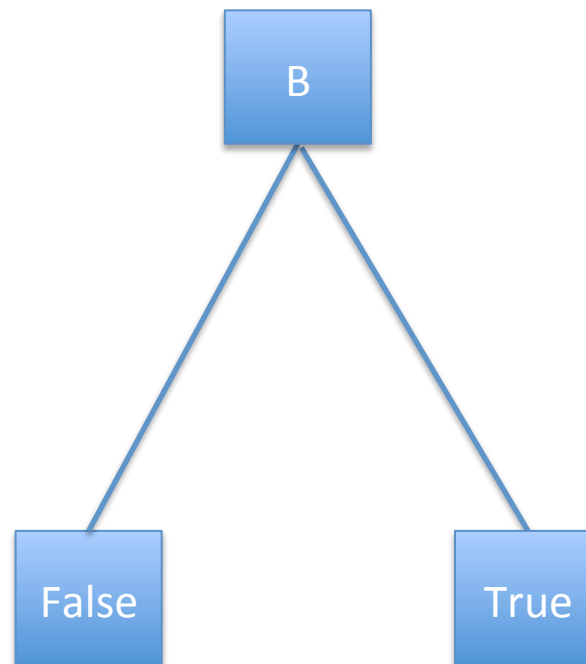
Example

- Find ROBDD for $(A \wedge B) \vee (\text{not } C)$

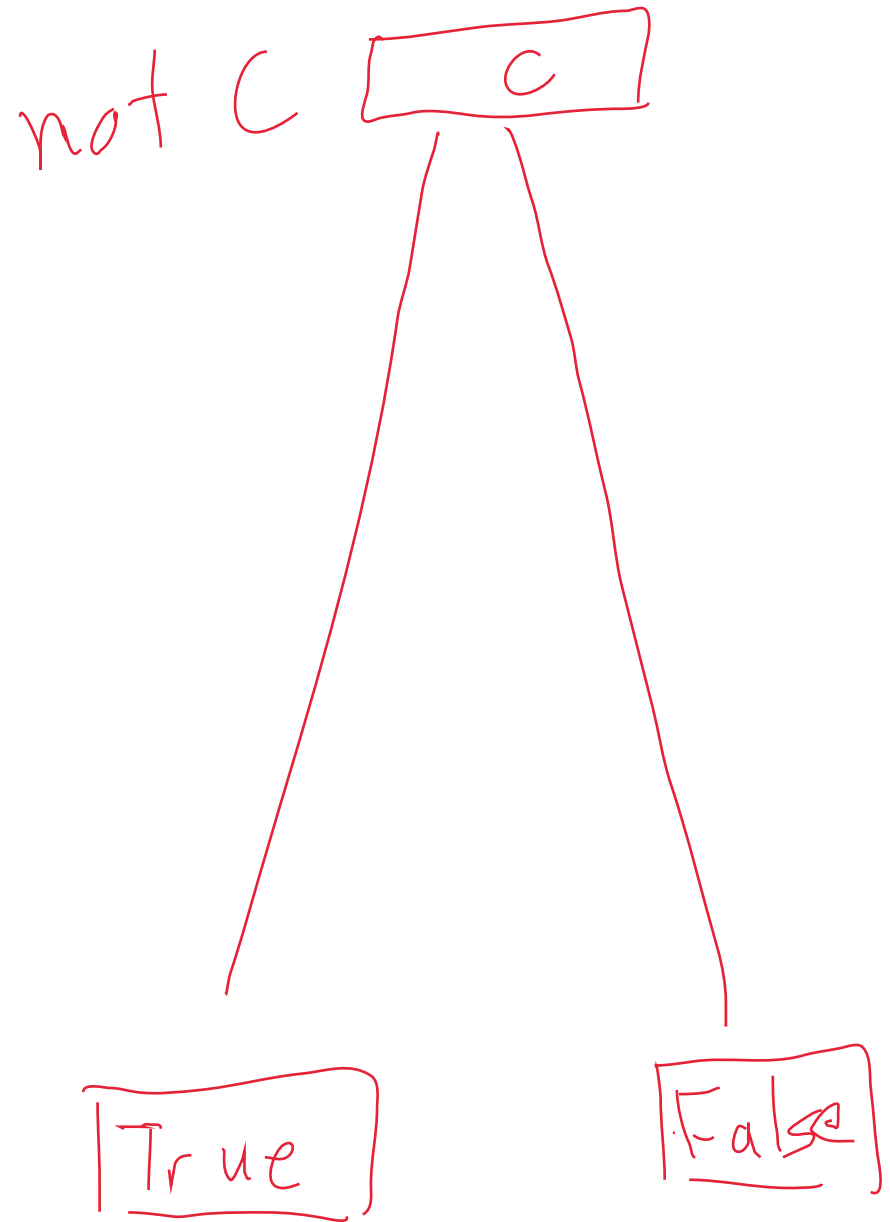
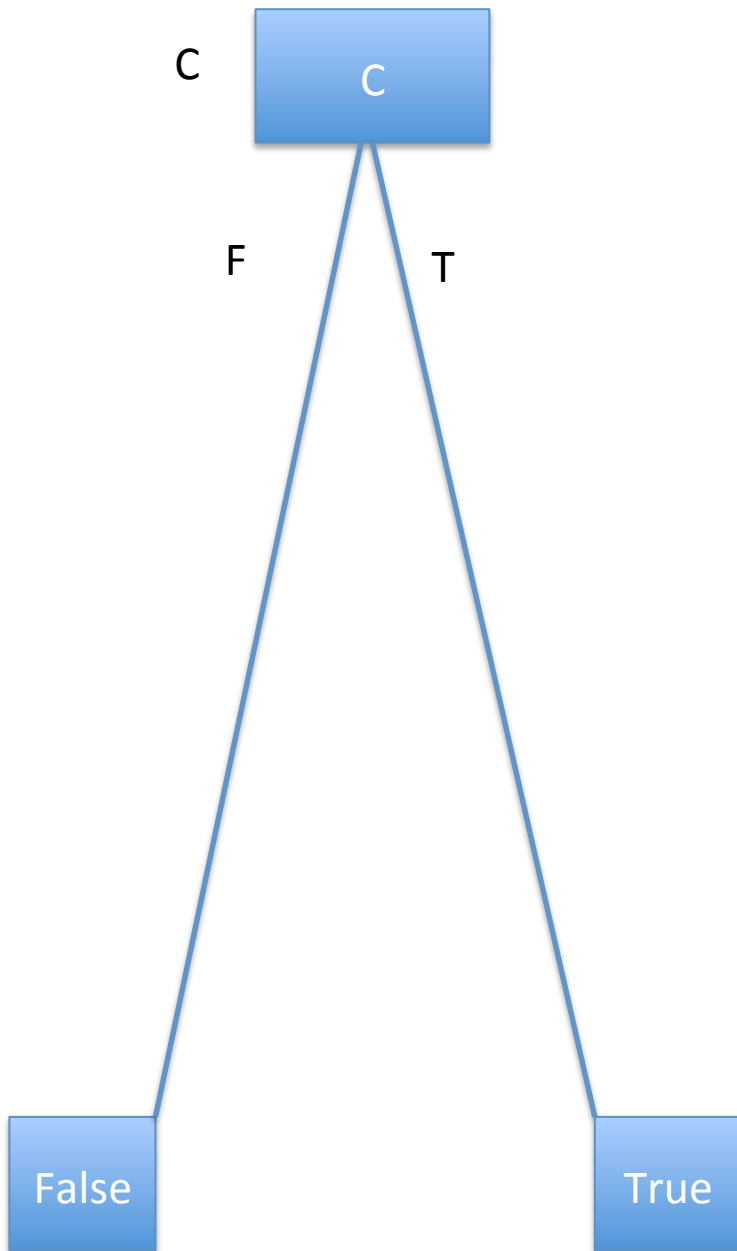
A Variables: $A < B < C$



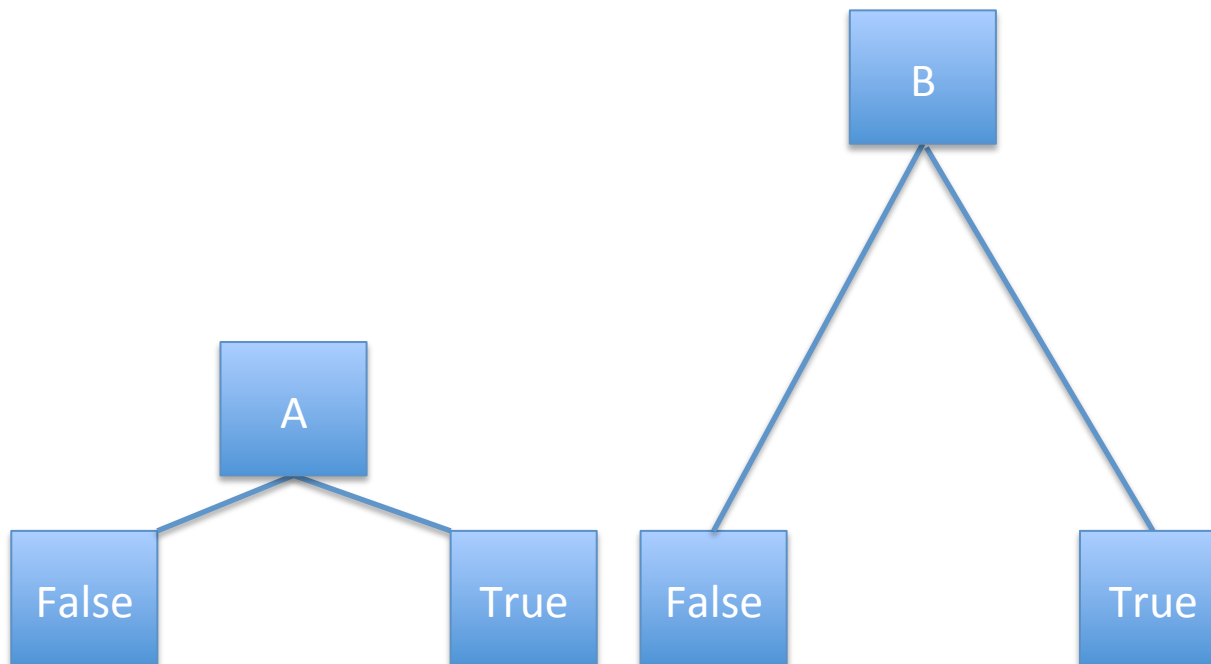
B Variables: $A < B < C$



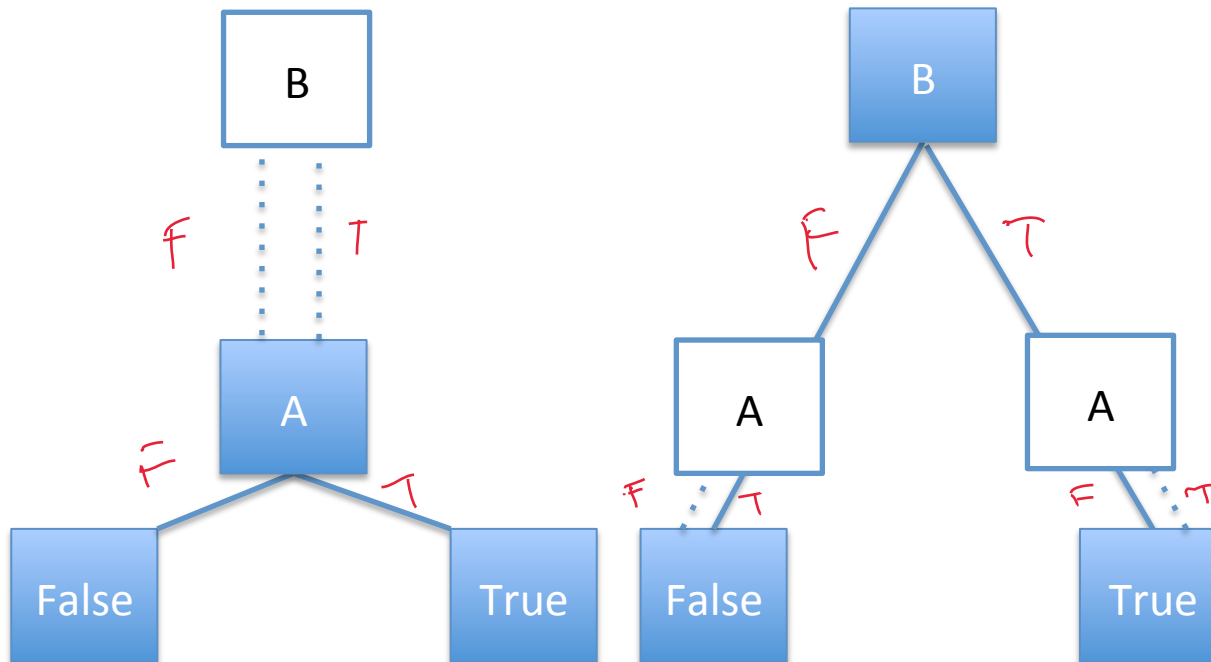
C Variables: $C > B > A$



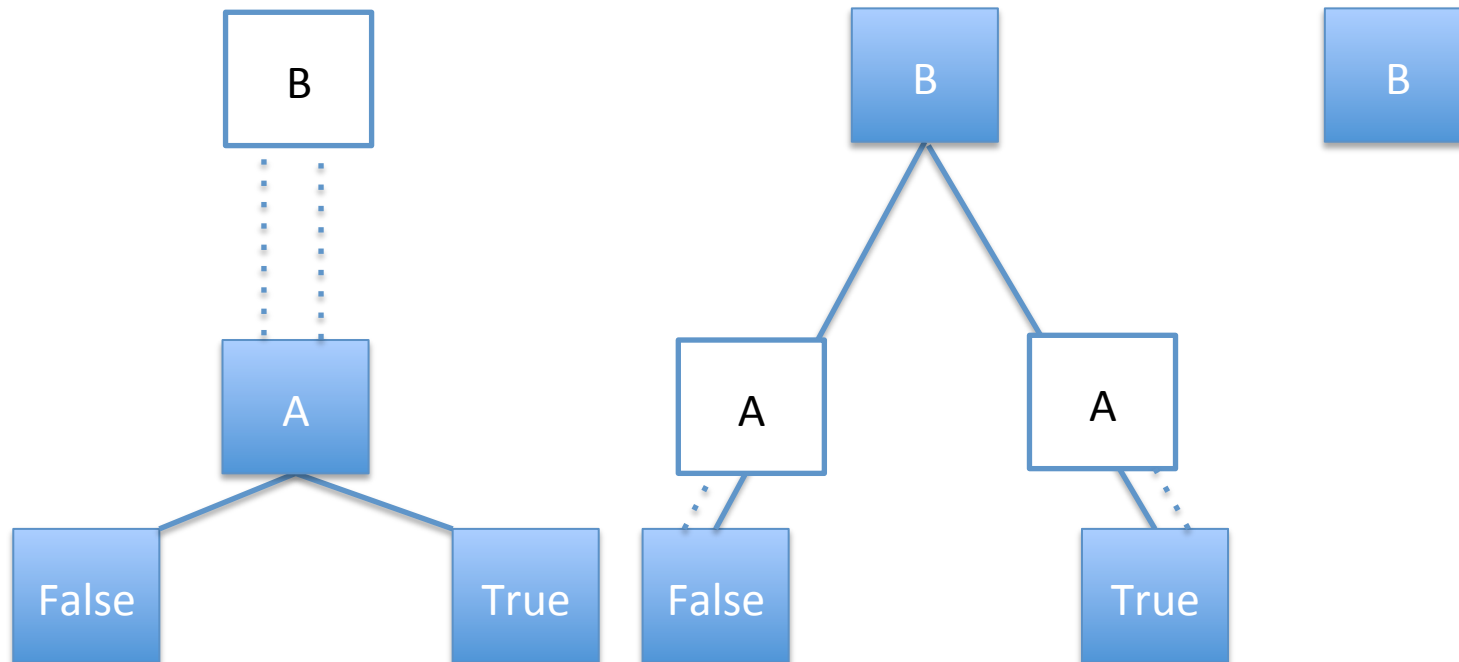
$(A \wedge B)$ Variables: $A < B < C$



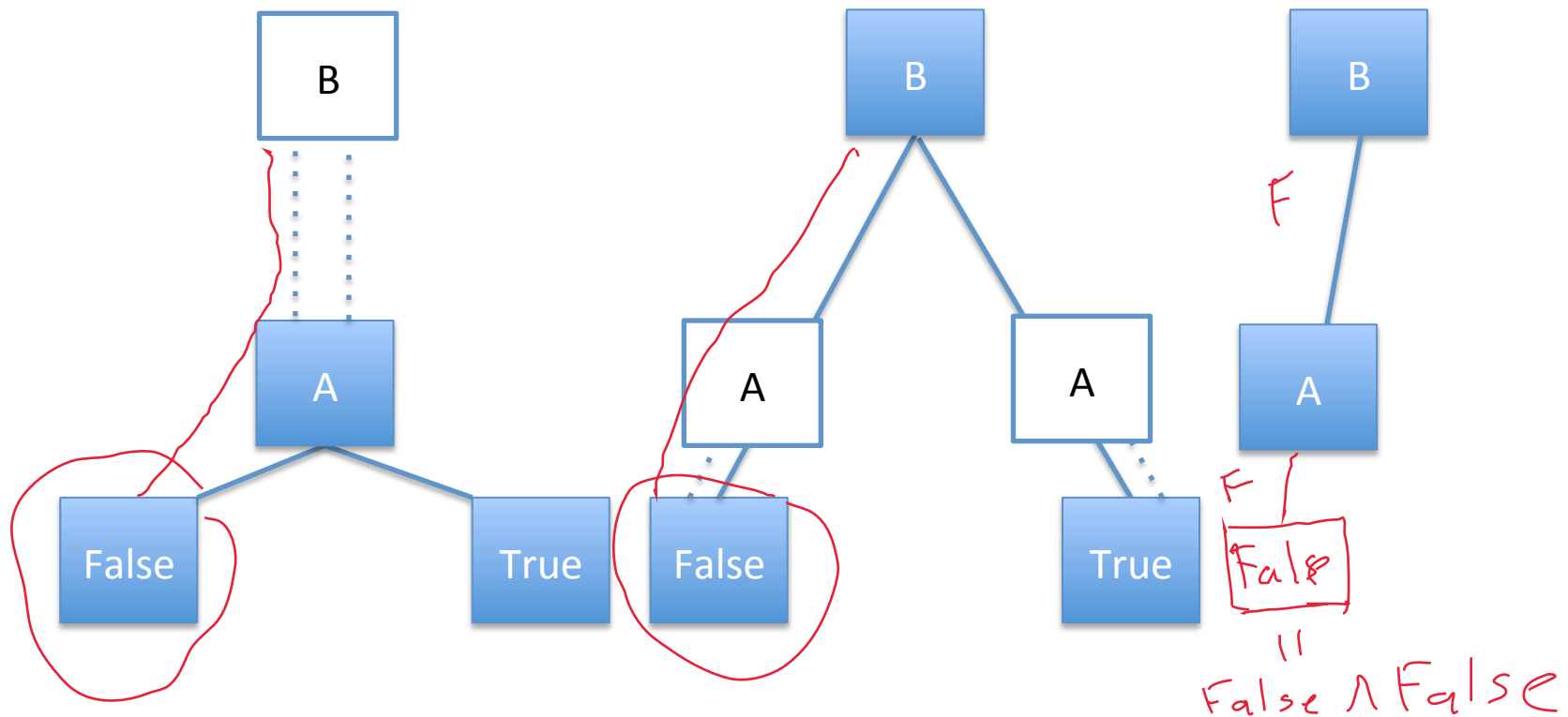
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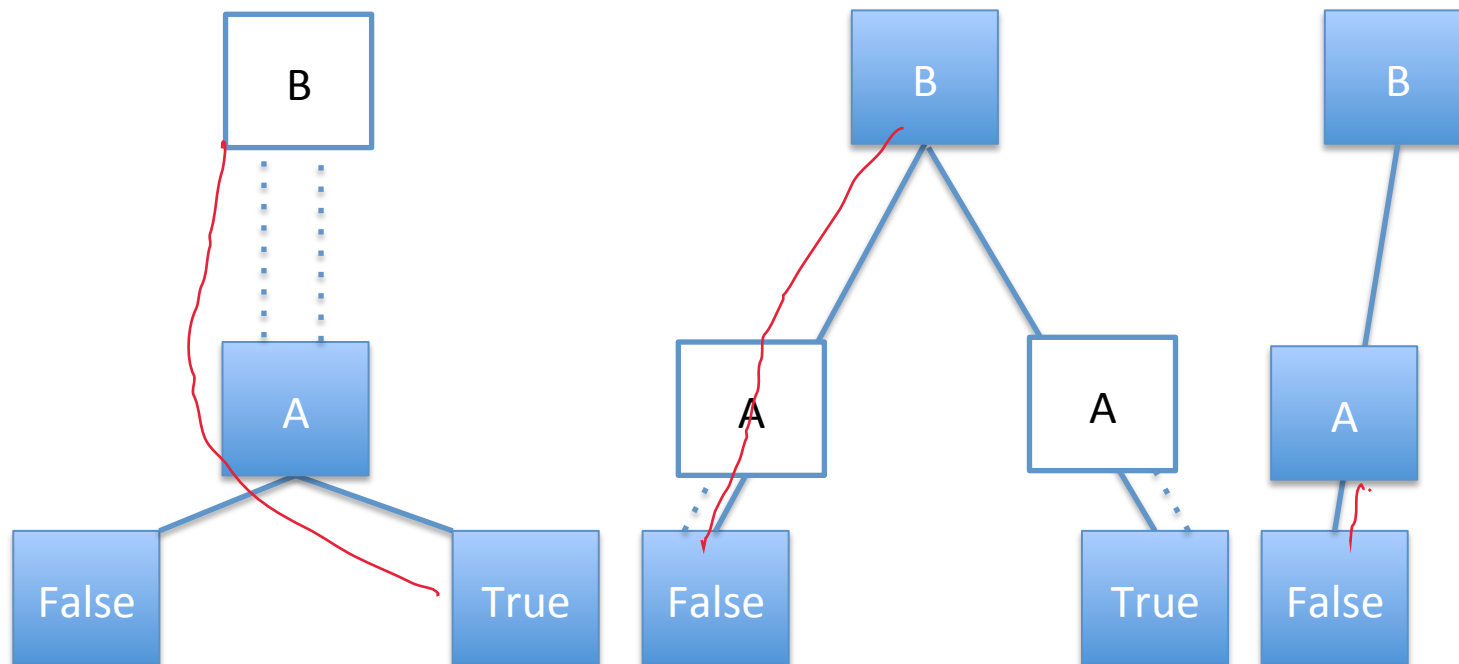
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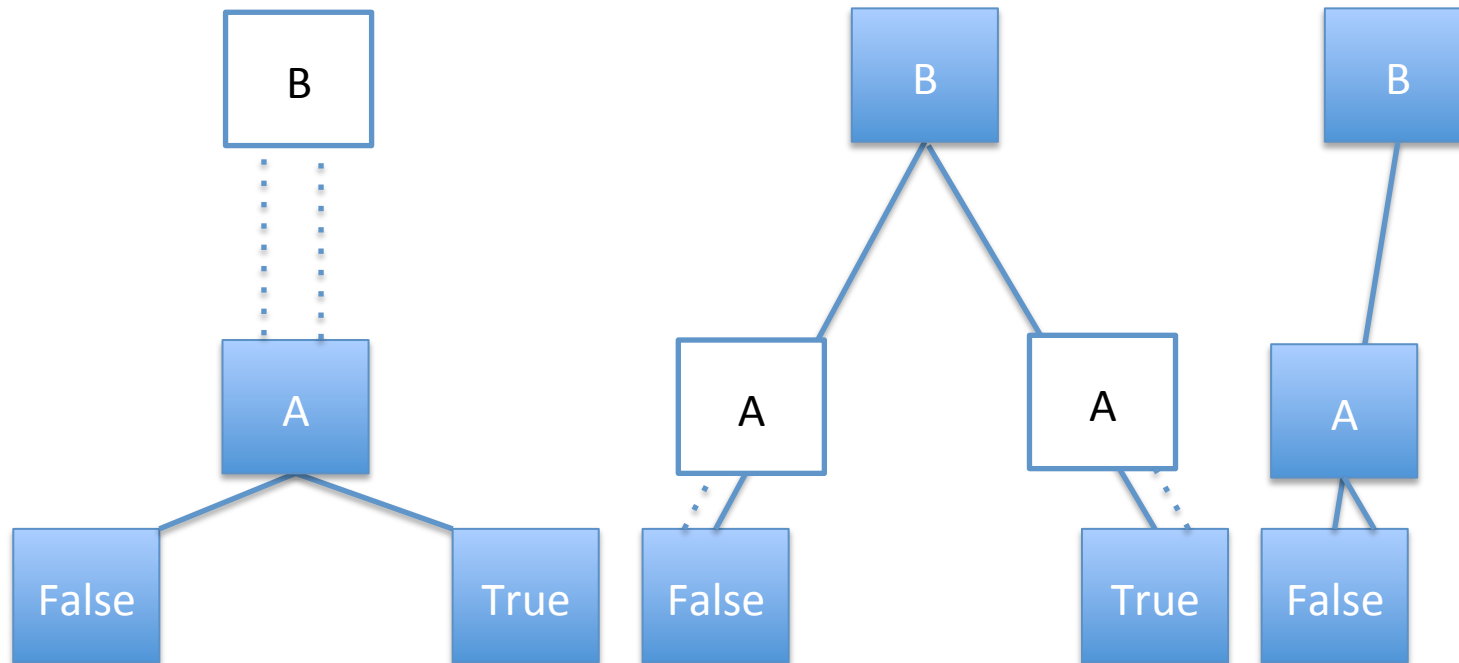
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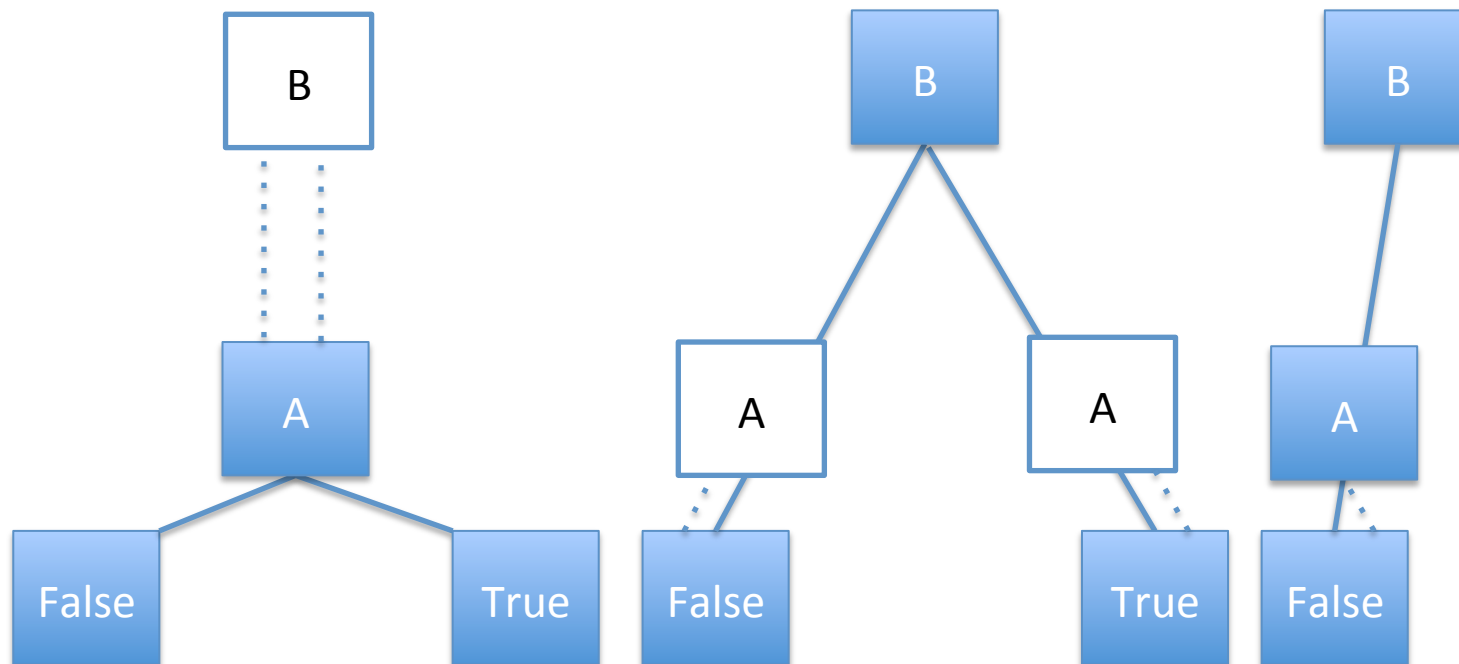
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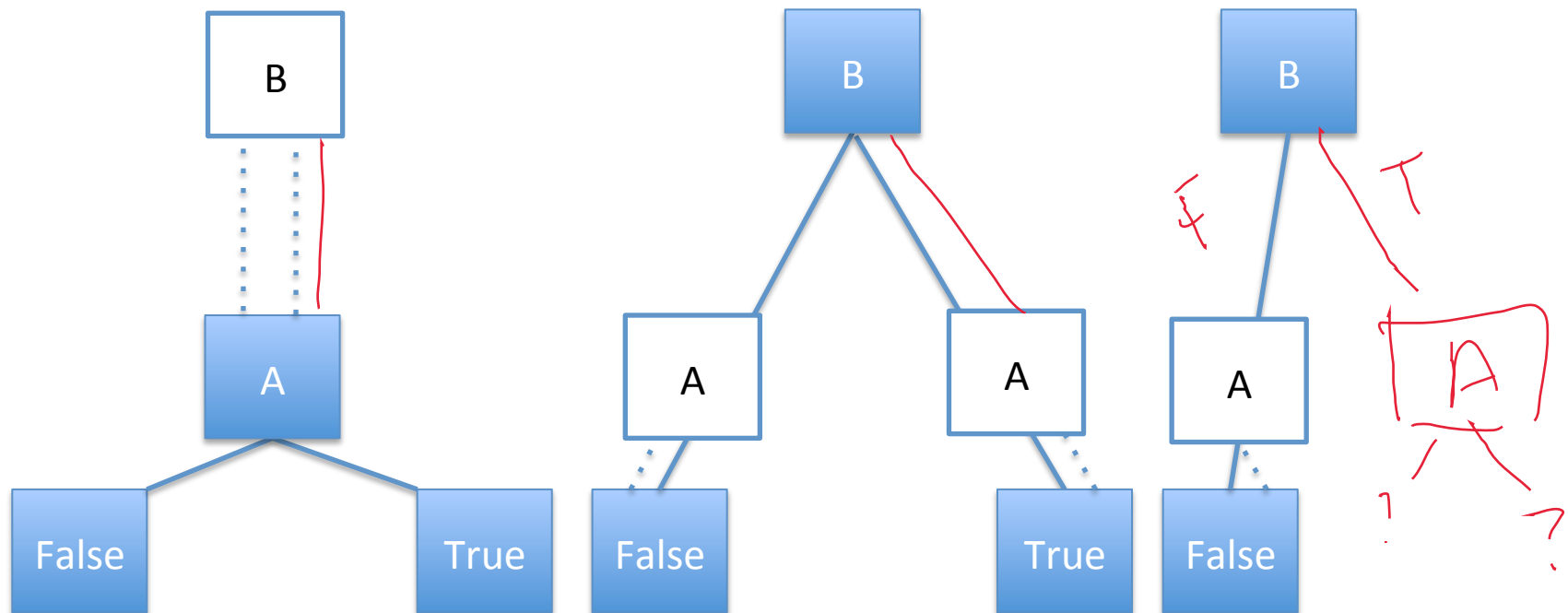
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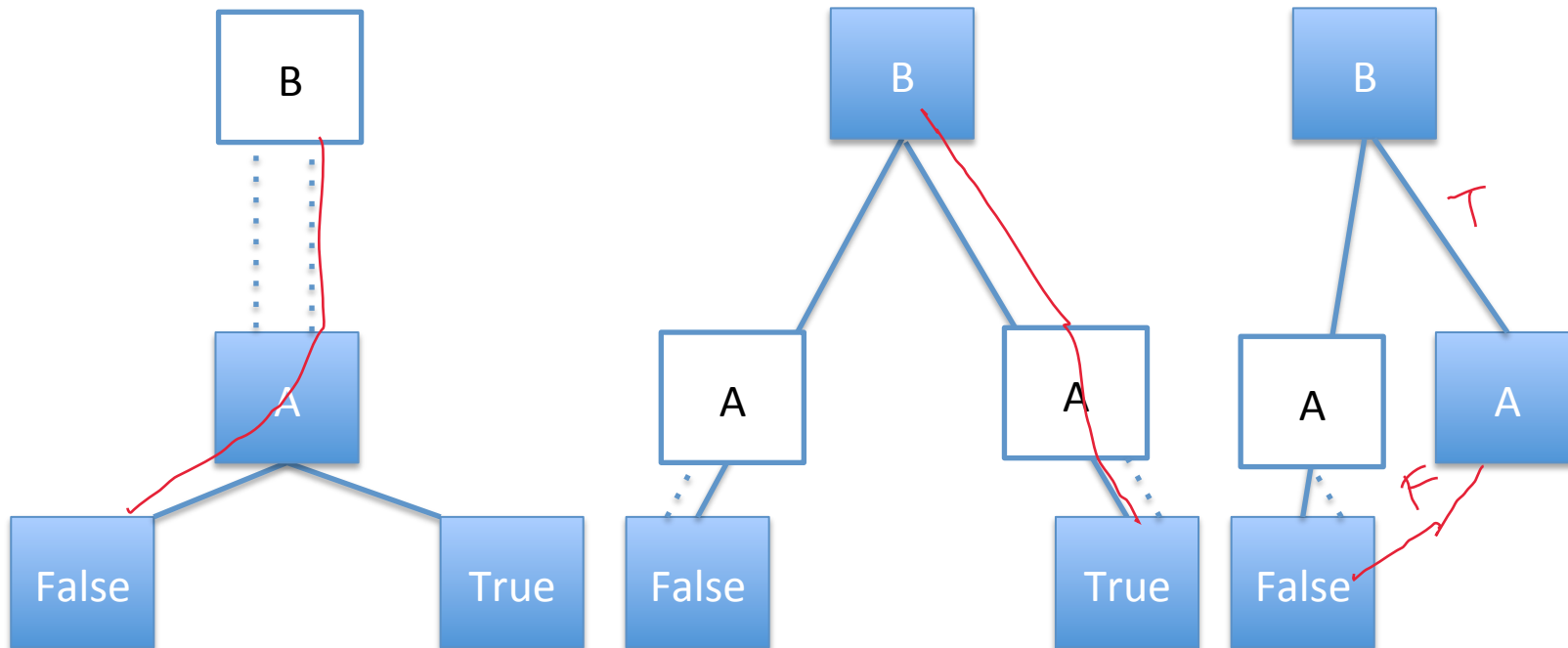
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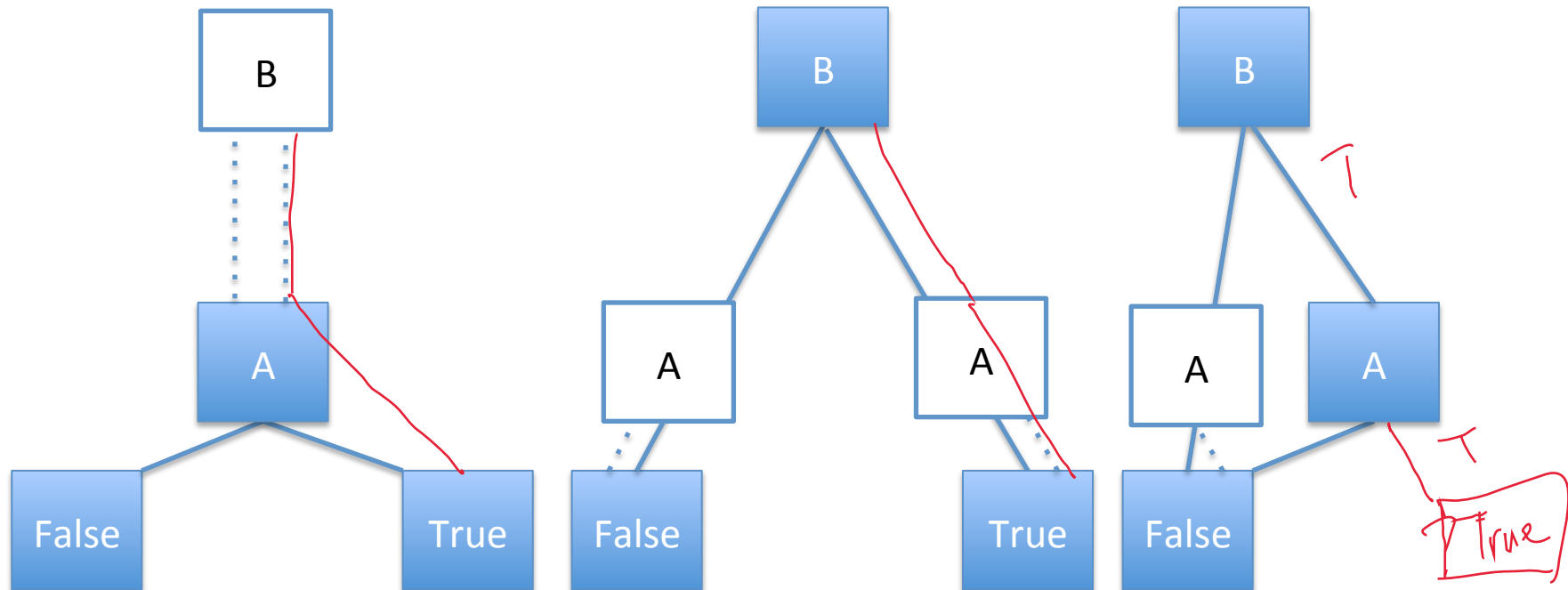
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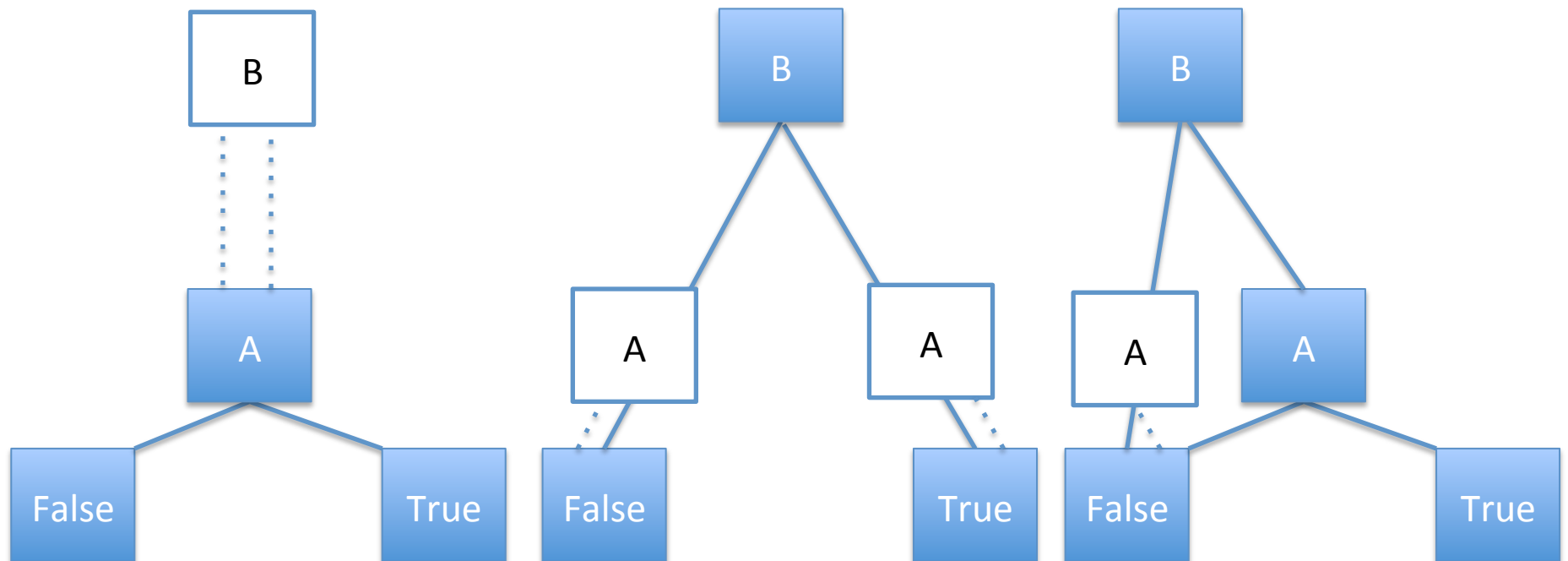
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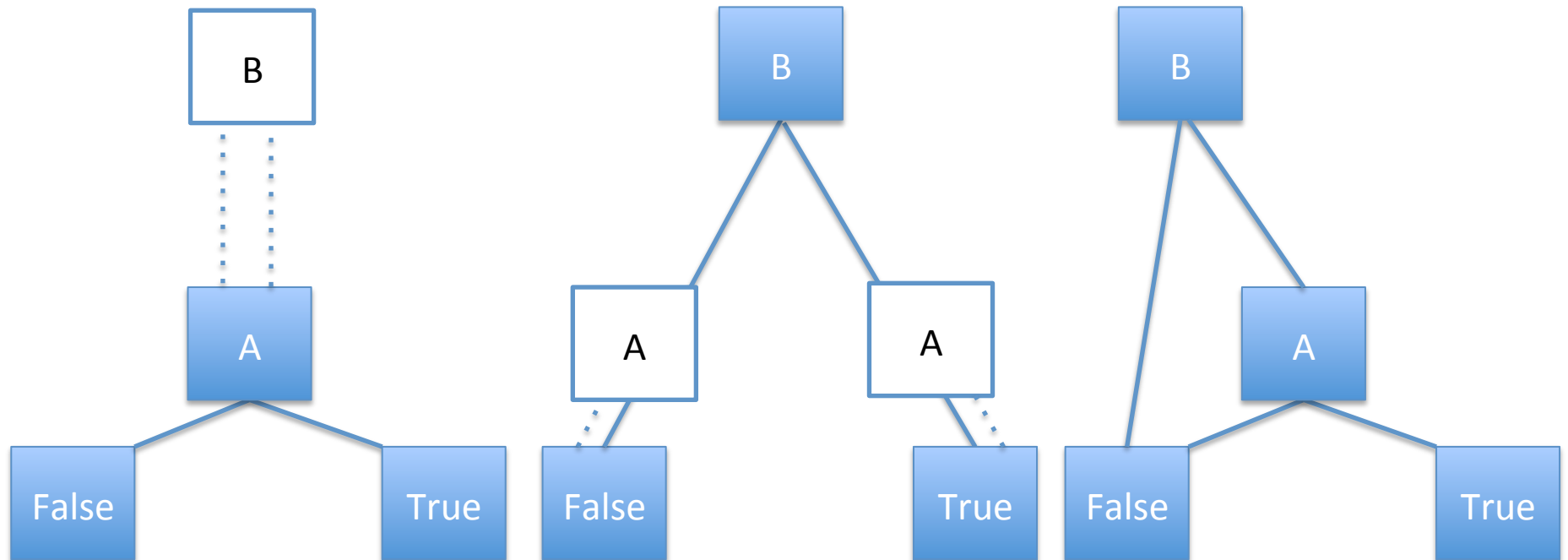
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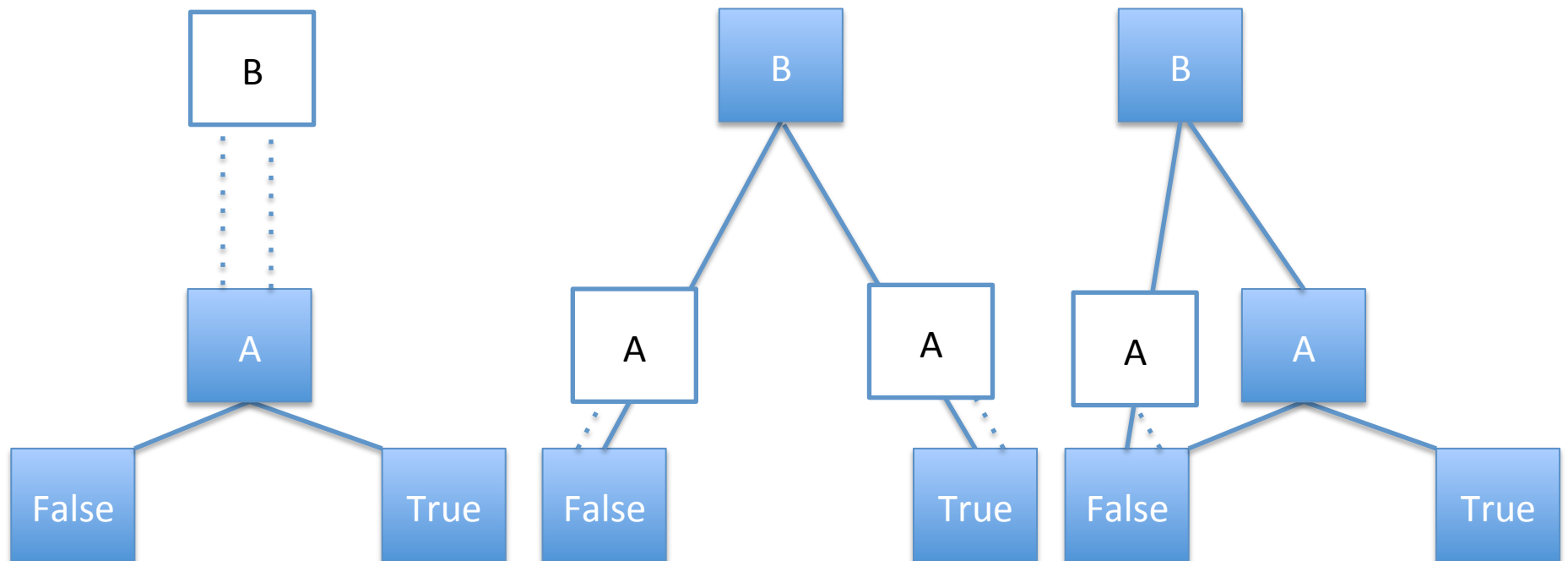
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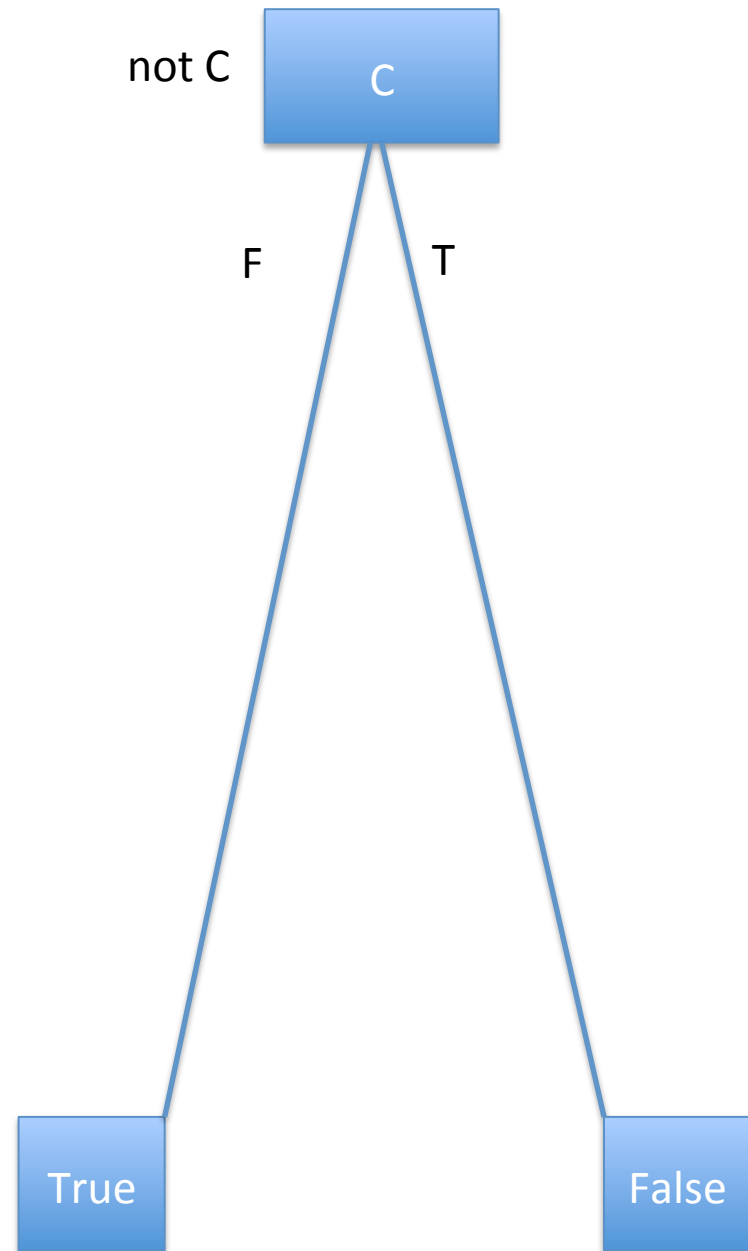
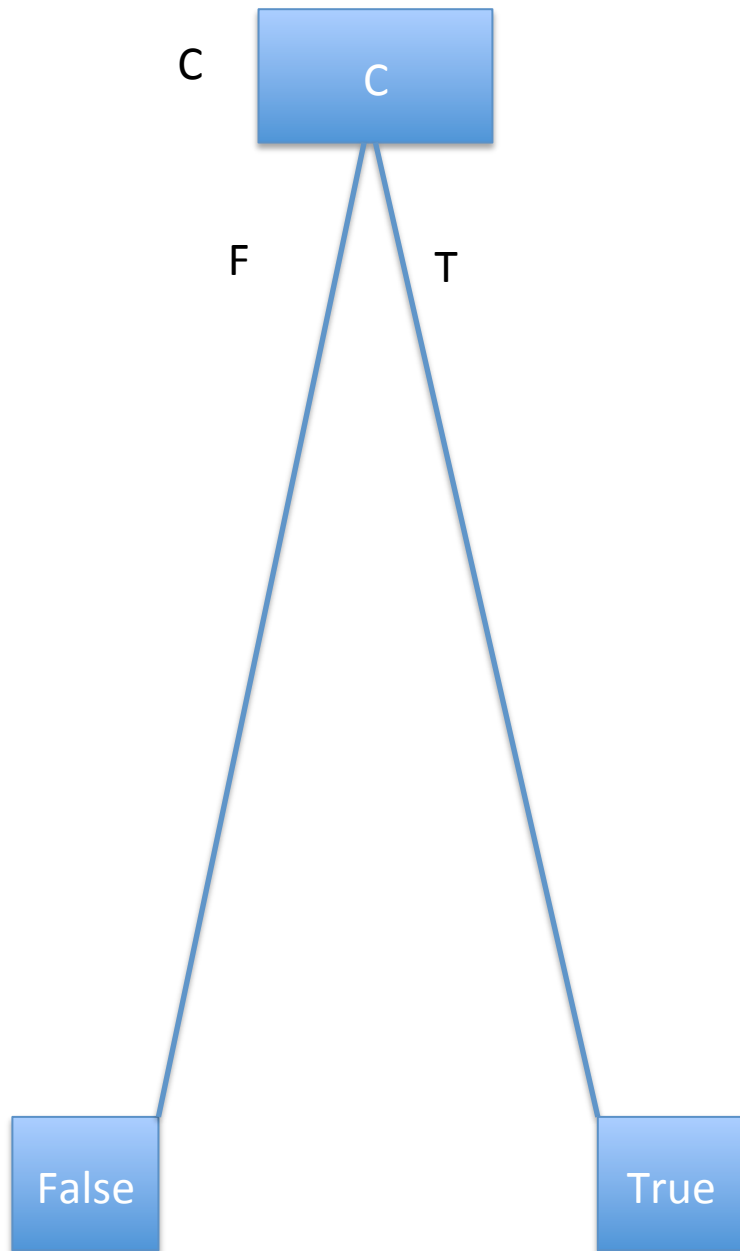


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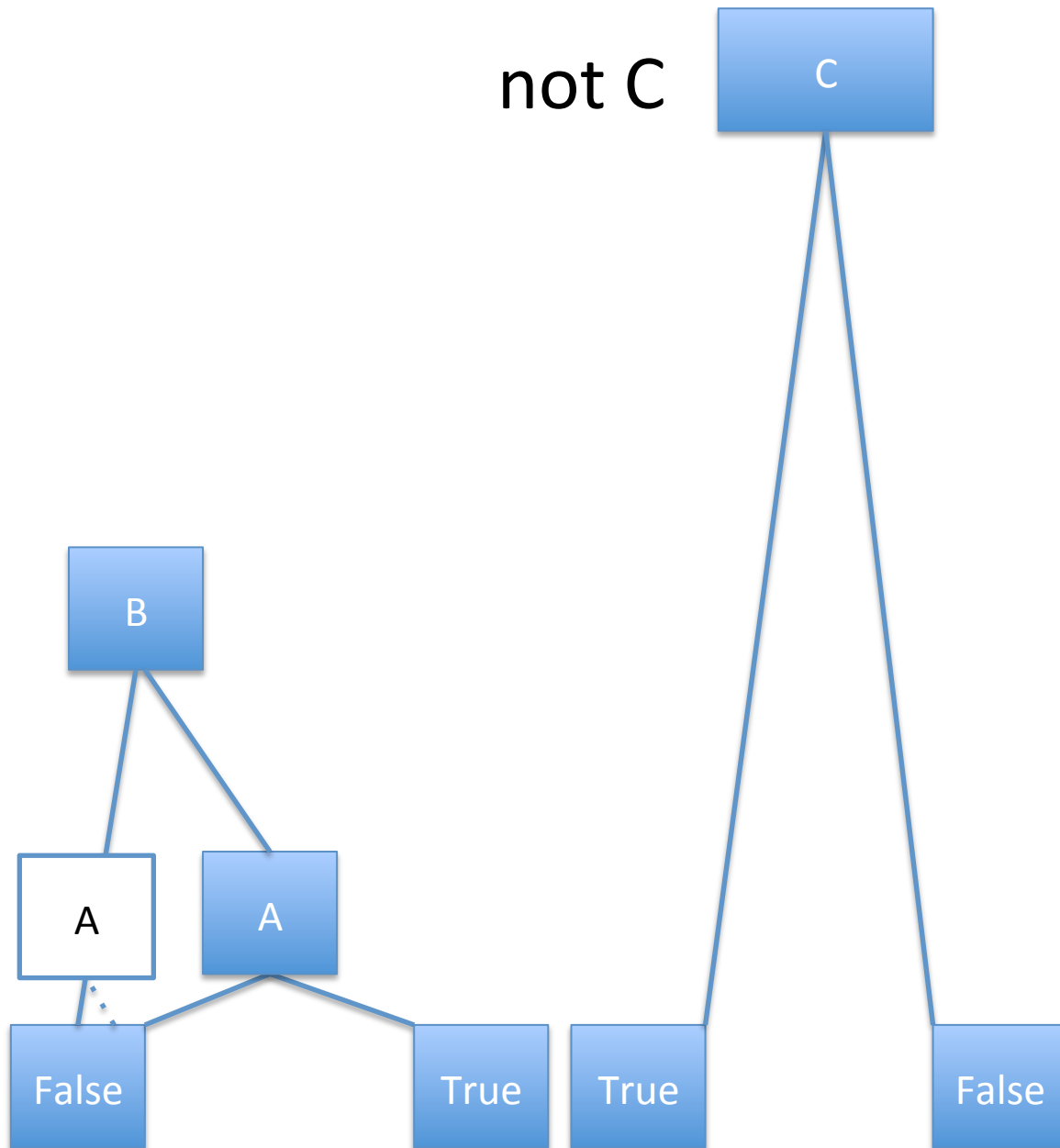


(not C)

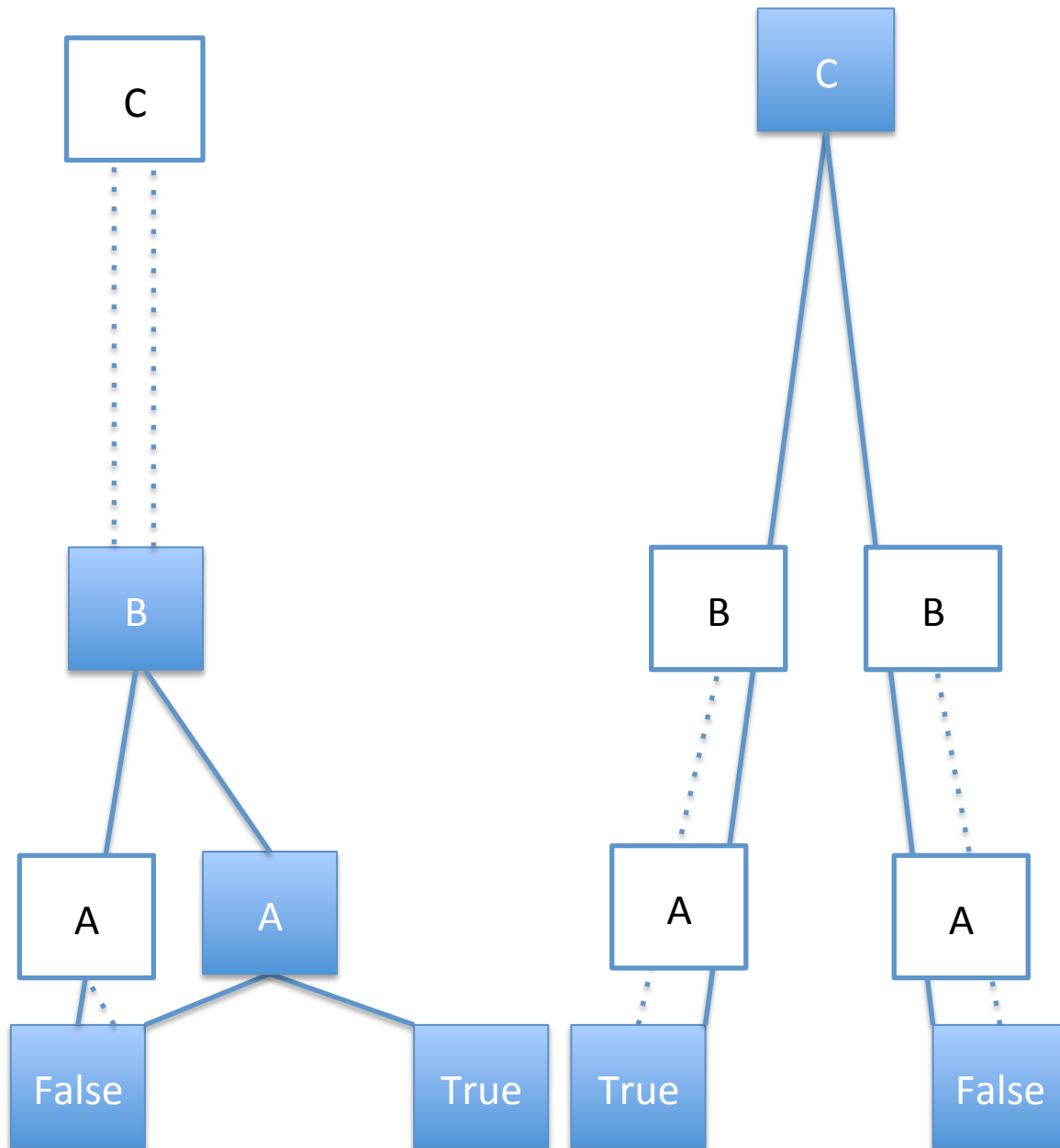
Variables: $C > B > A$



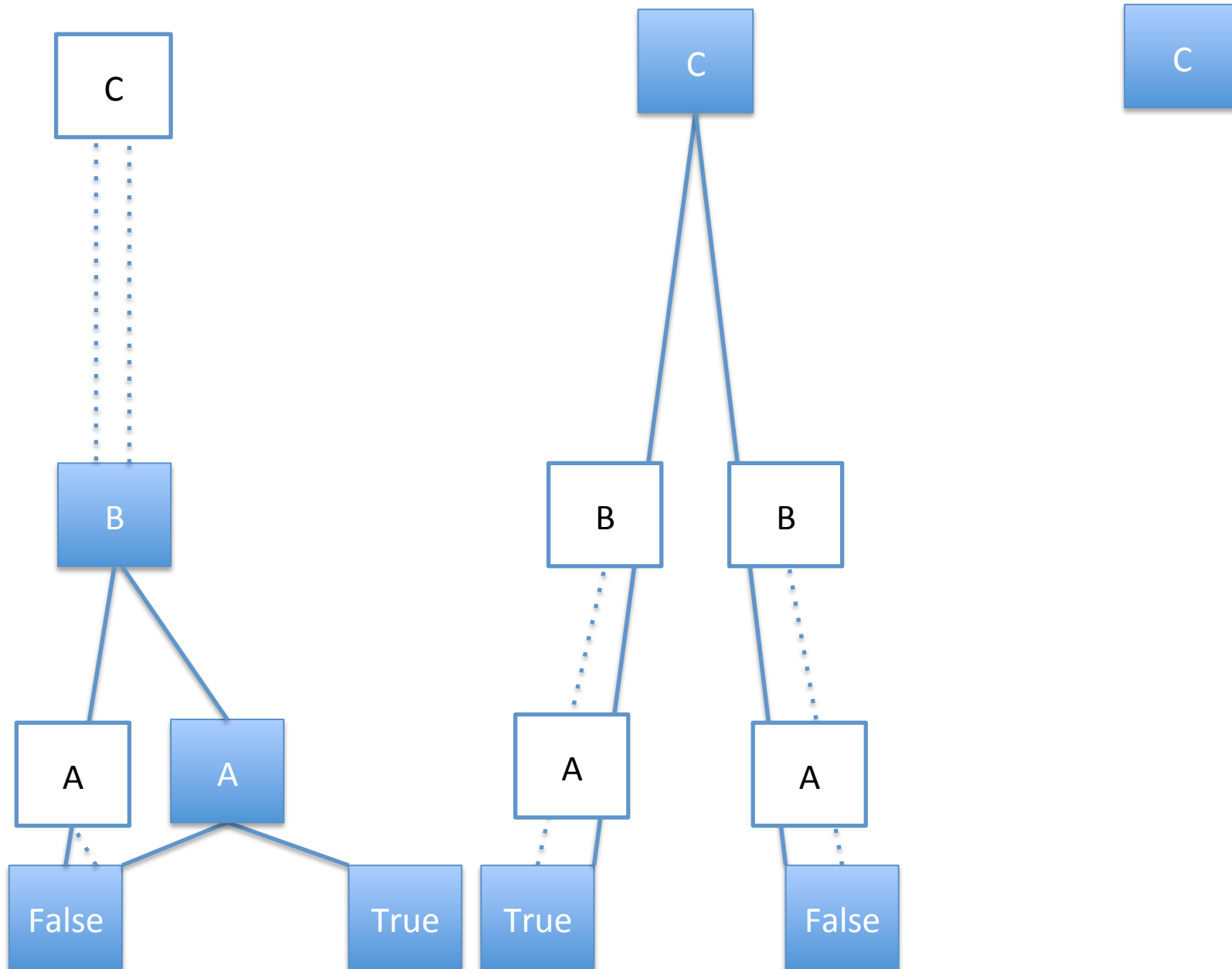
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$(A \wedge B) \vee (\text{not } C)$ Variables: $C > B > A$

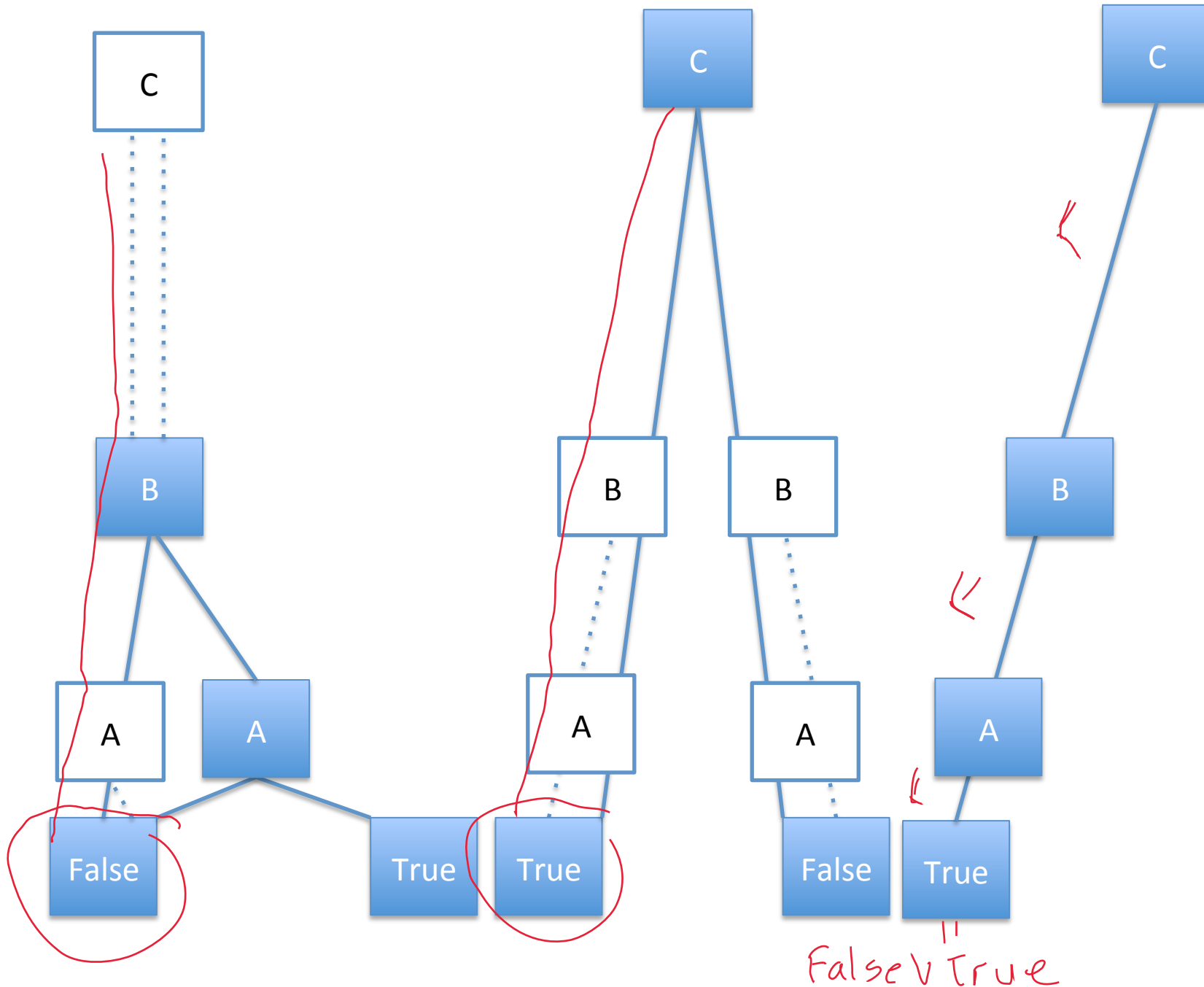


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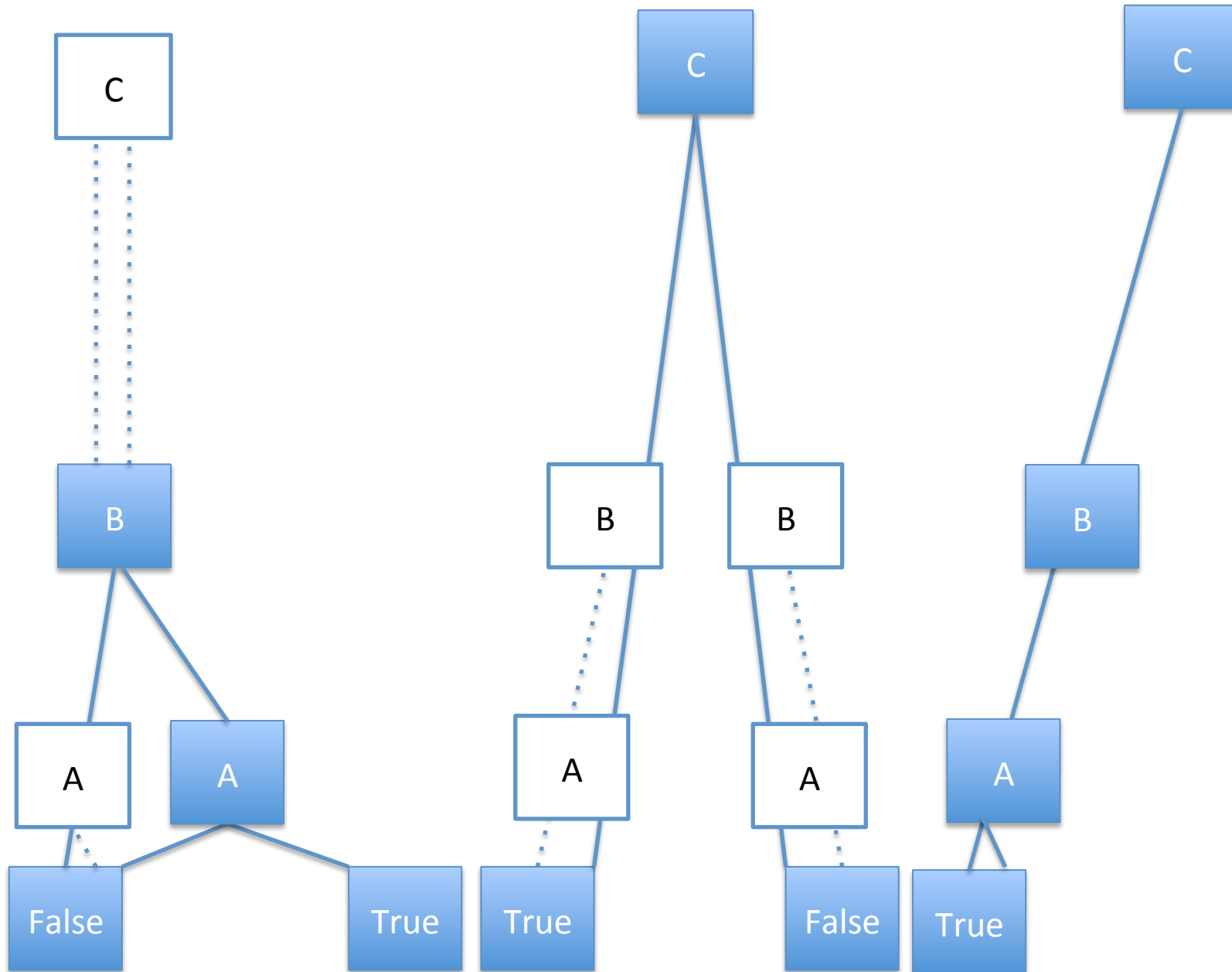


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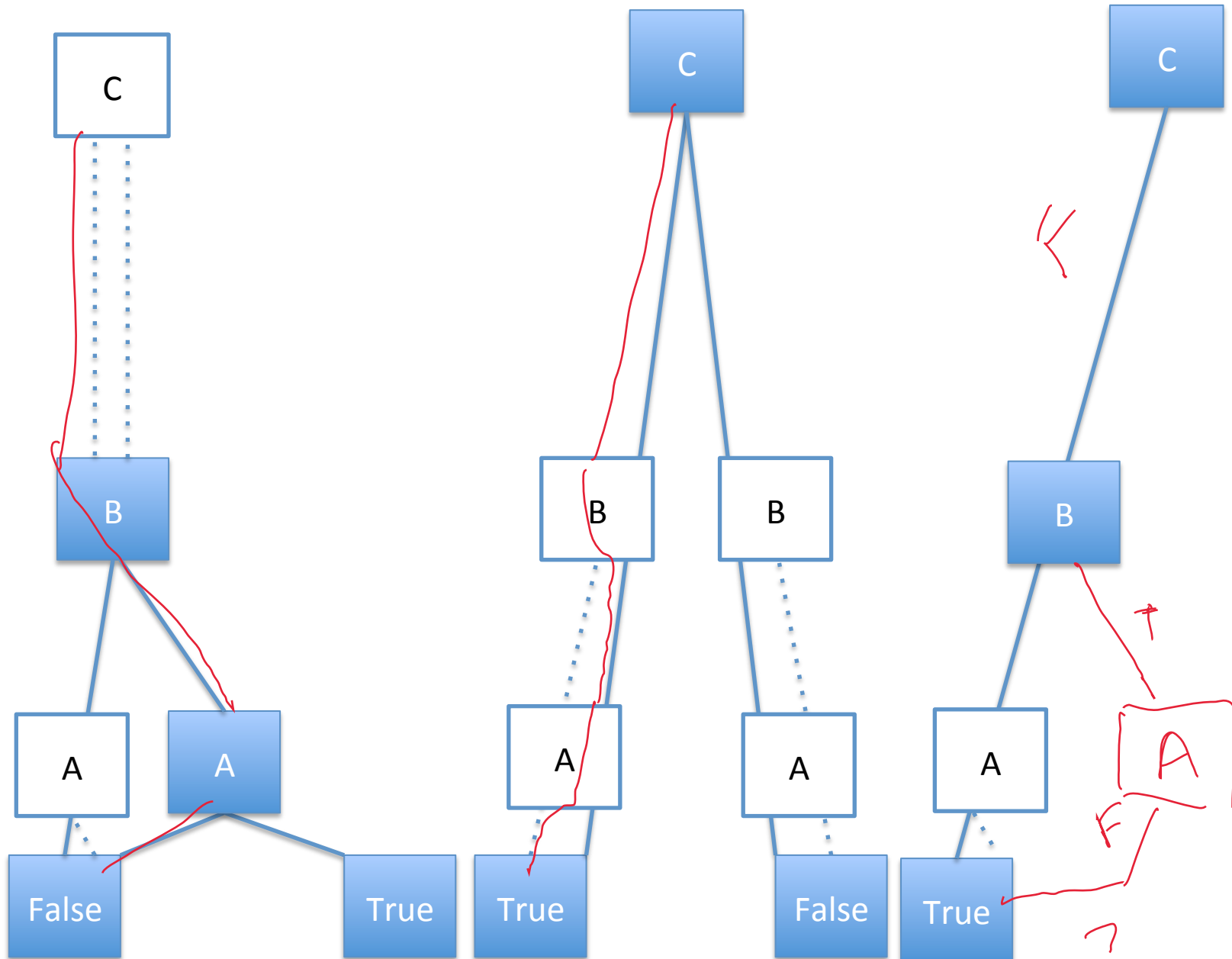
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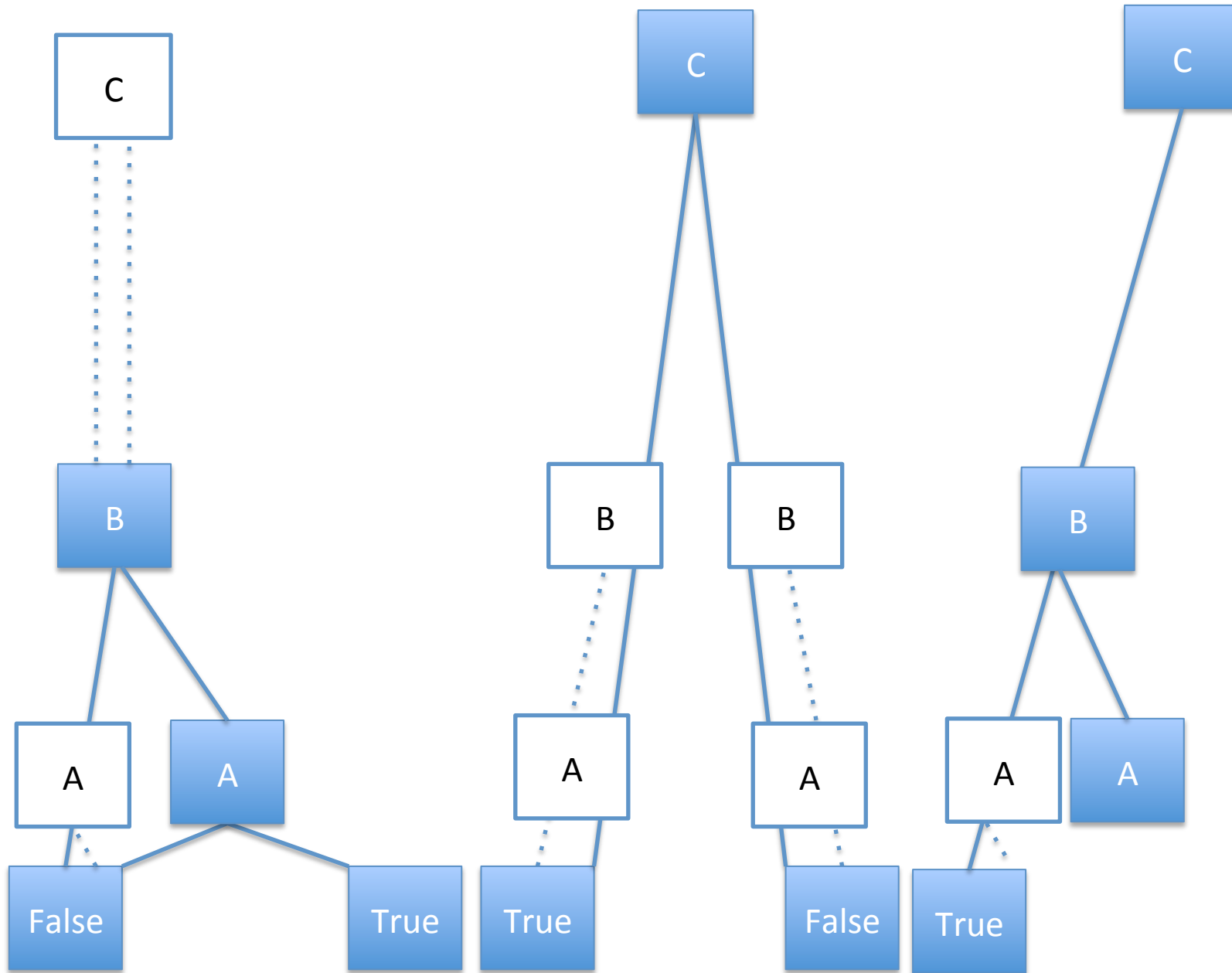
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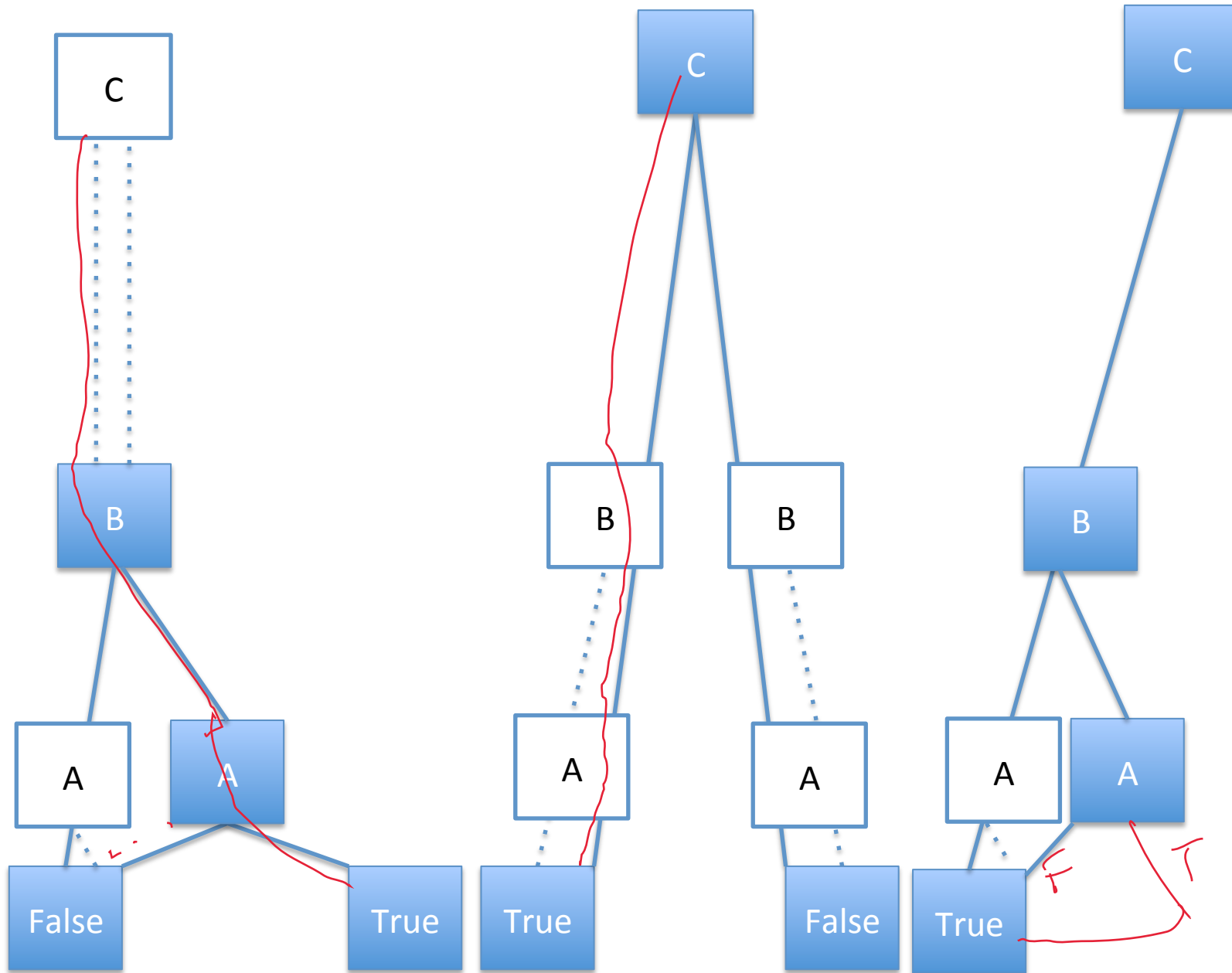
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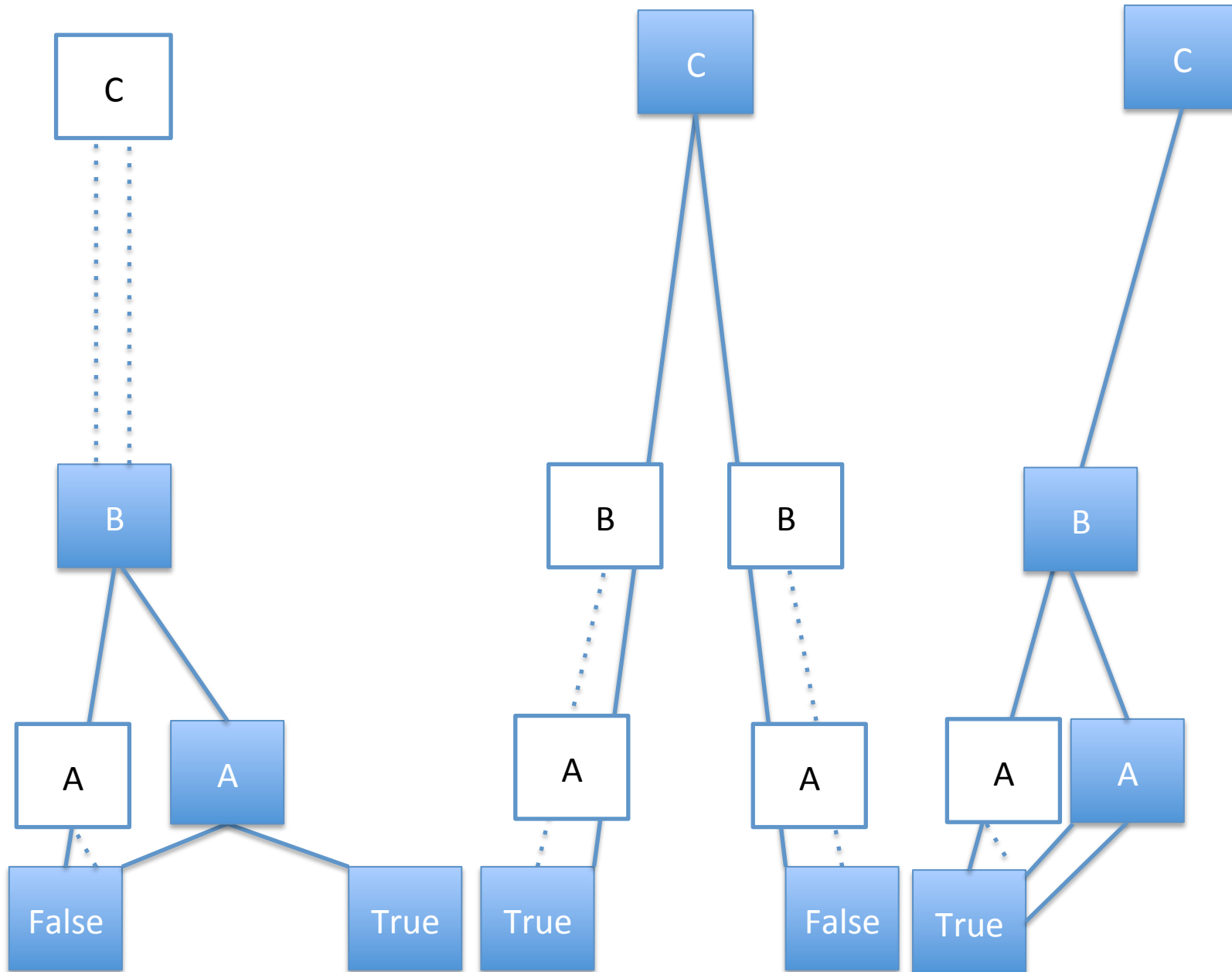
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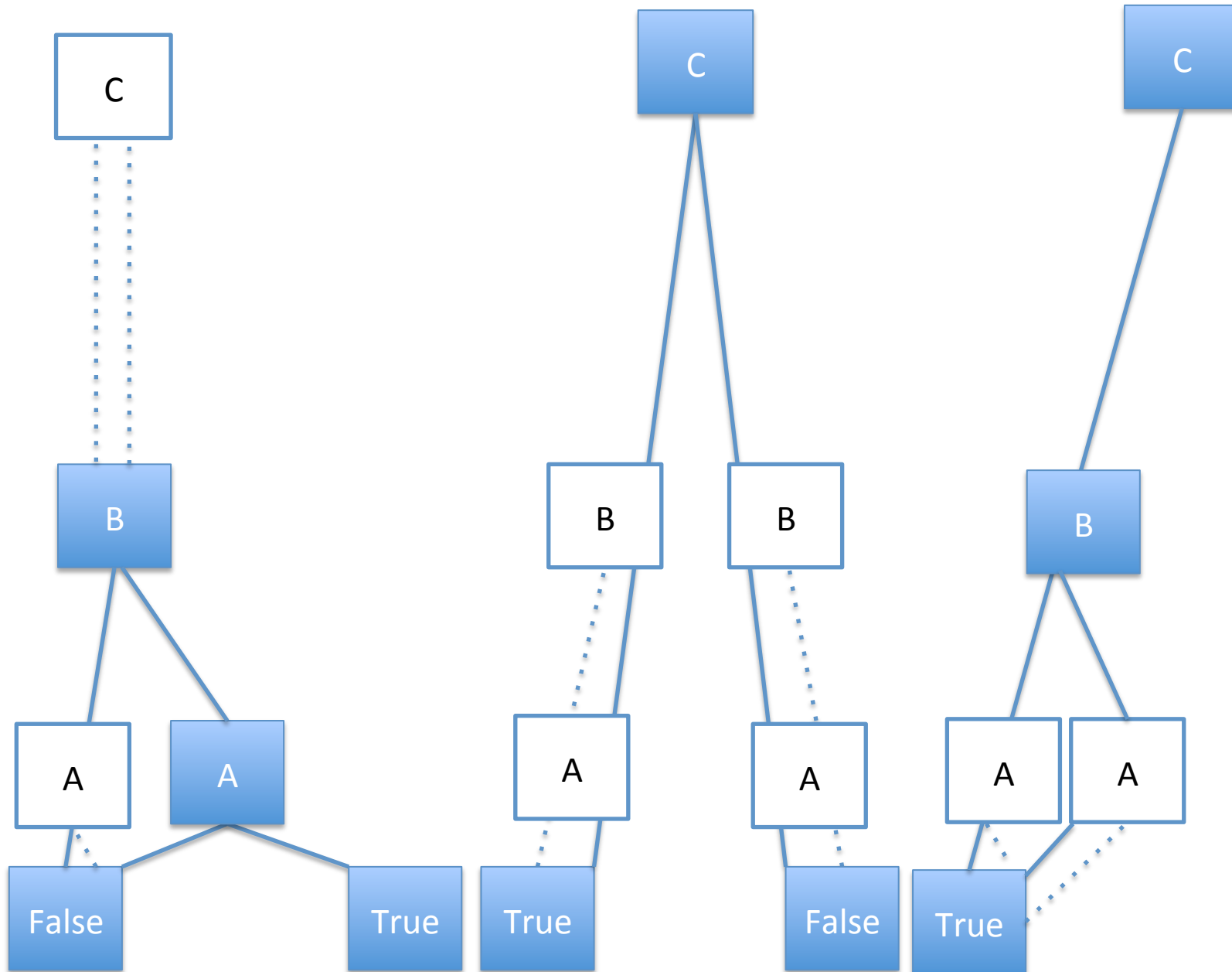
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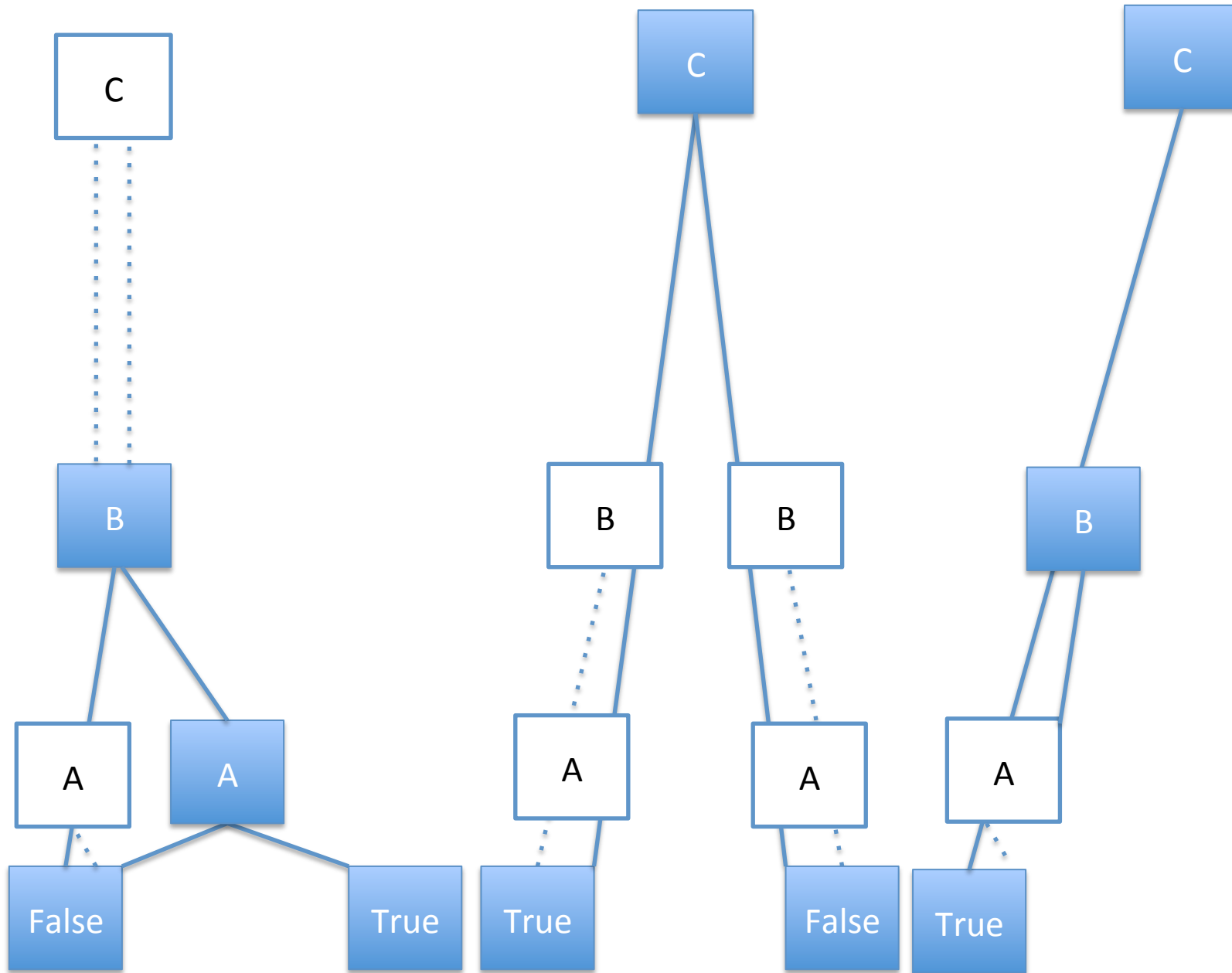
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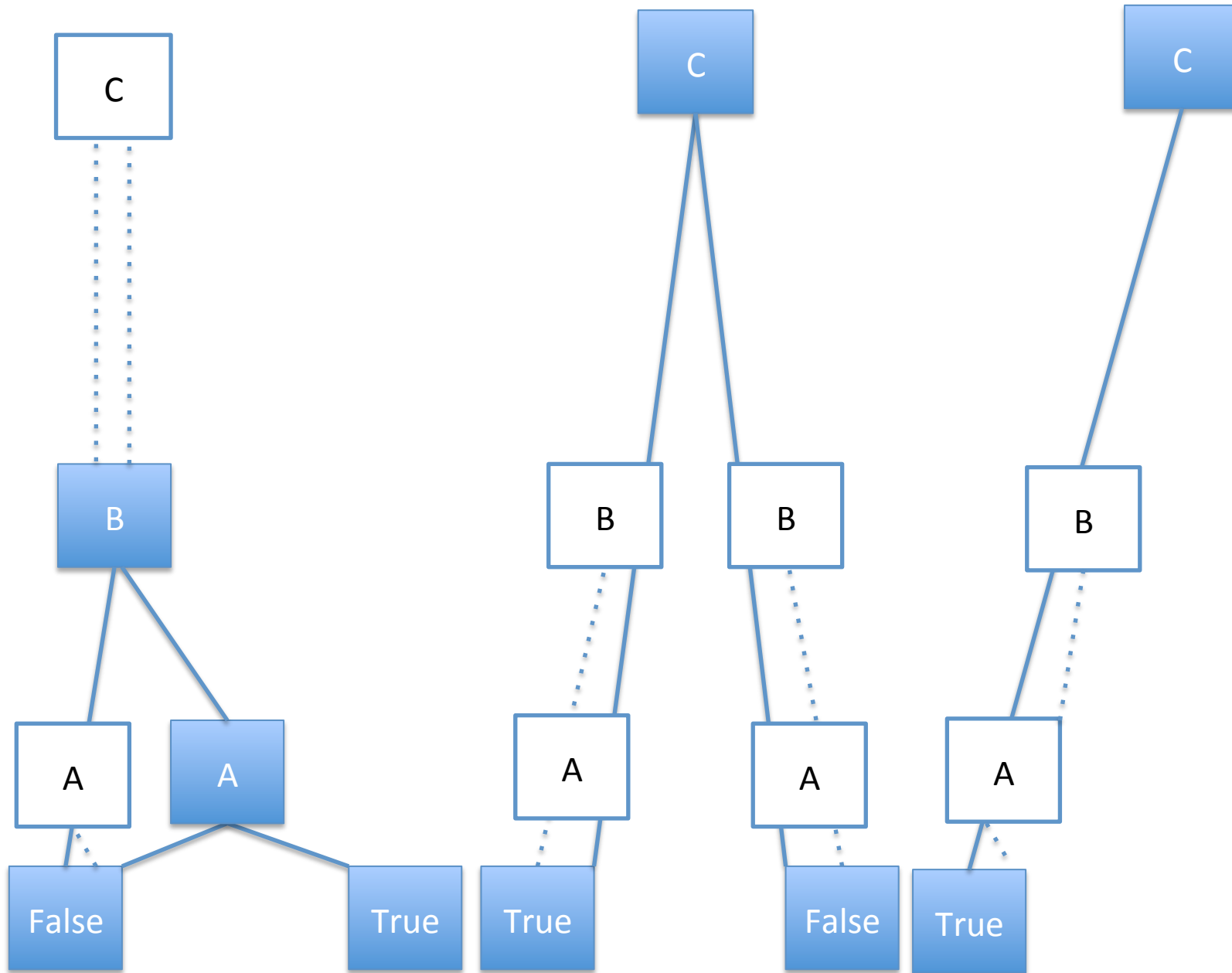
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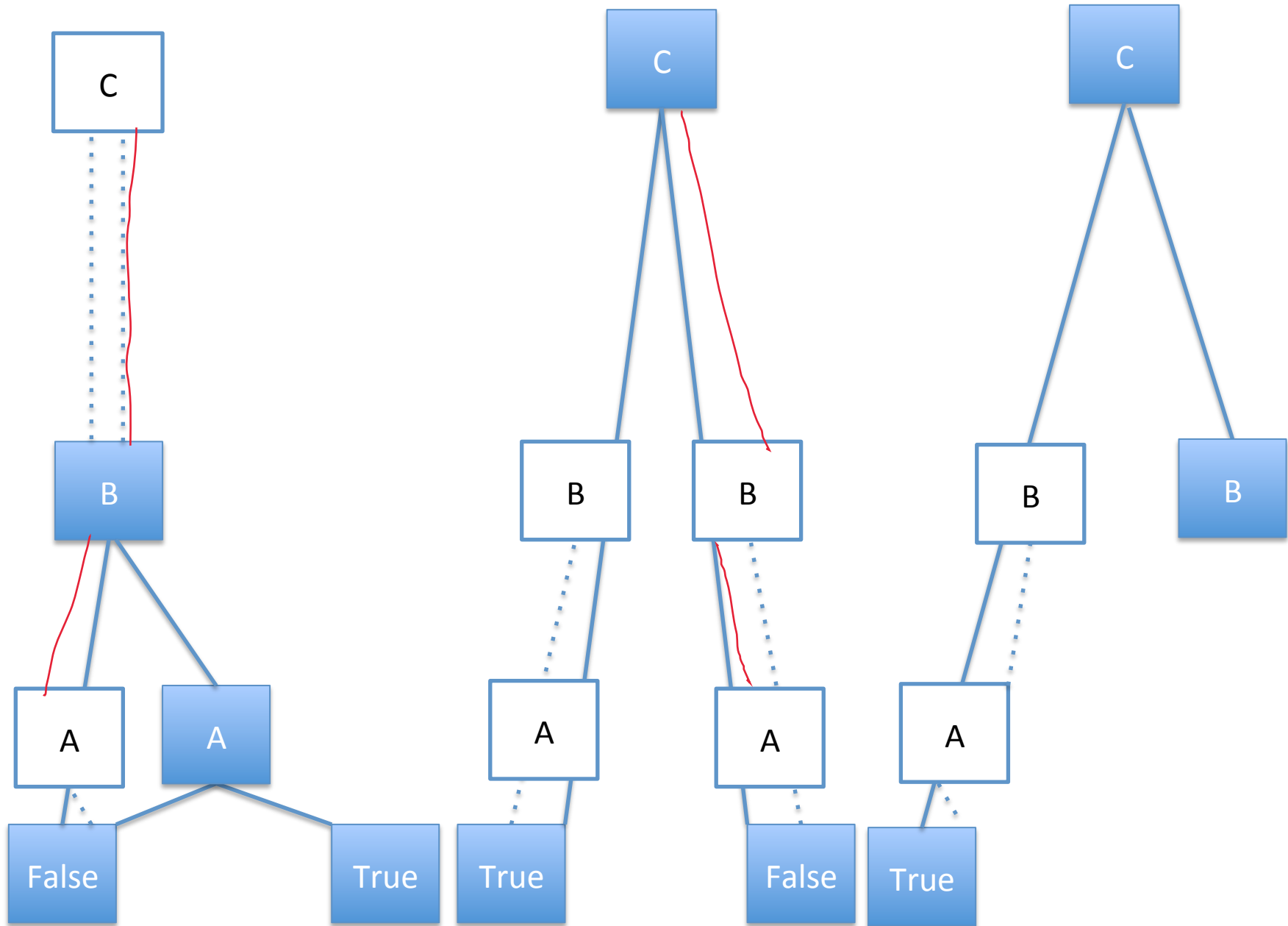
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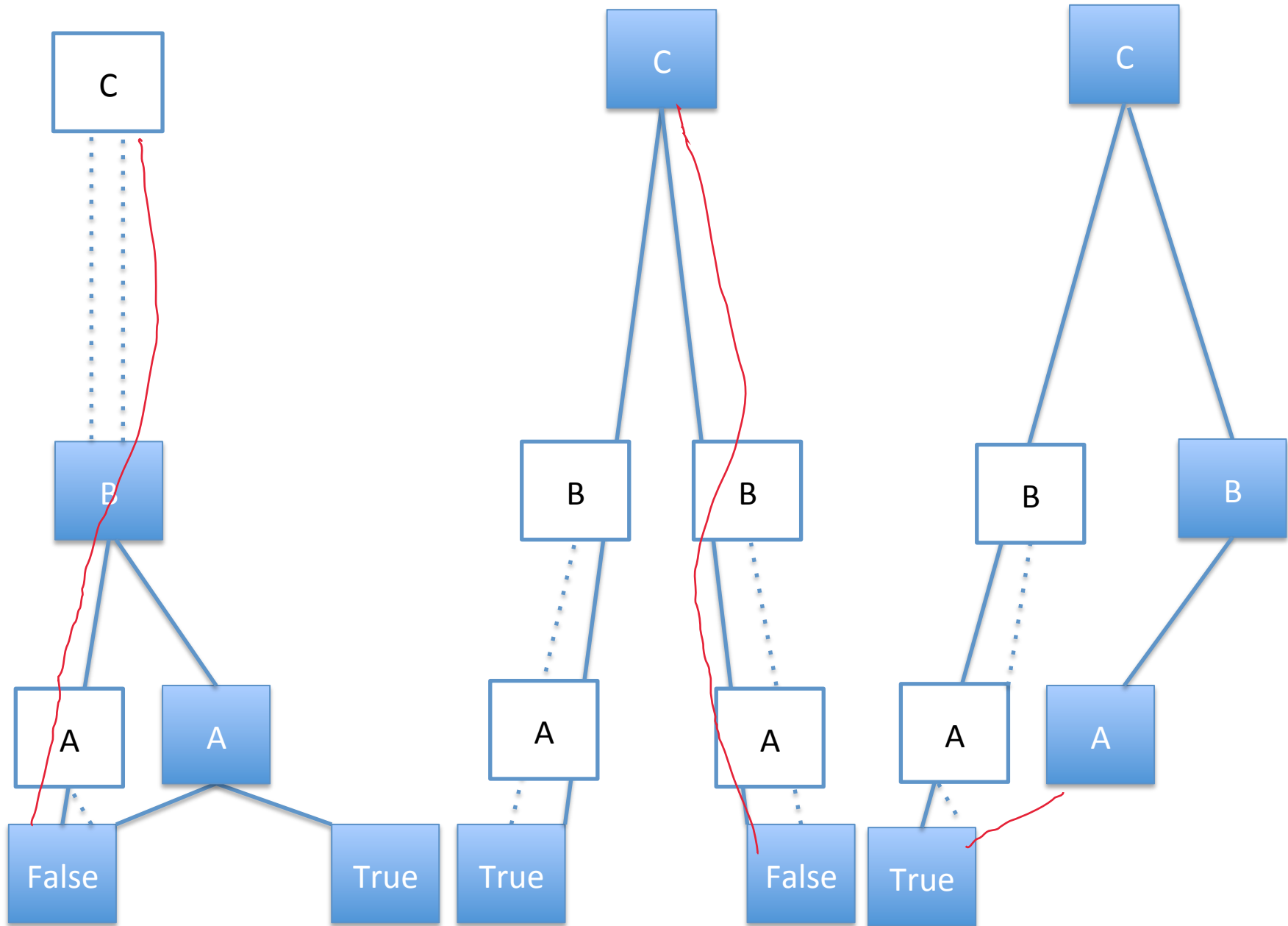
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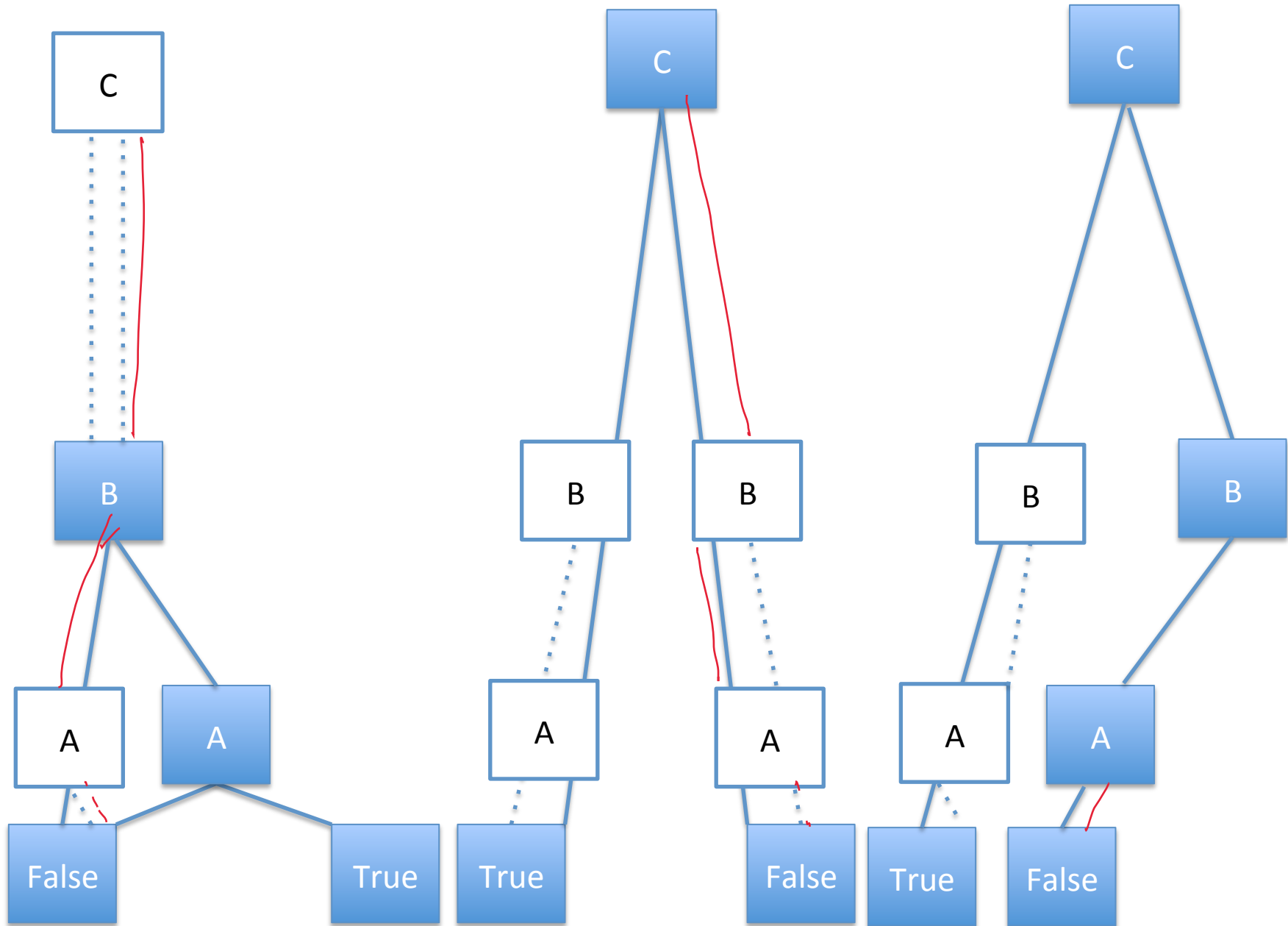
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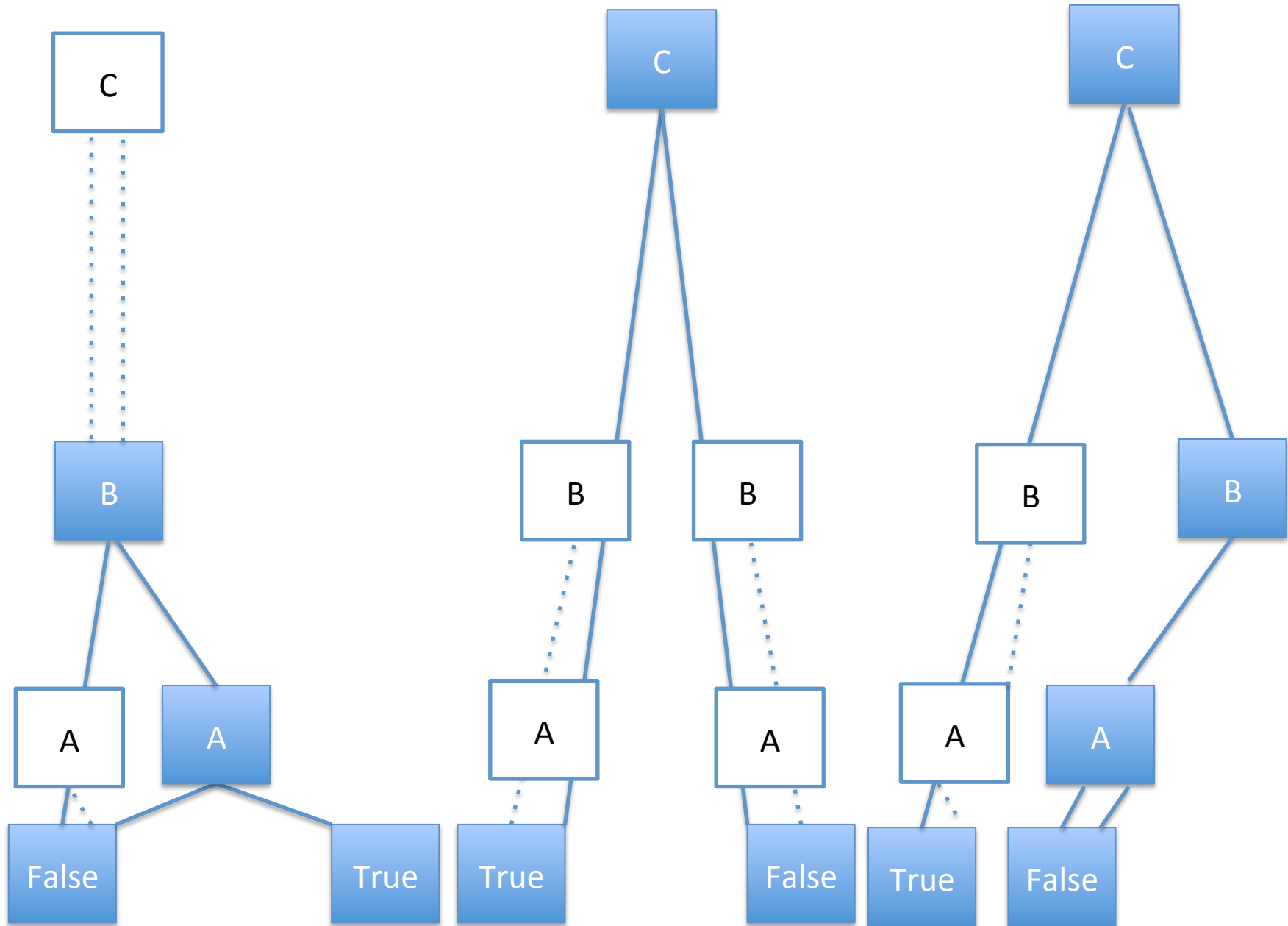


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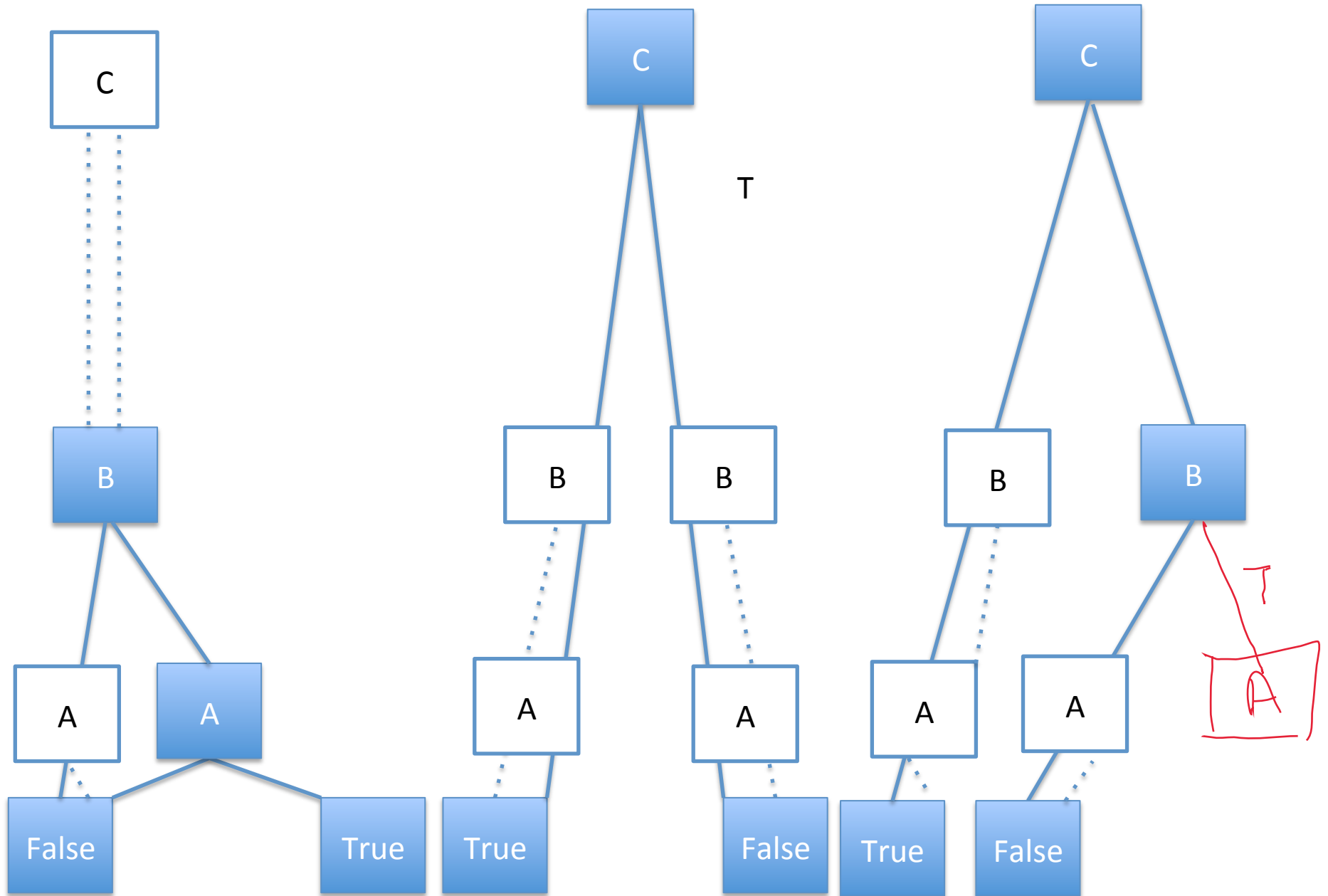


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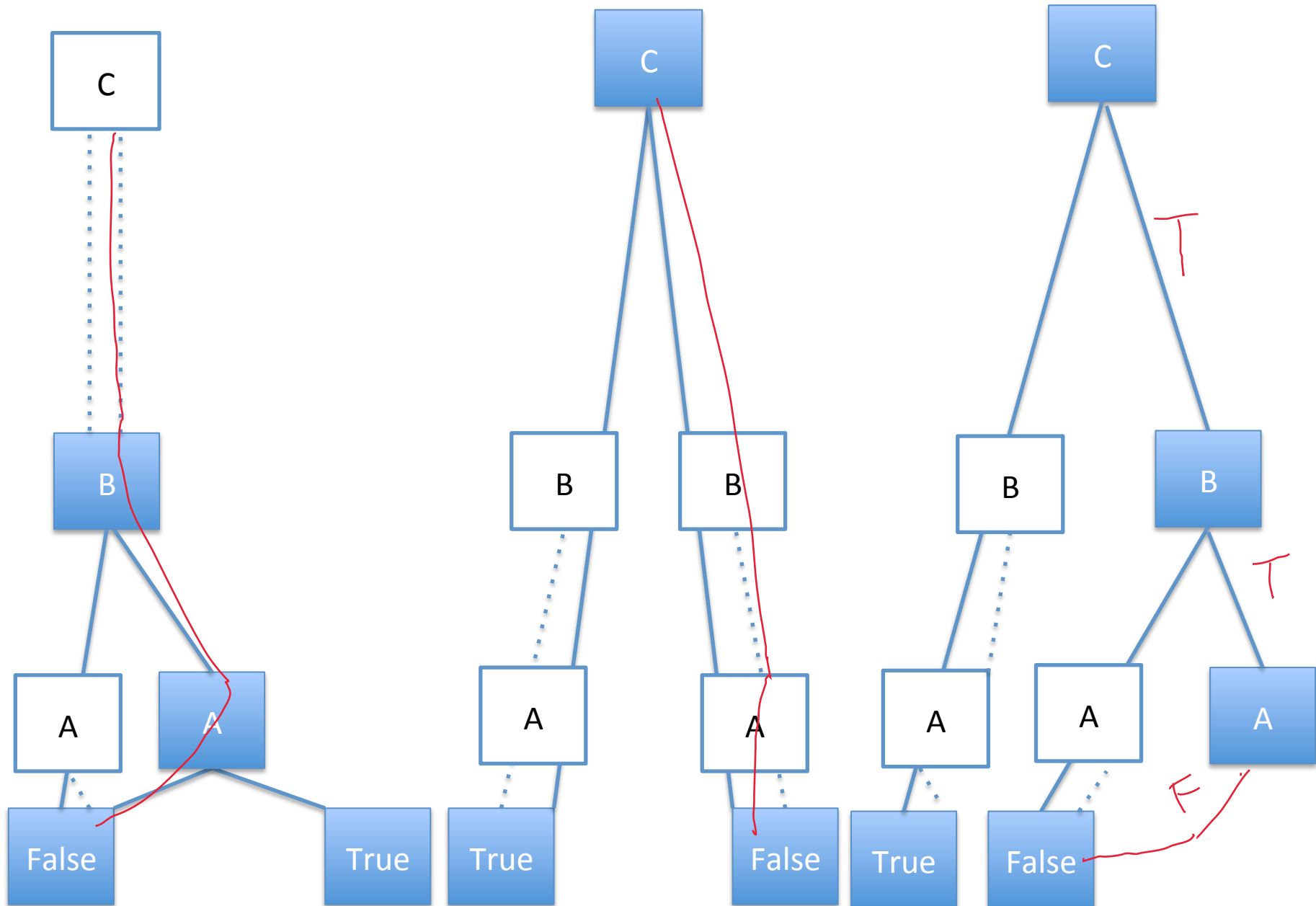
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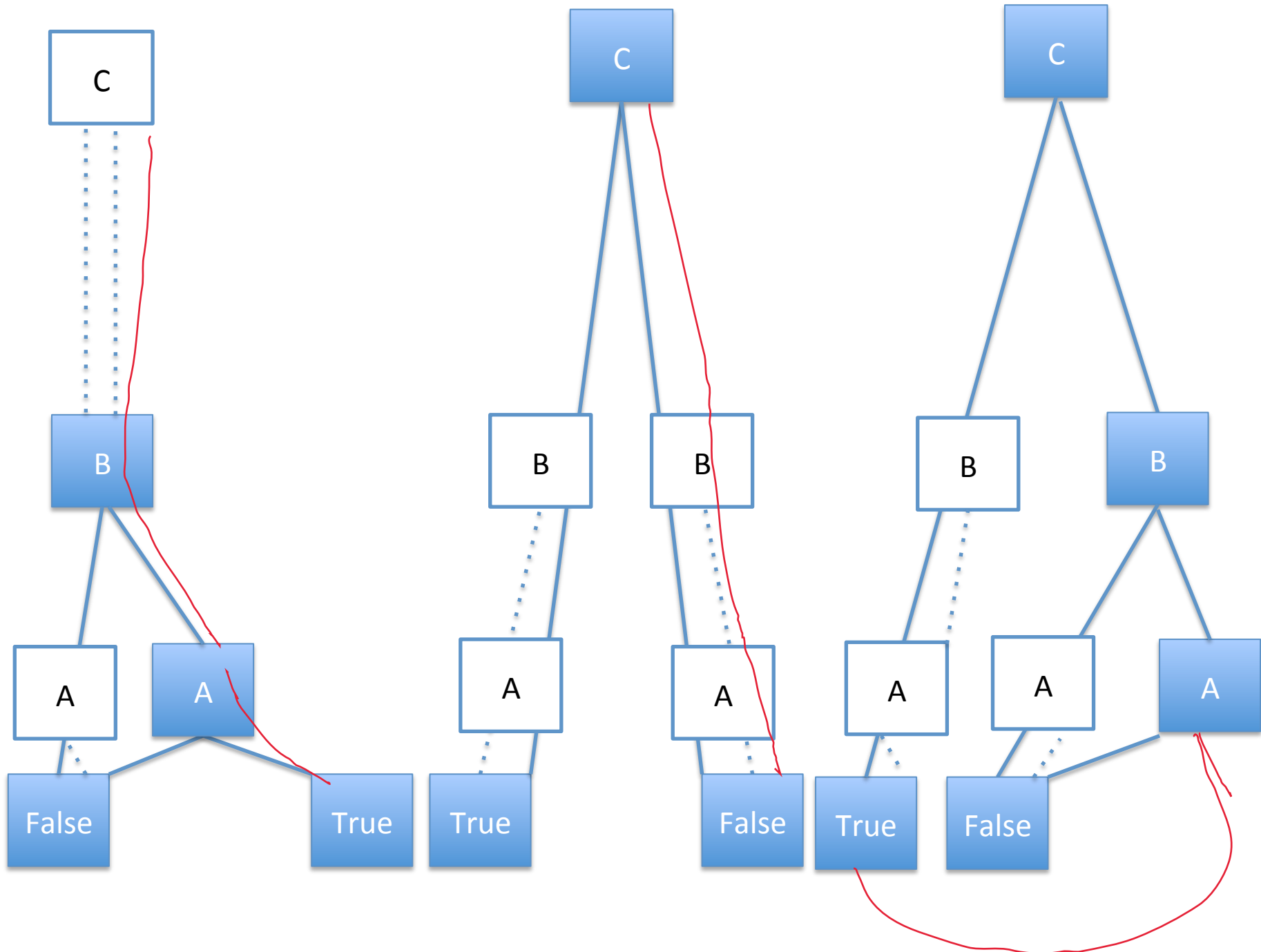
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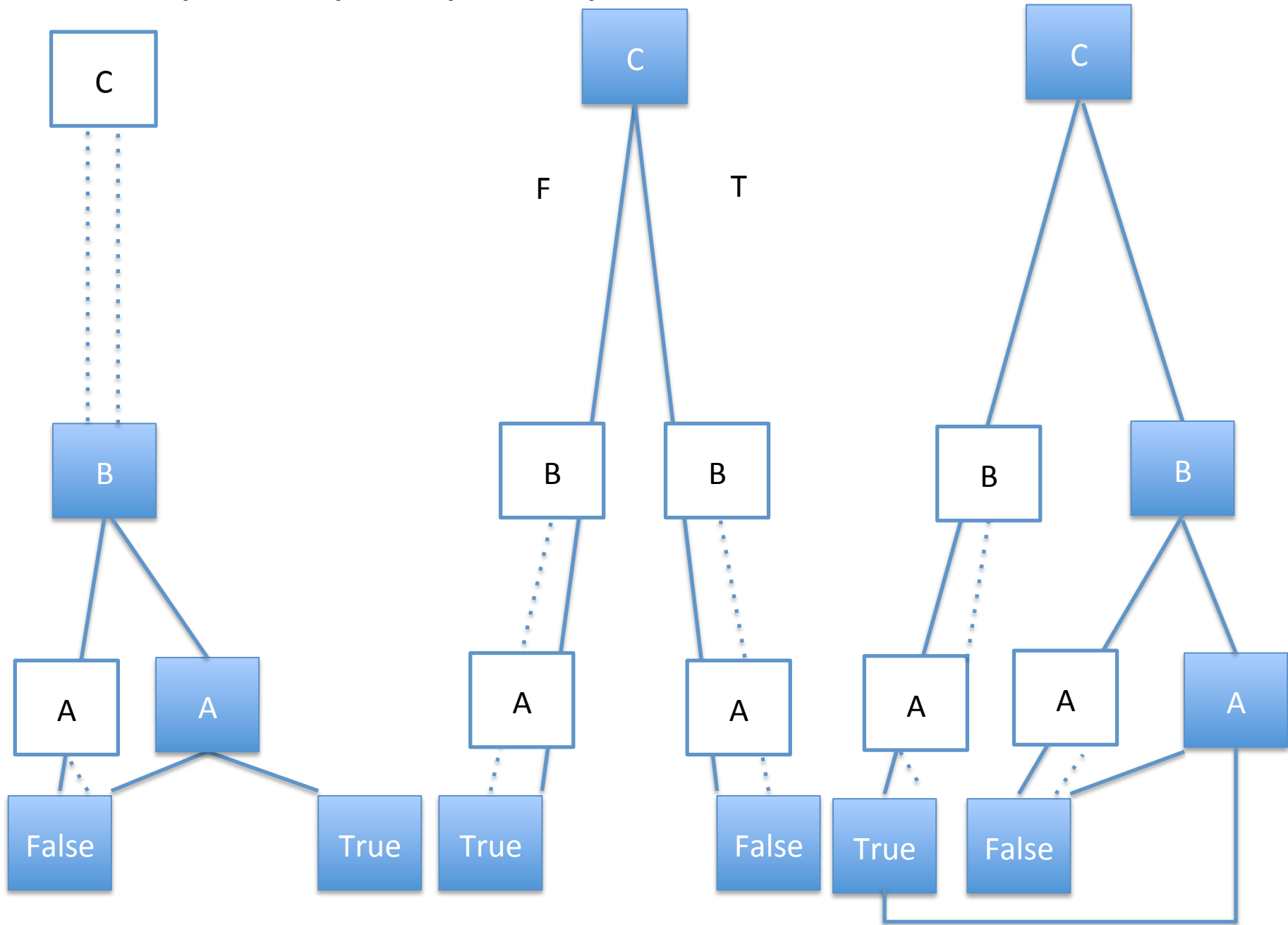
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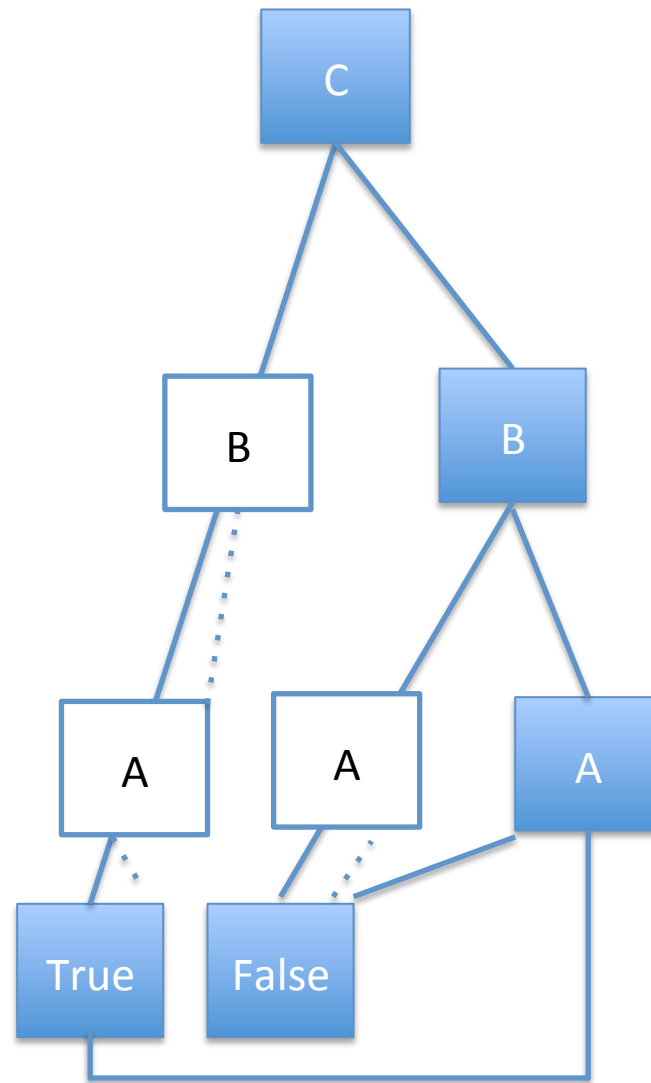
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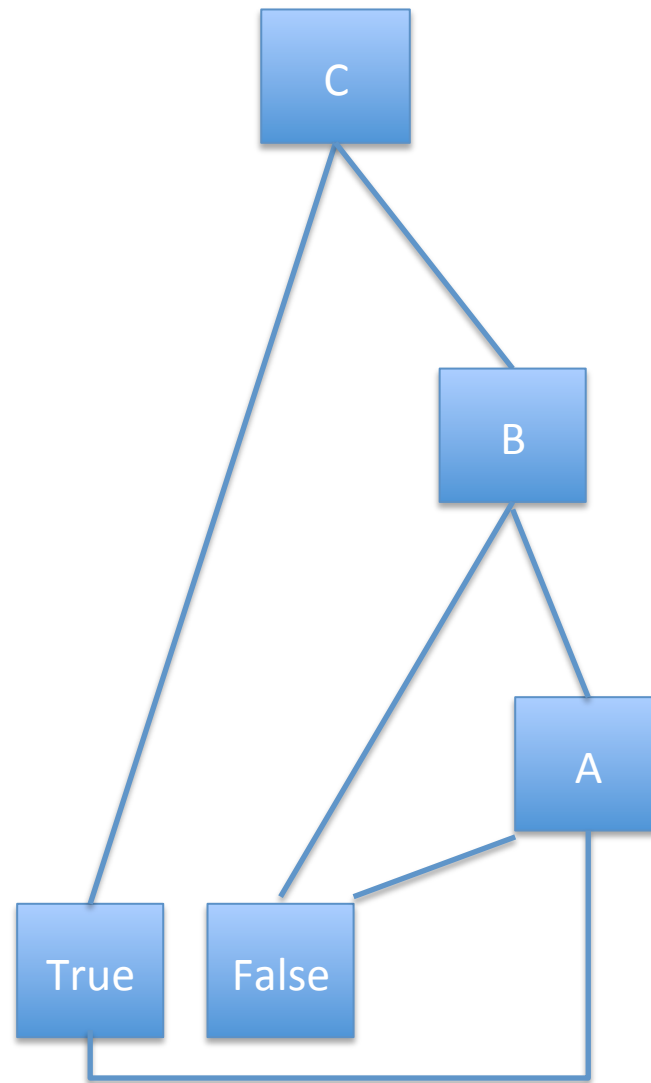
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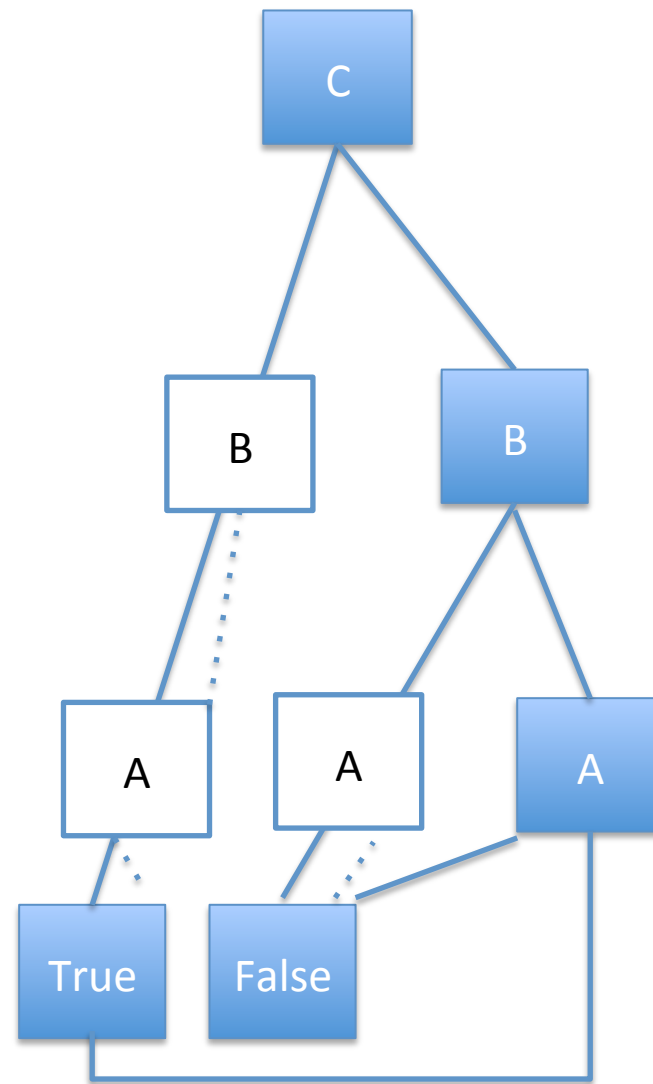
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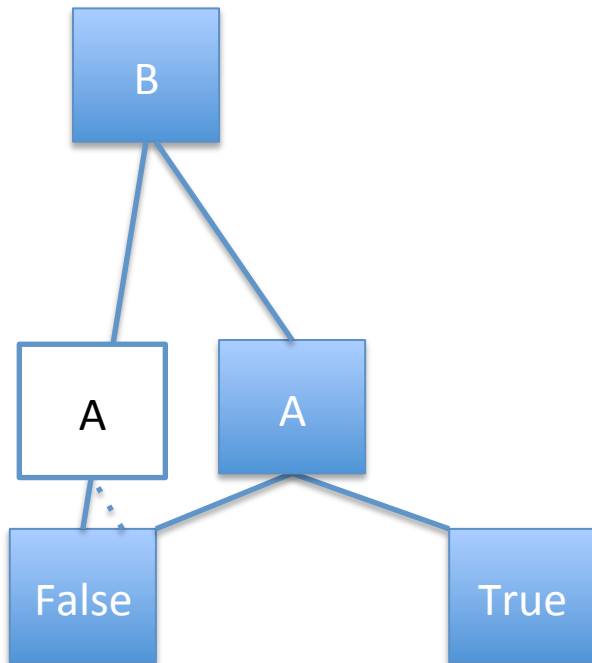
Example

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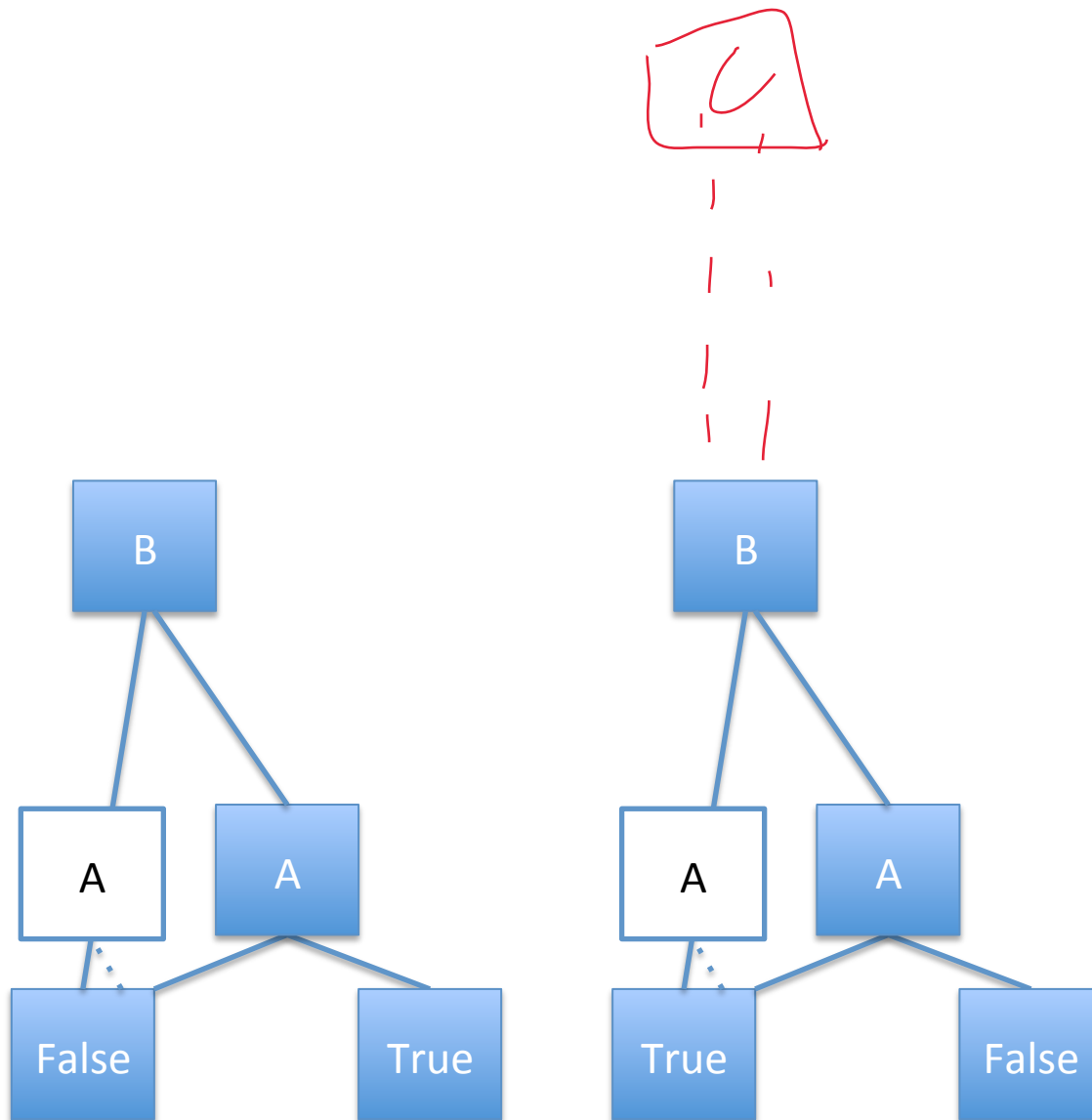
$((A \wedge B) \vee (\text{not } C))$ Variables: $C > B > A$



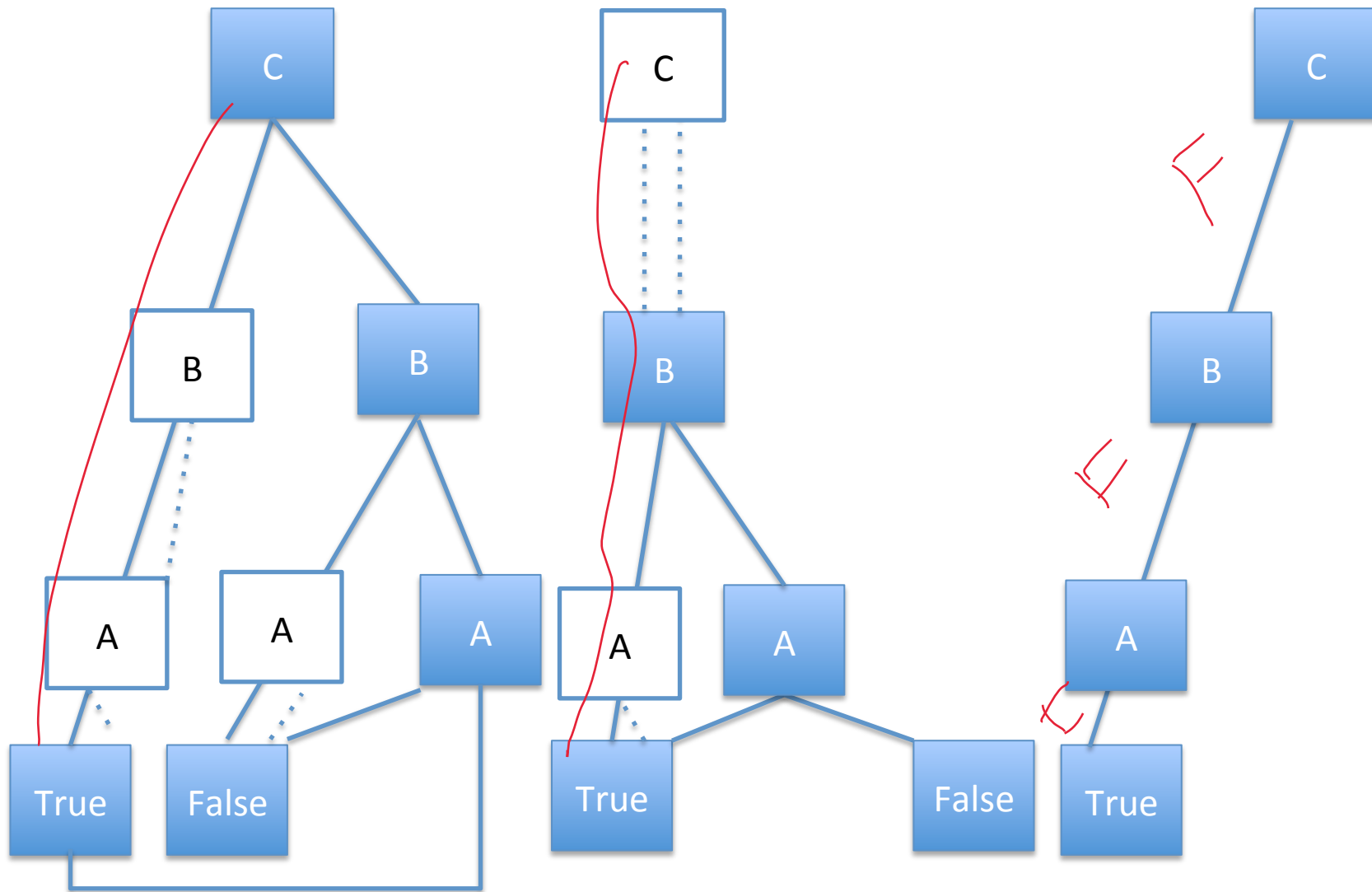
$(A \wedge B)$ Variables: $C > B > A$



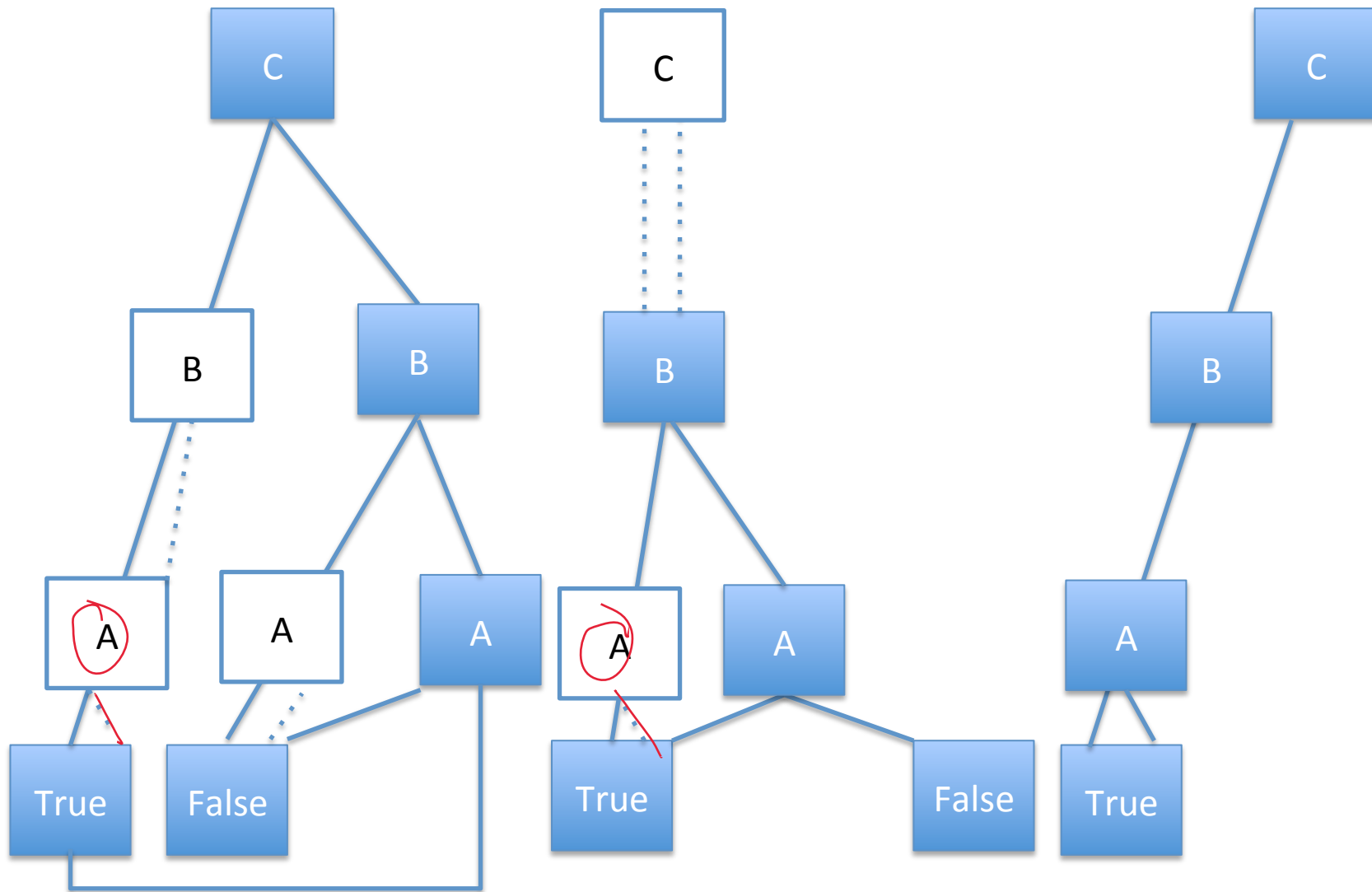
not ($A \wedge B$) Variables: $C > B > A$



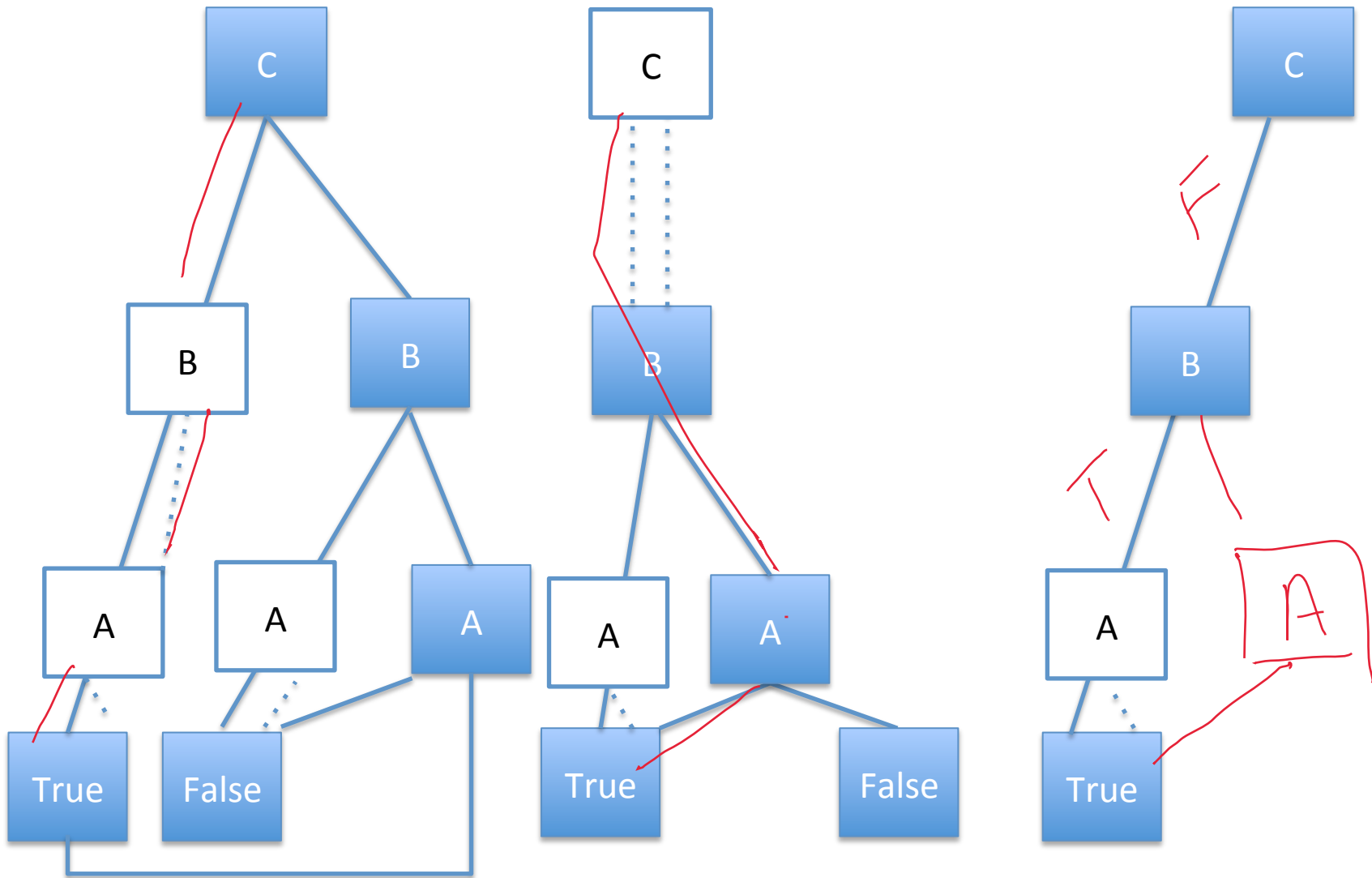
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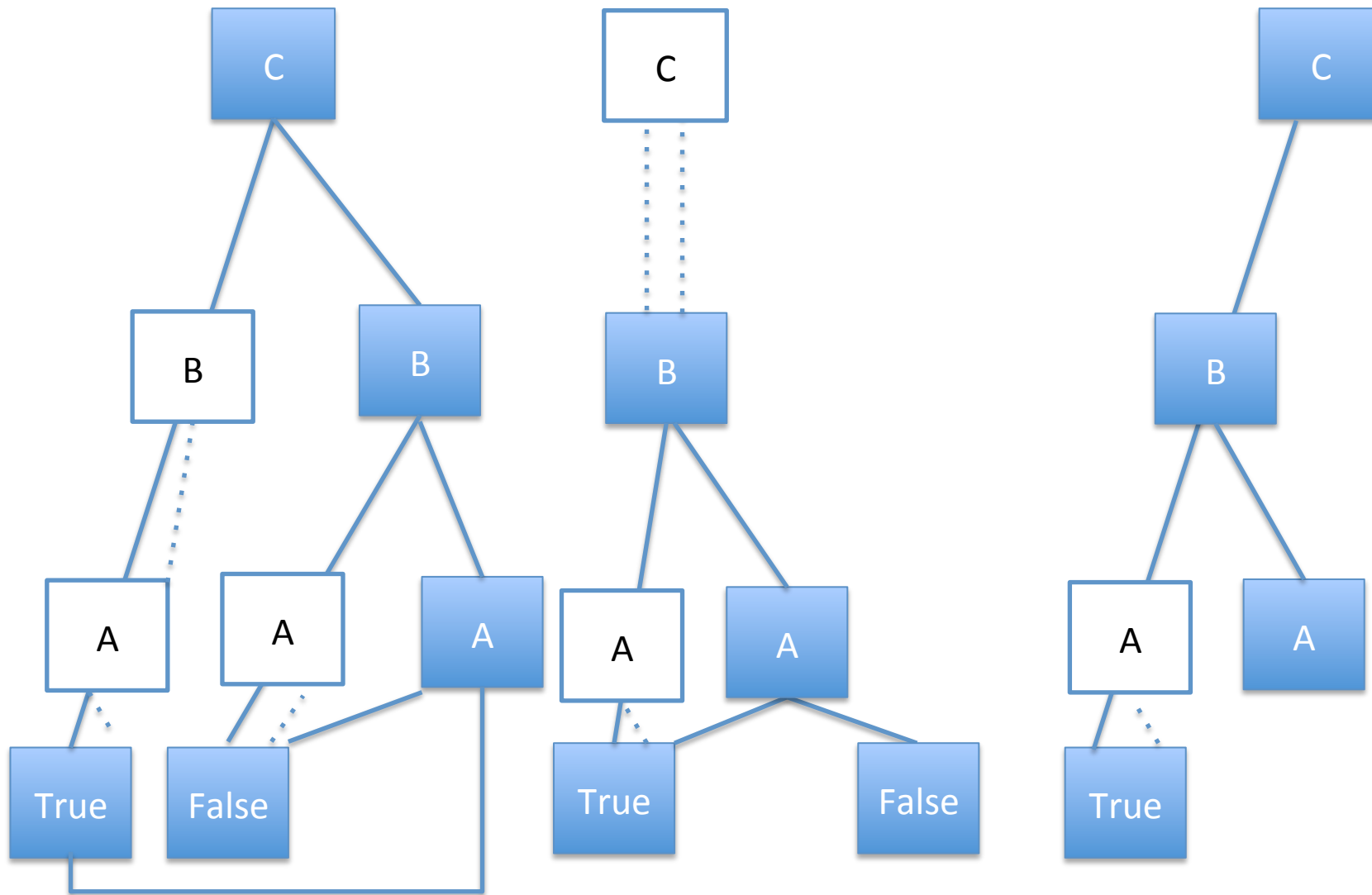
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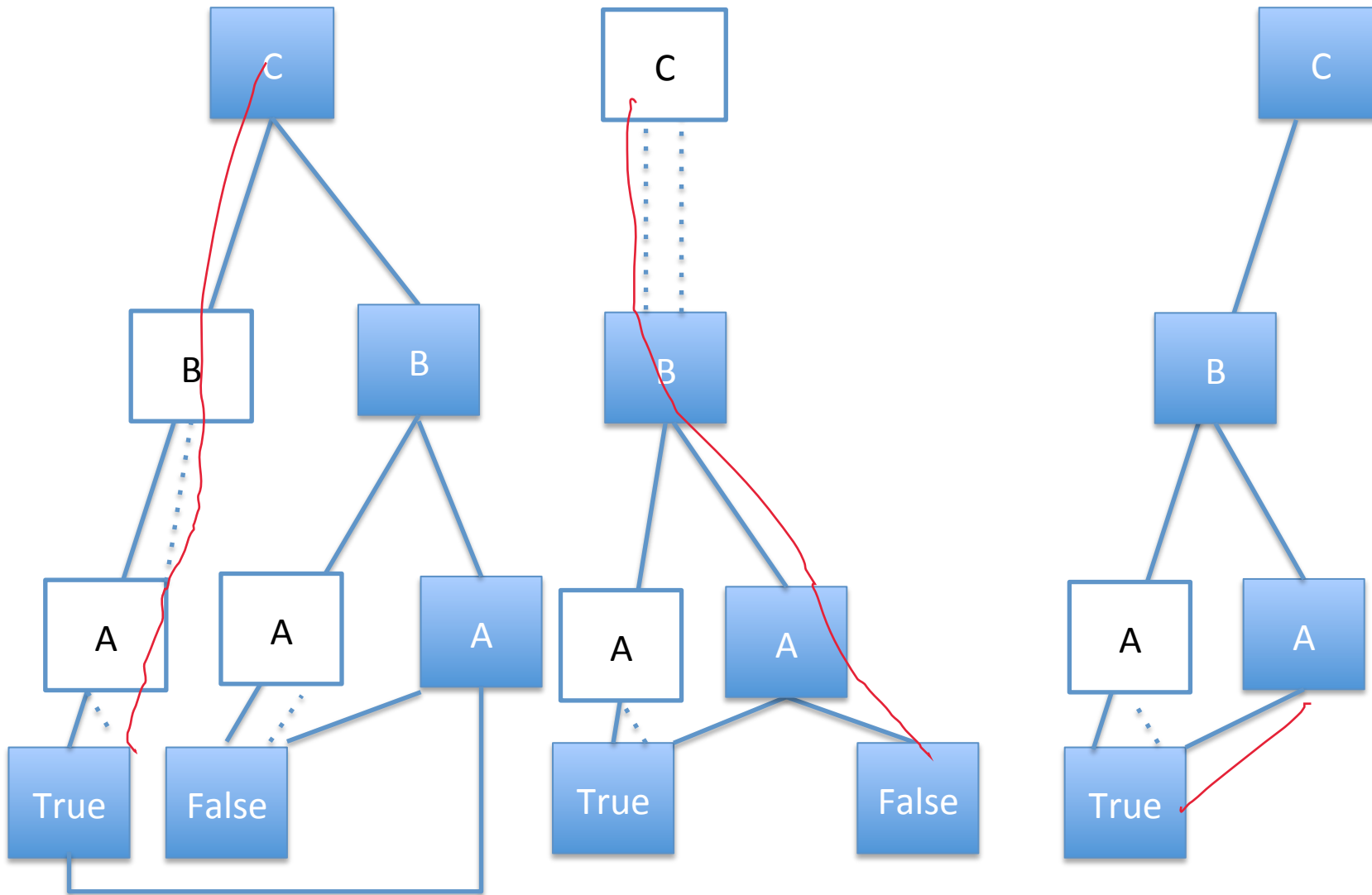
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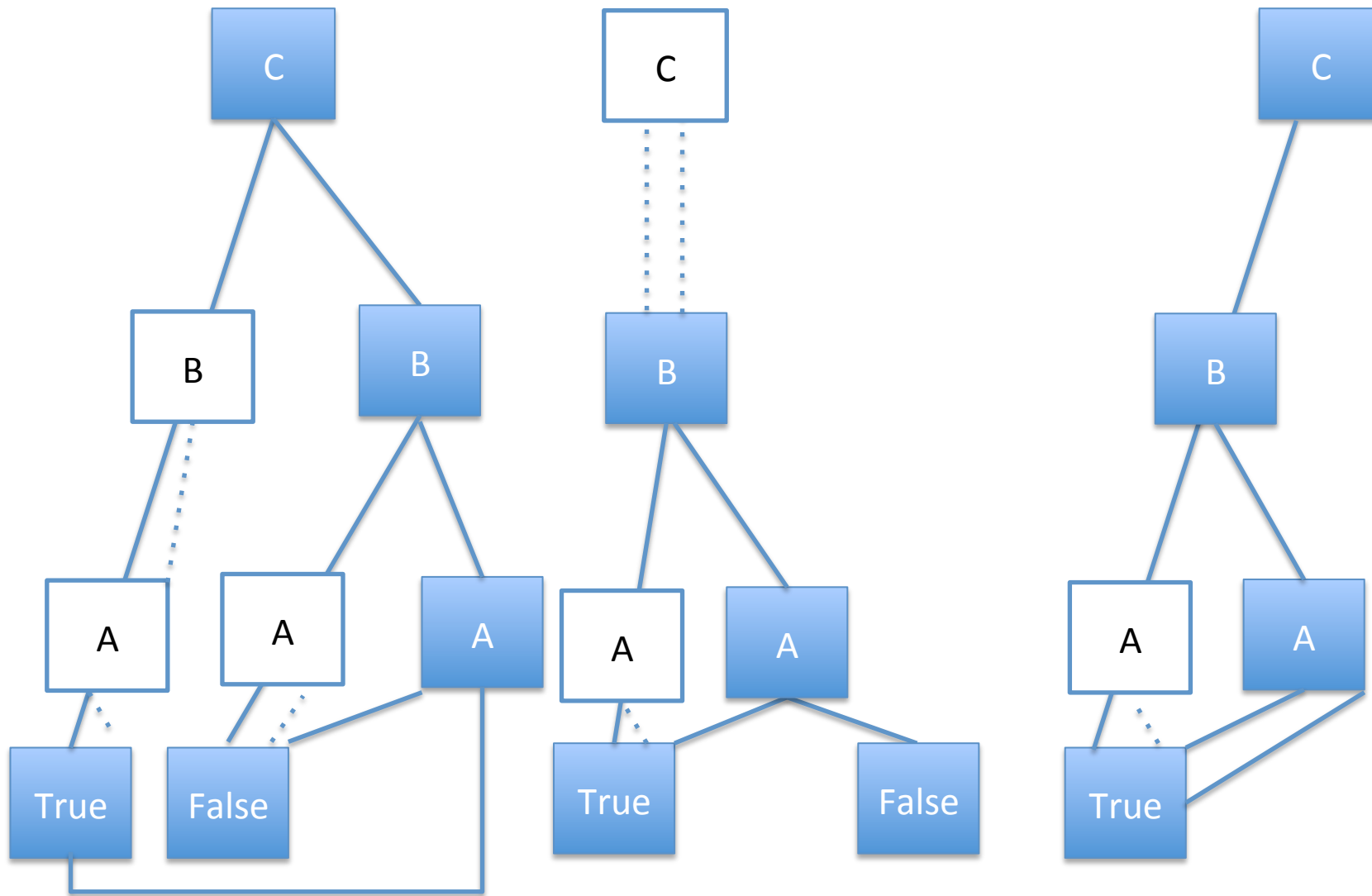
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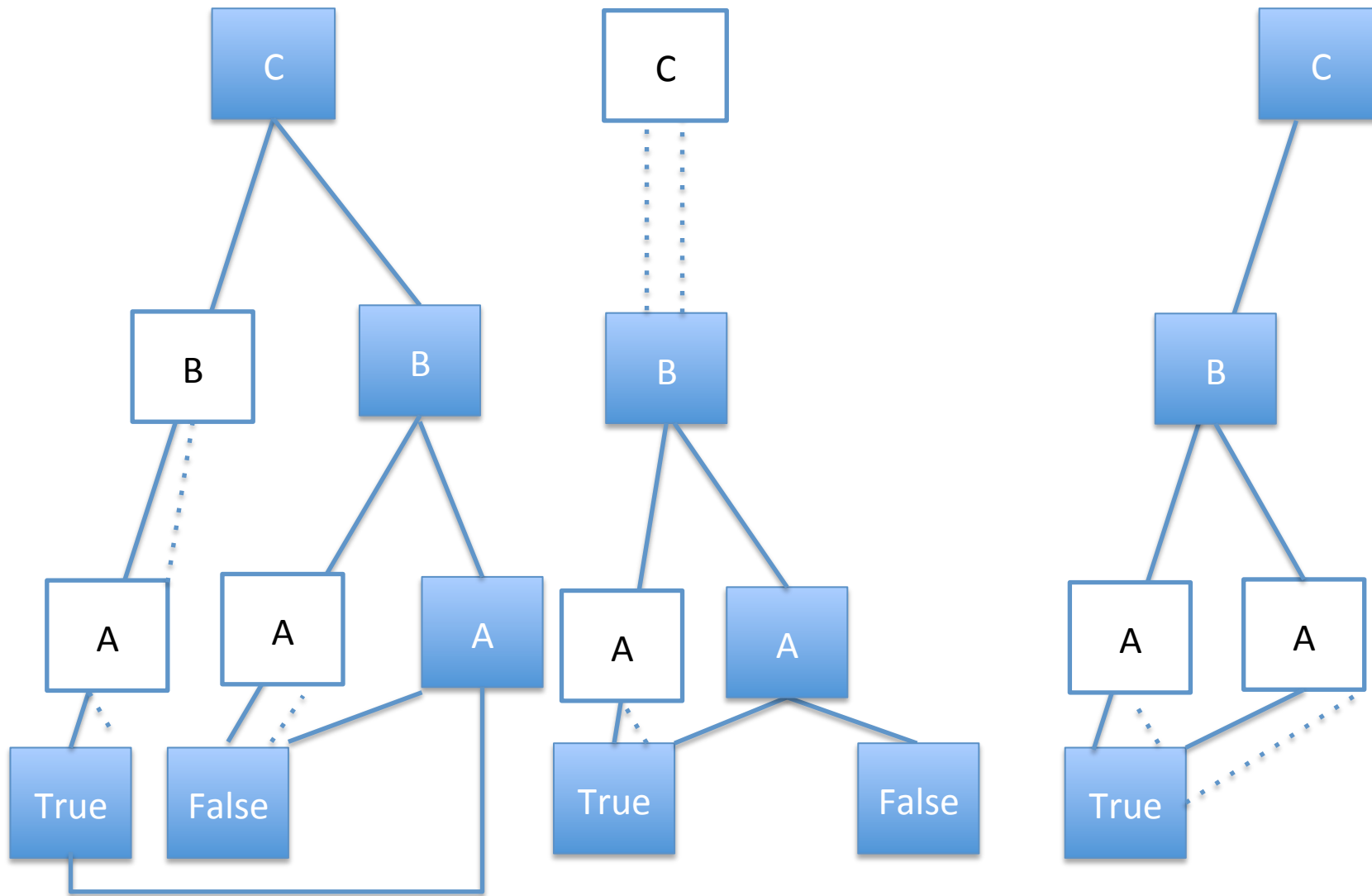
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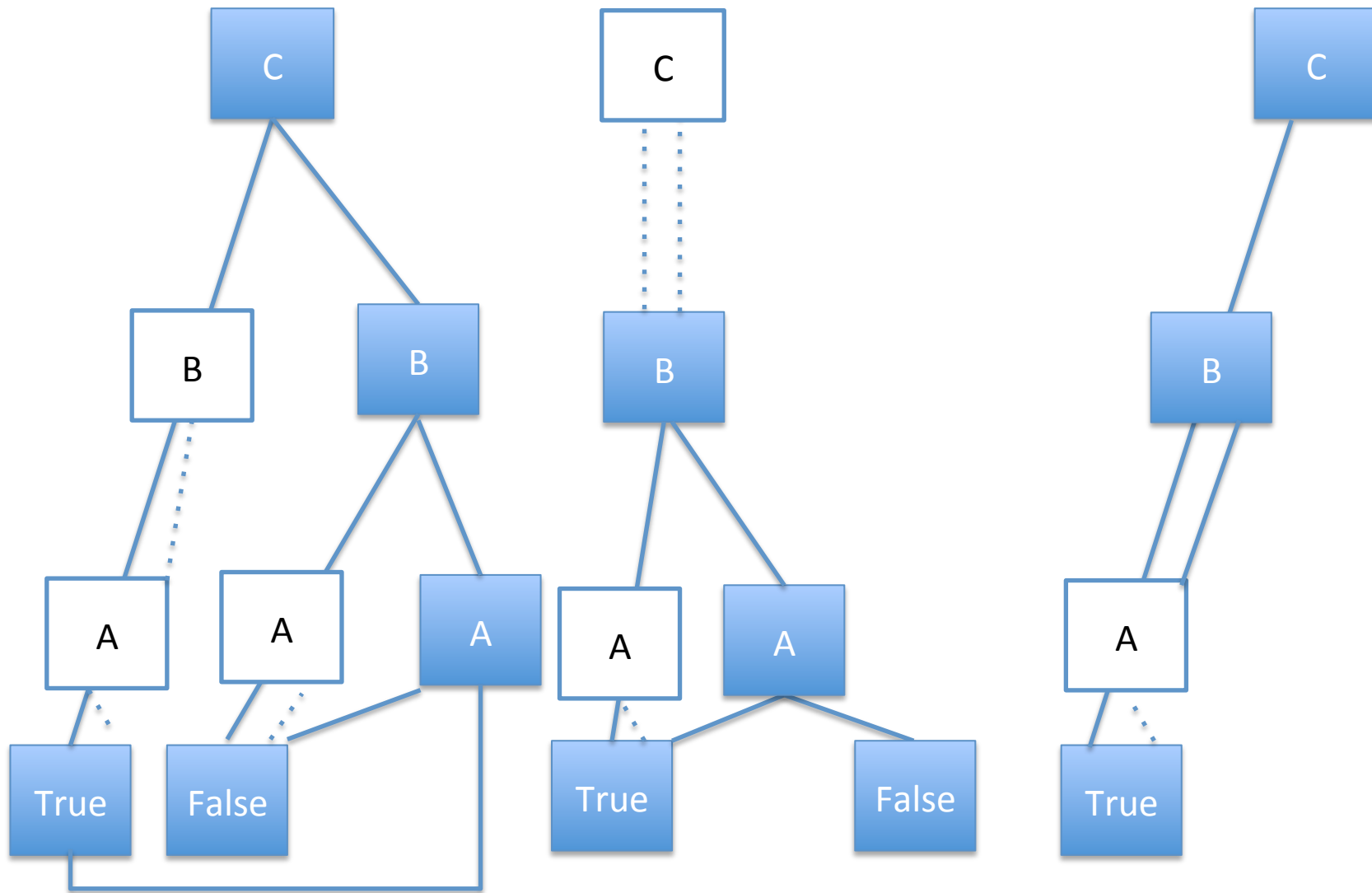
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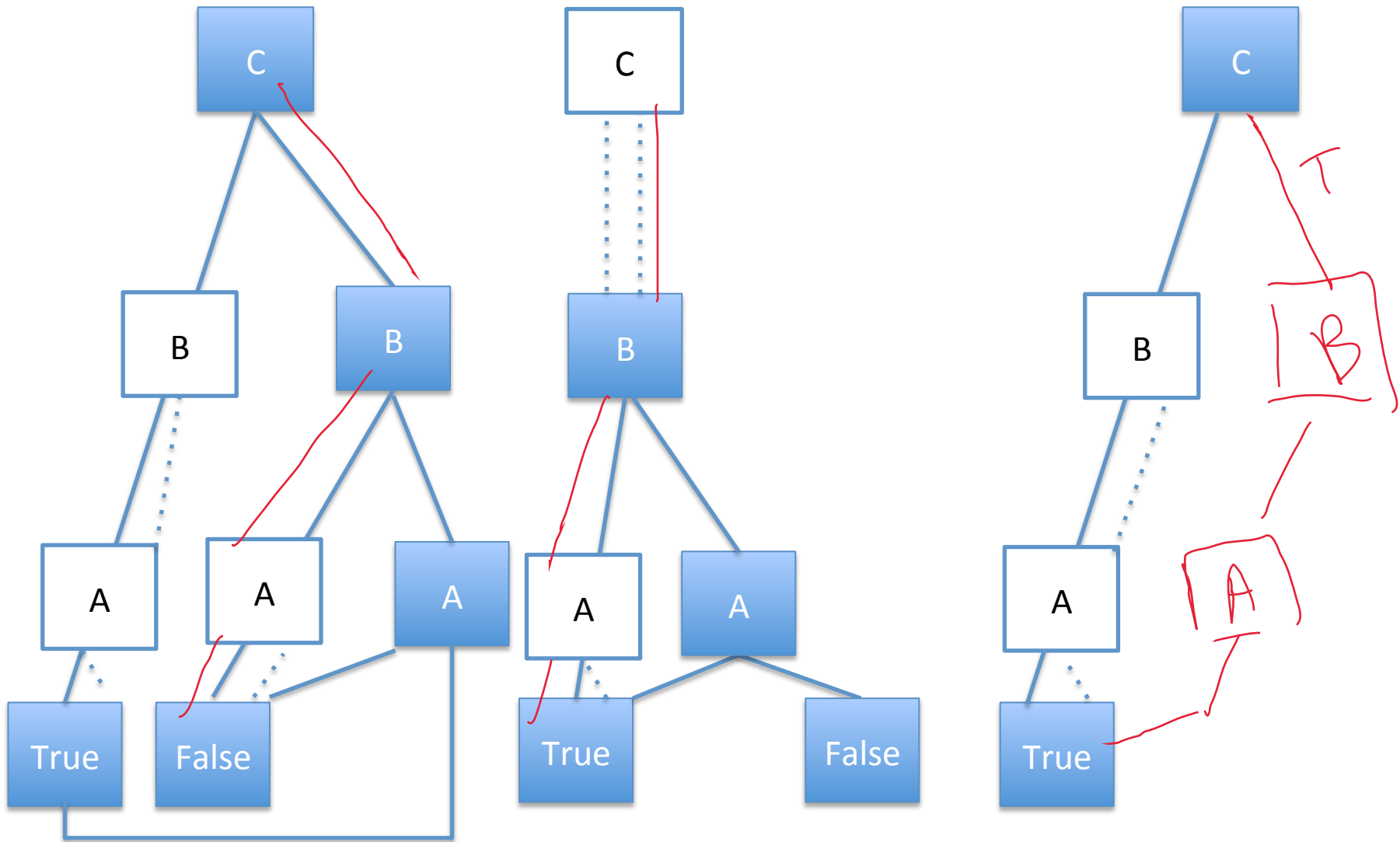
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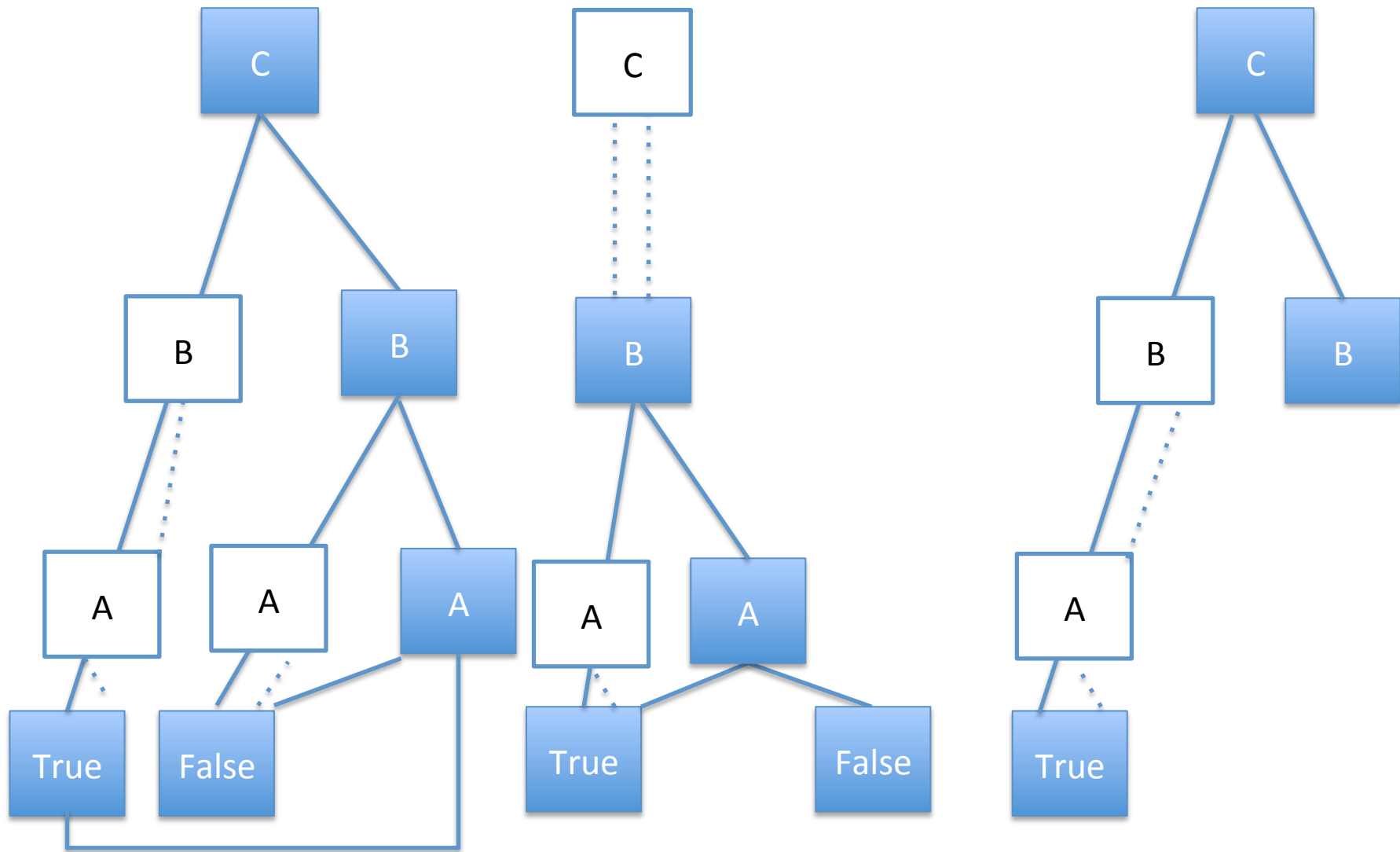
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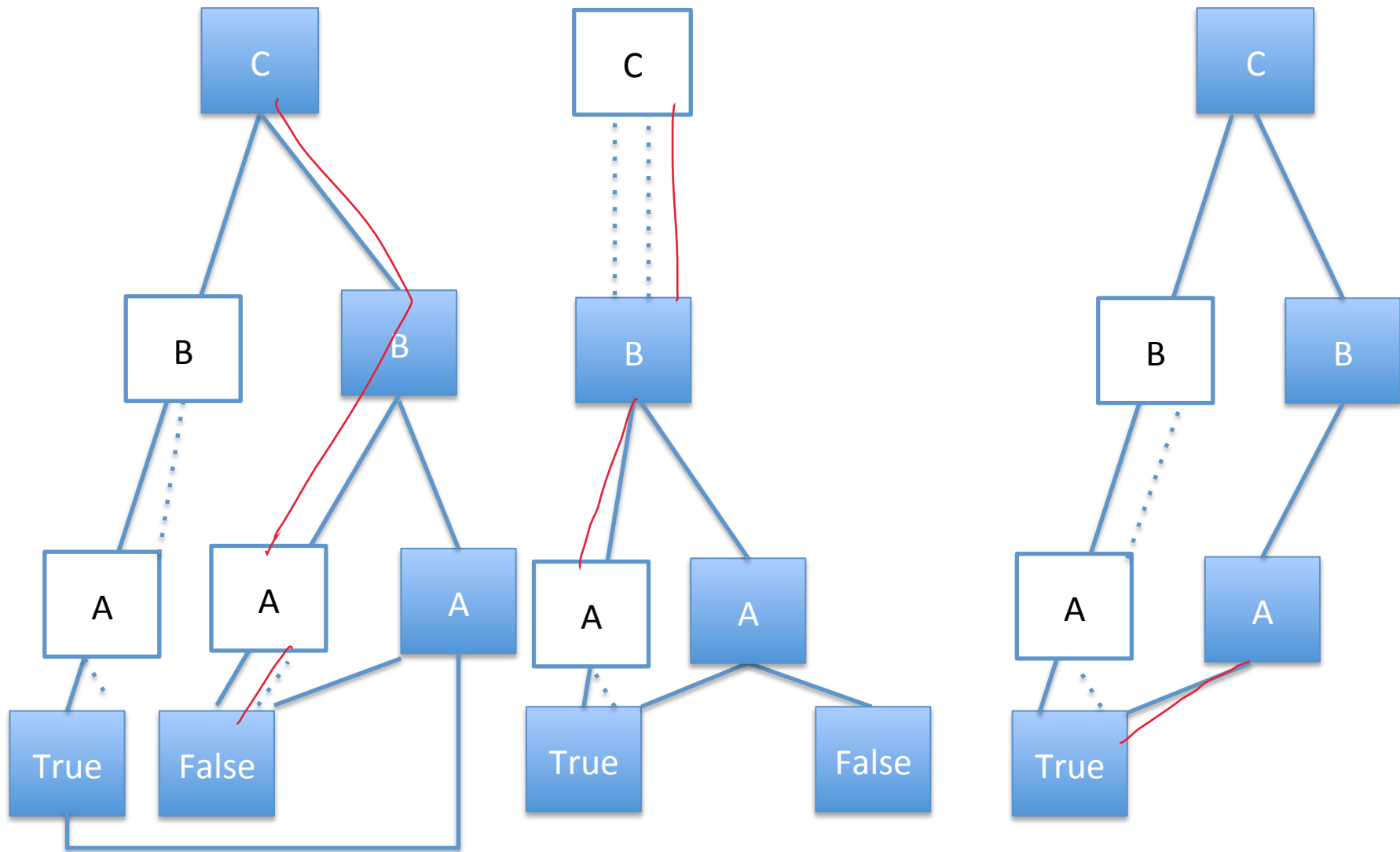
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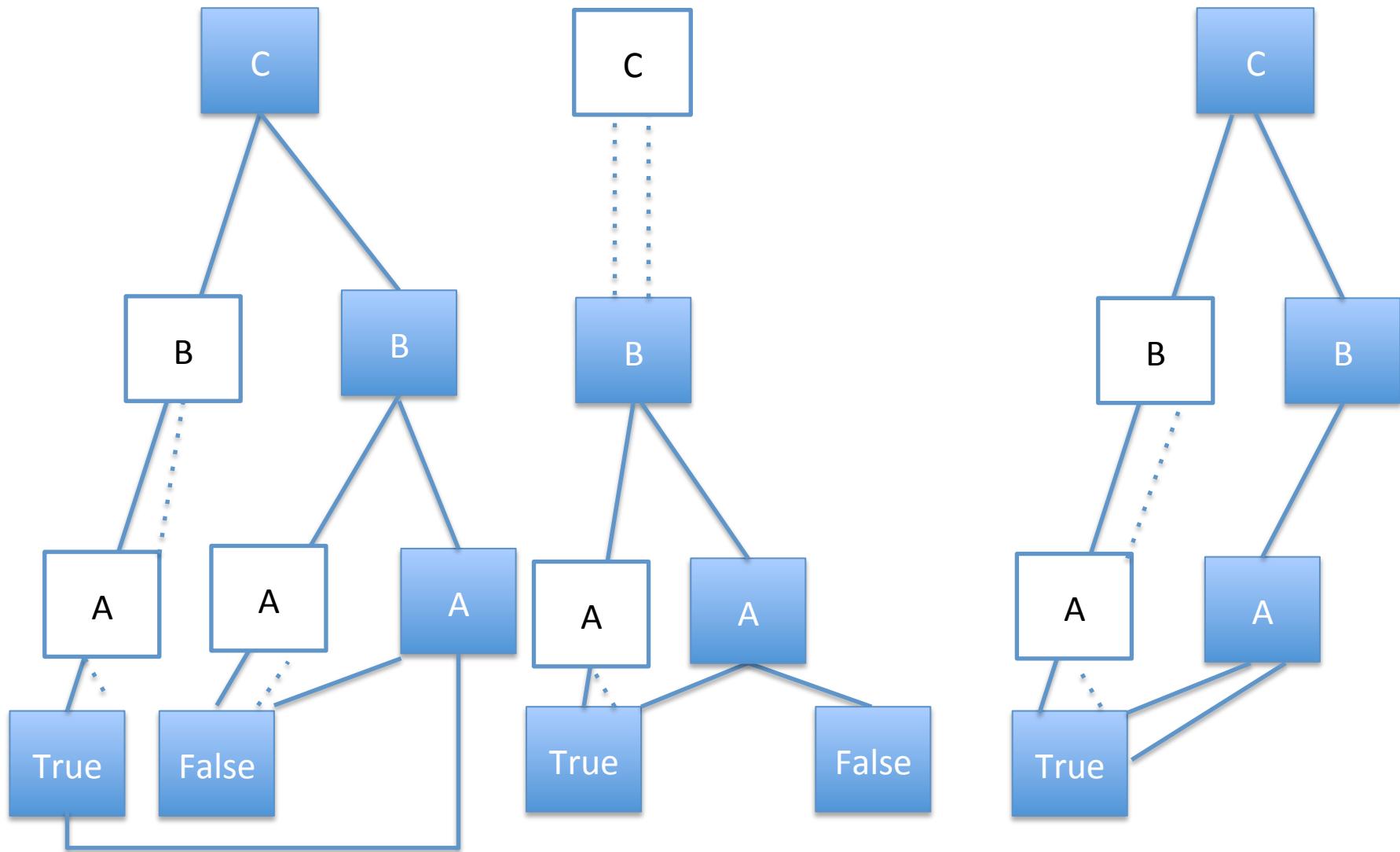
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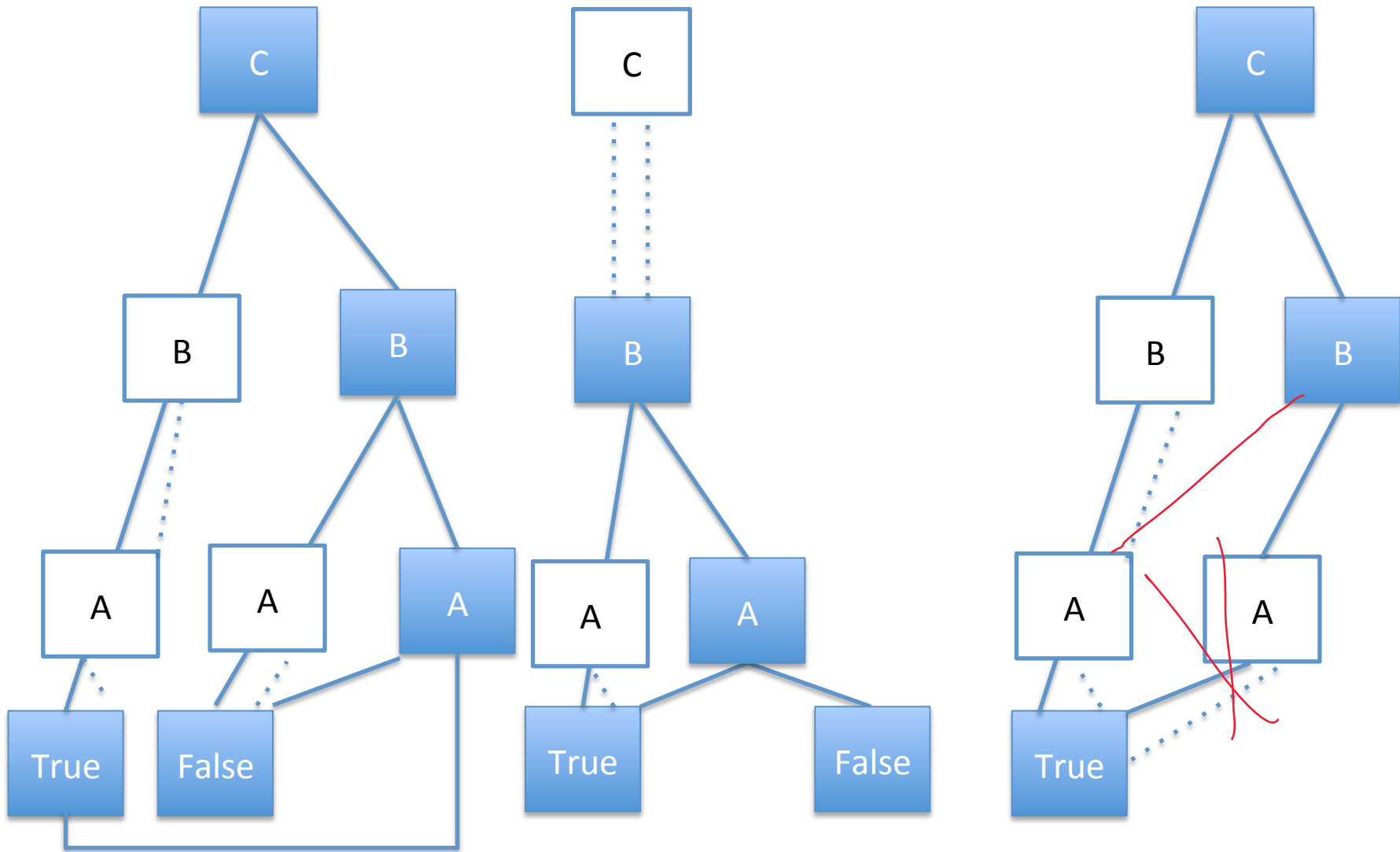
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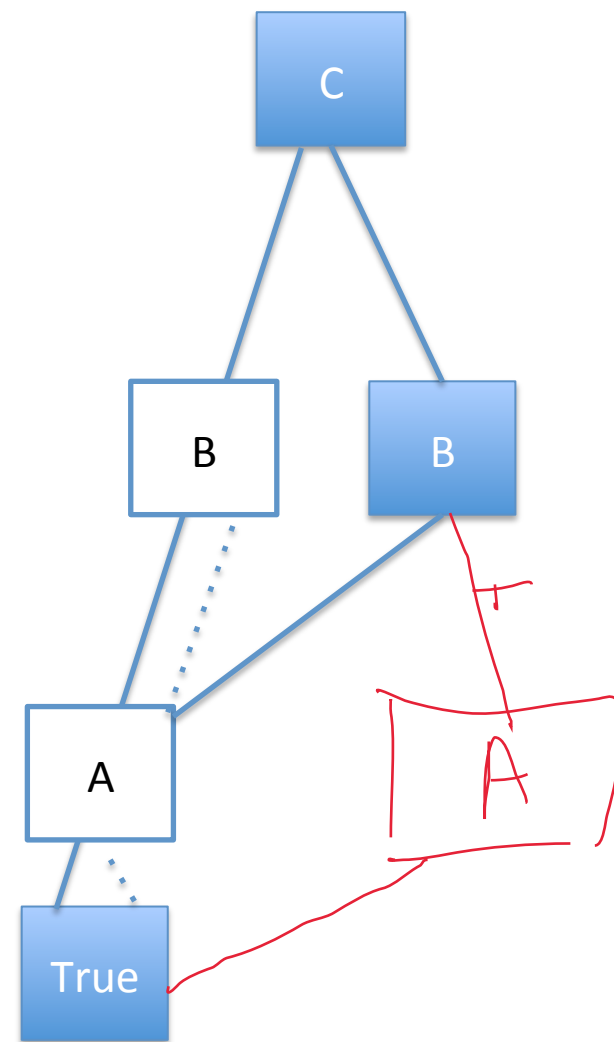
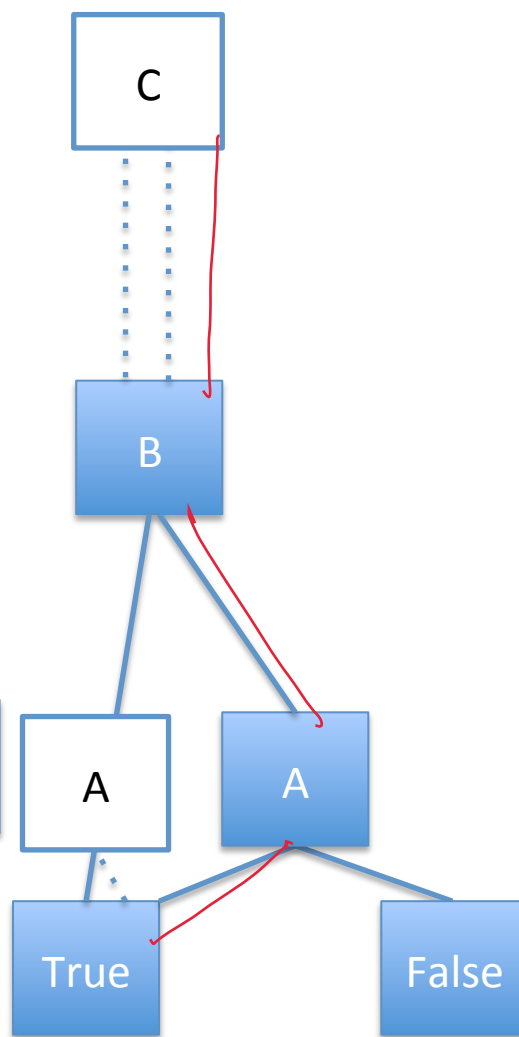
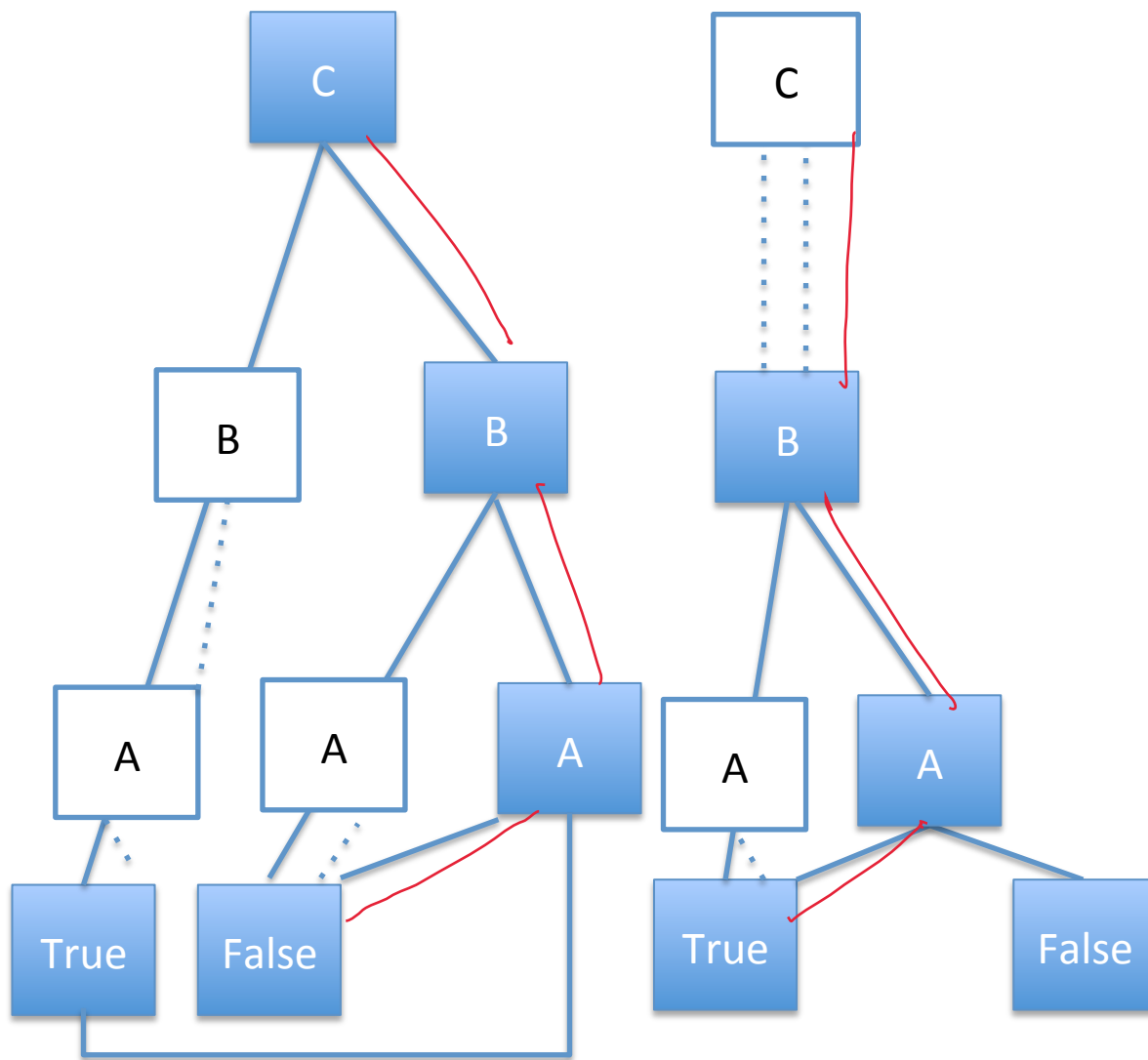


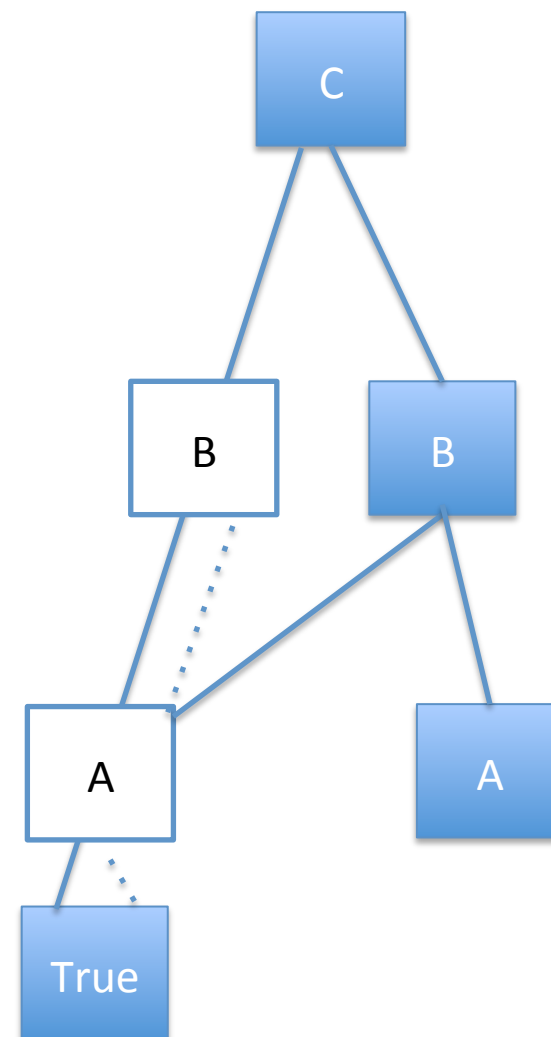
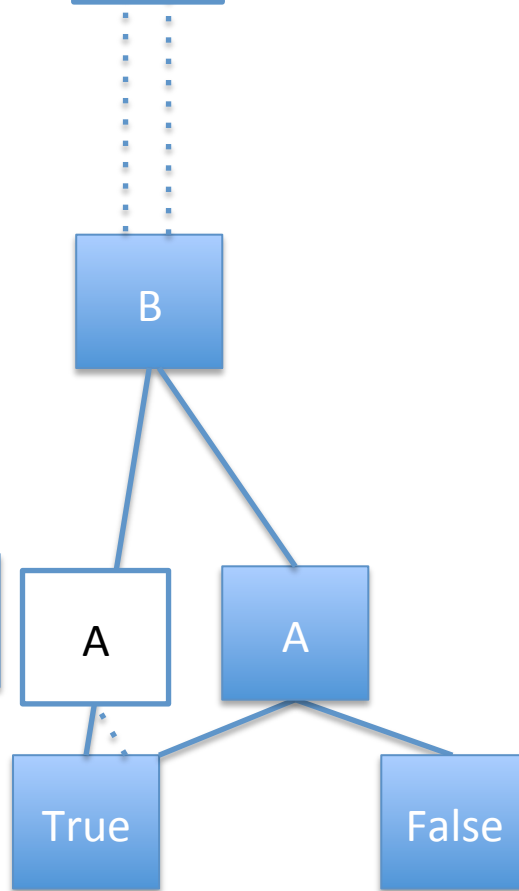
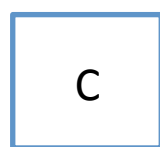
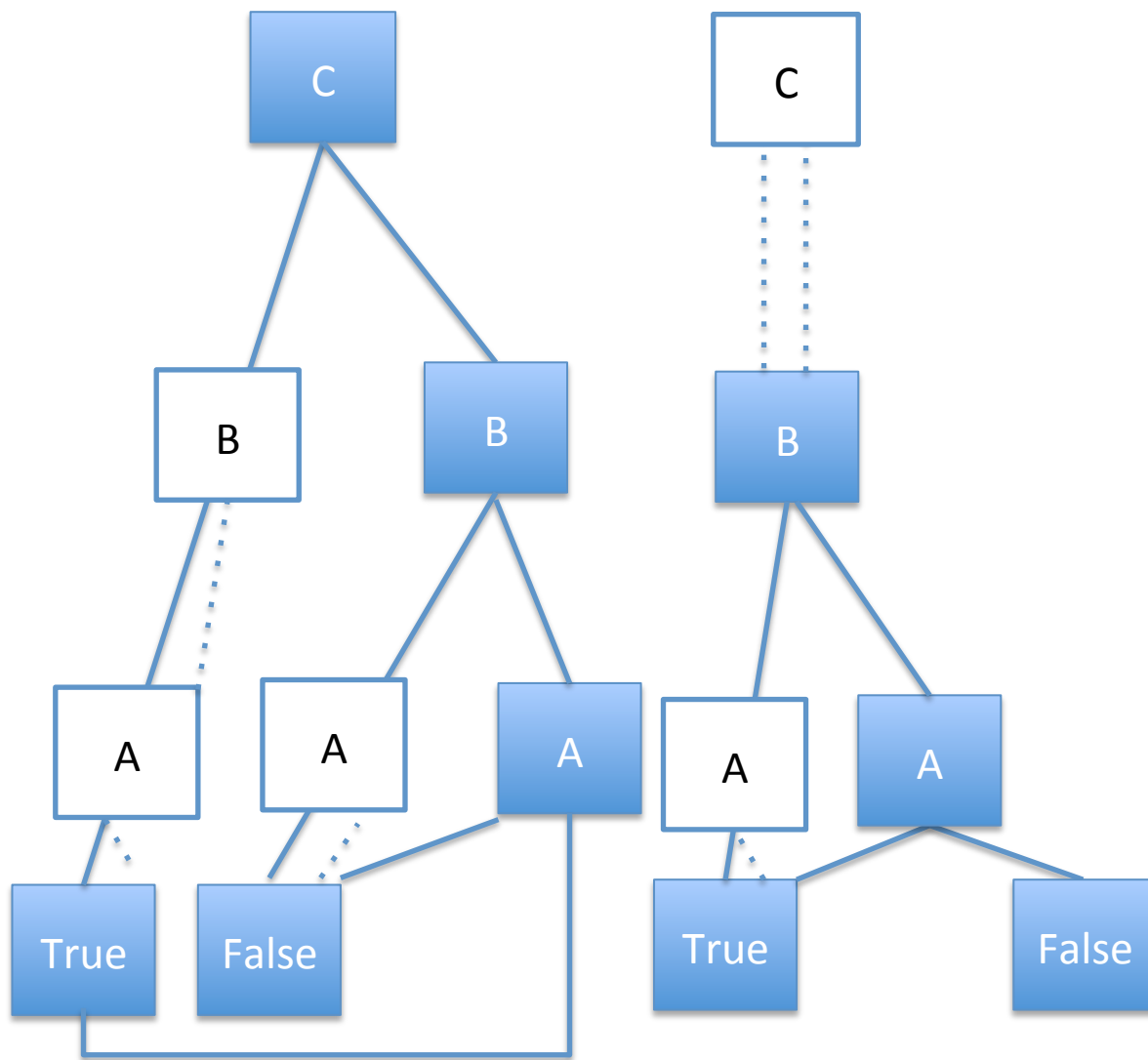
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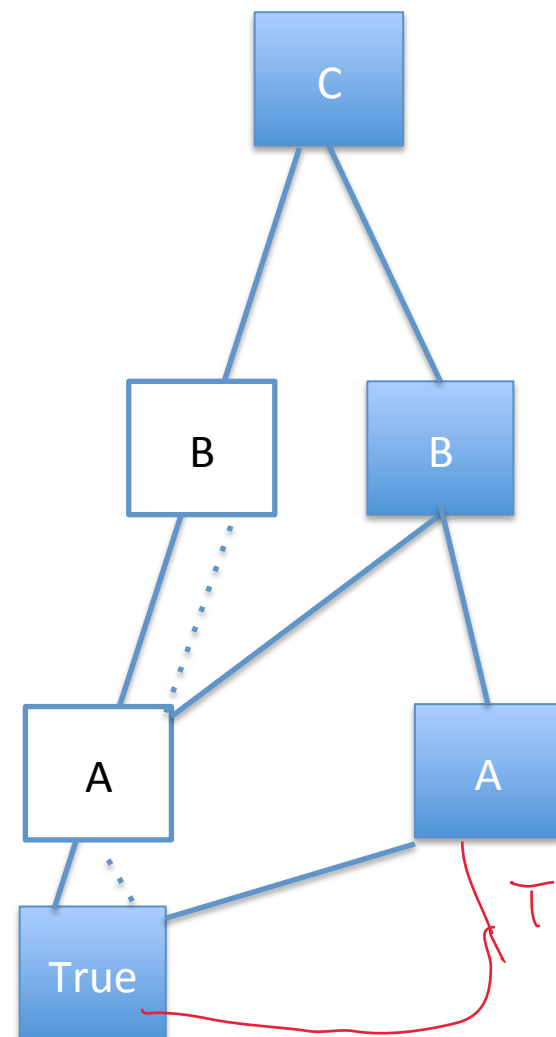
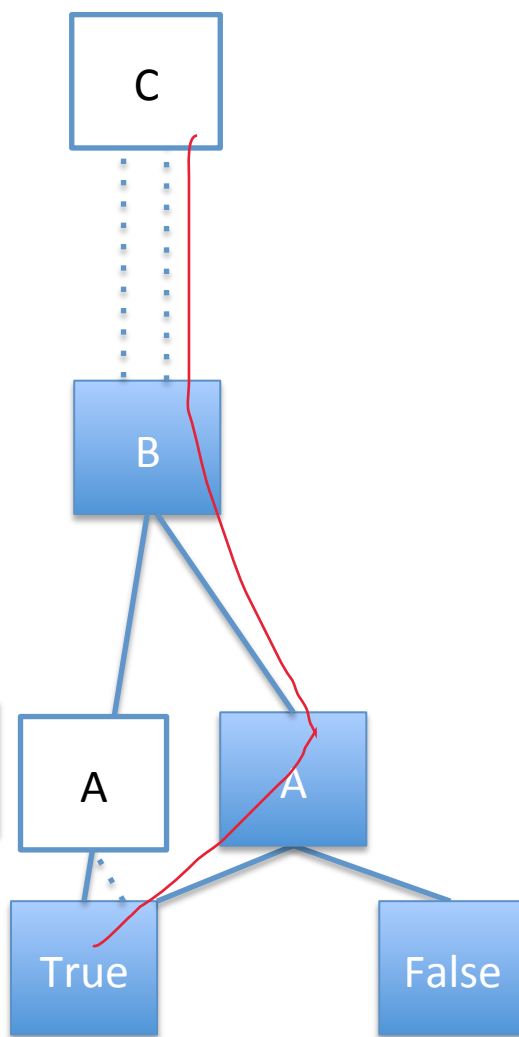
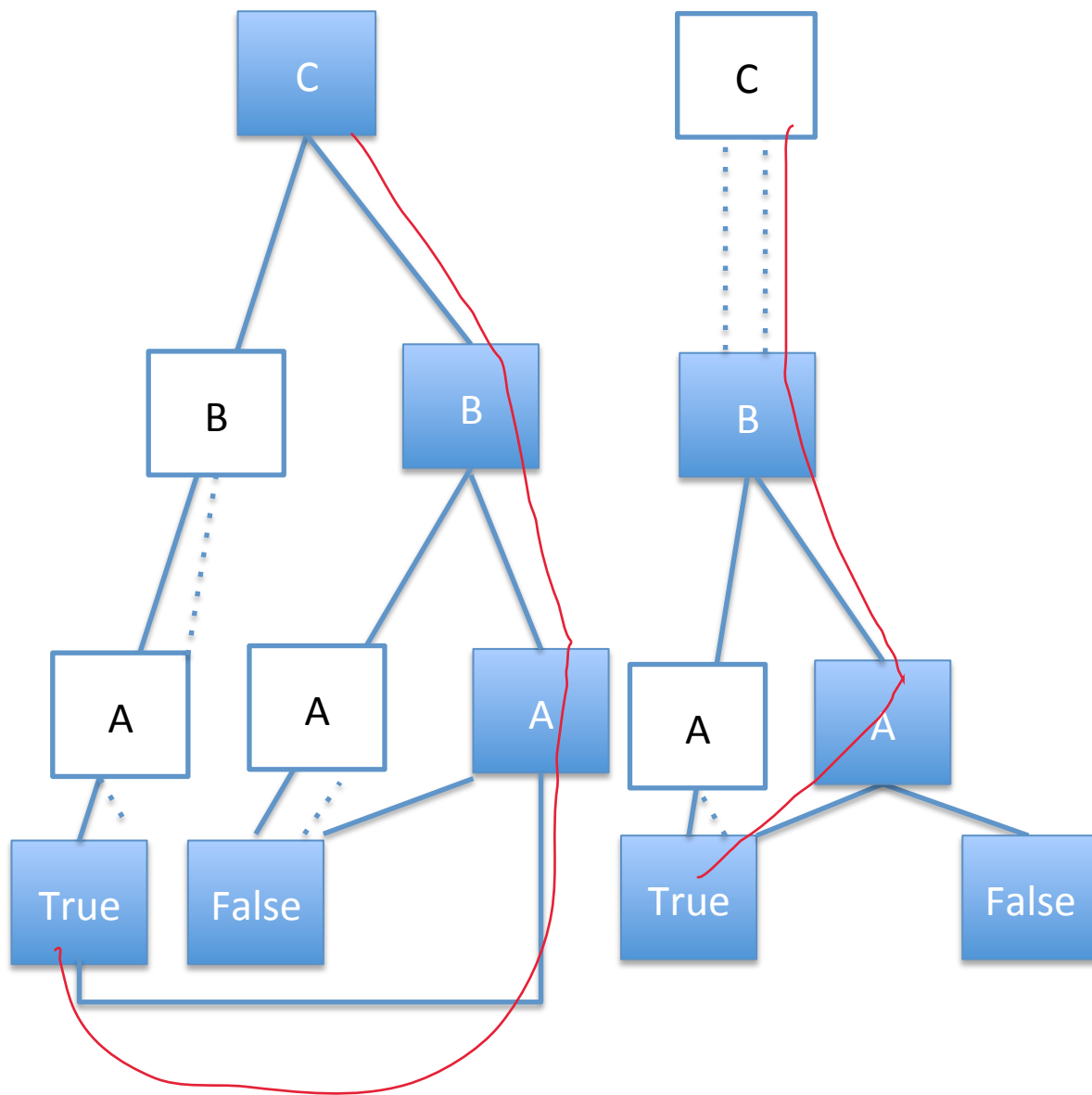


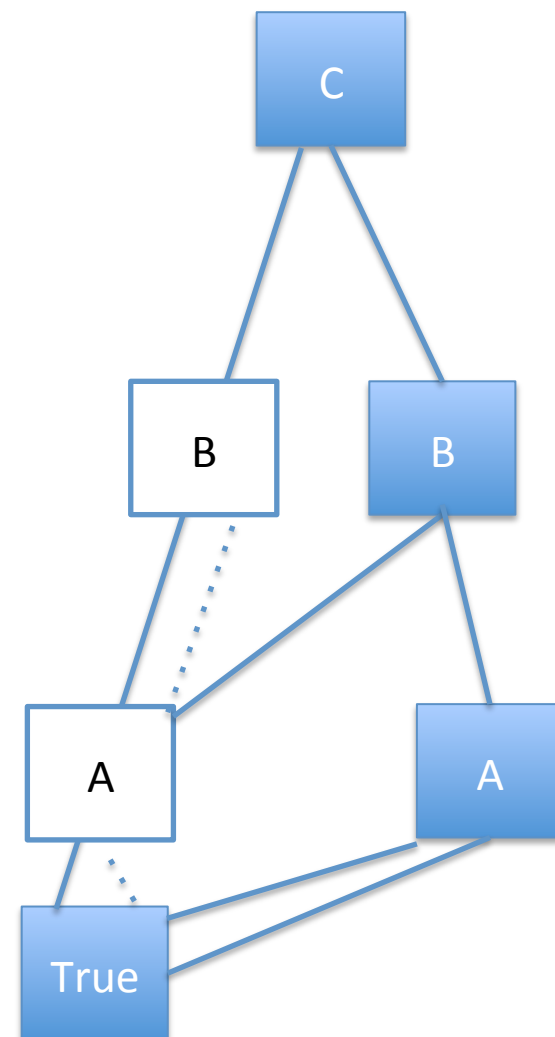
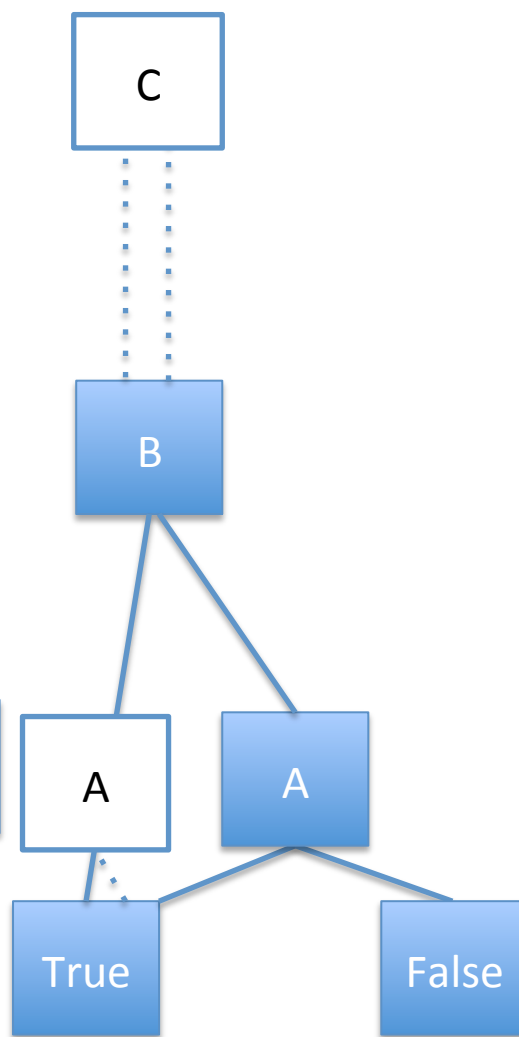
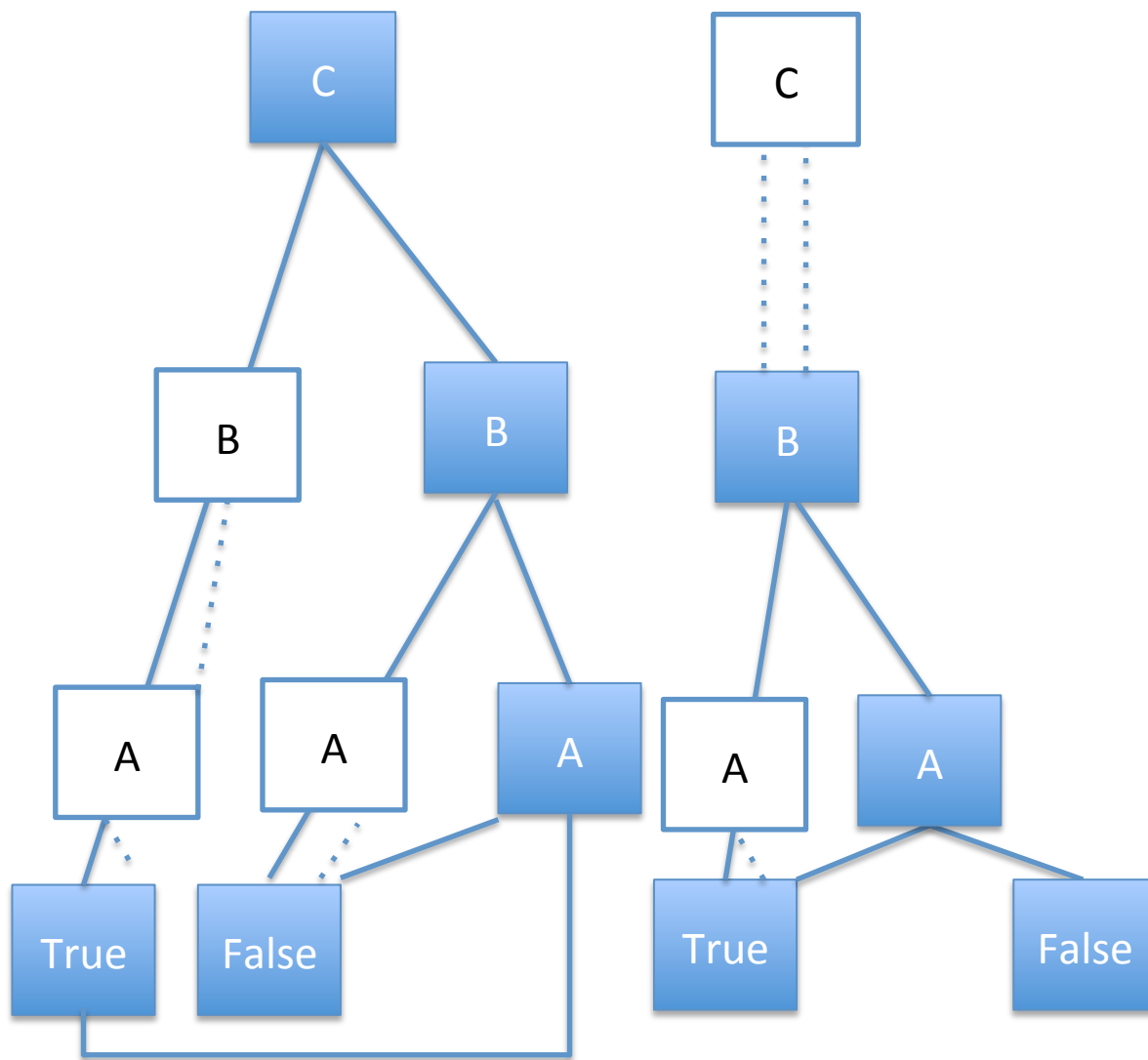
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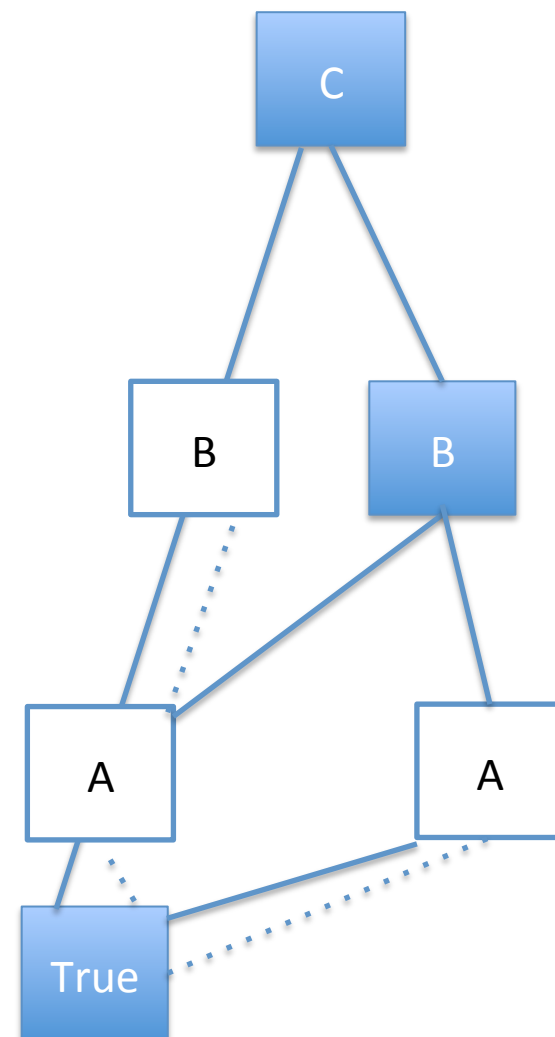
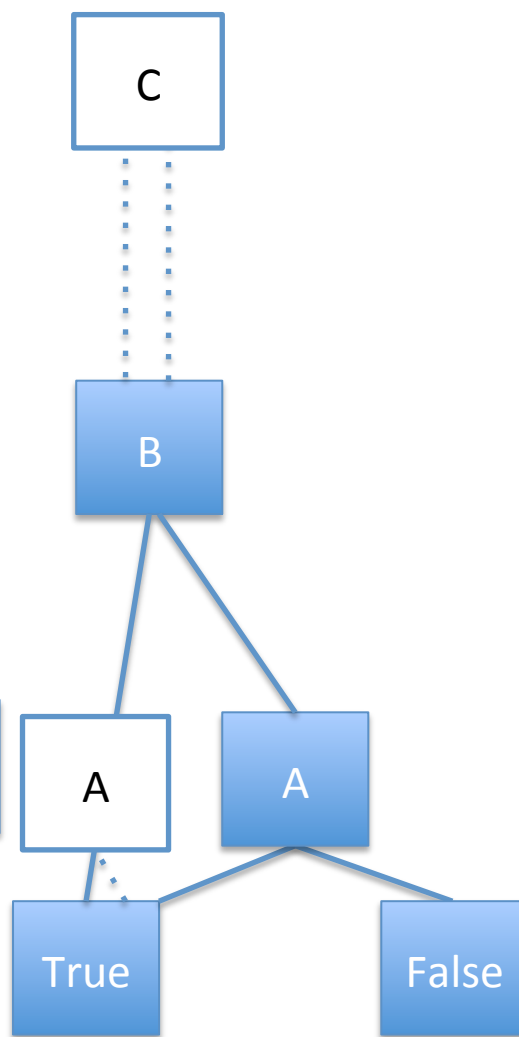
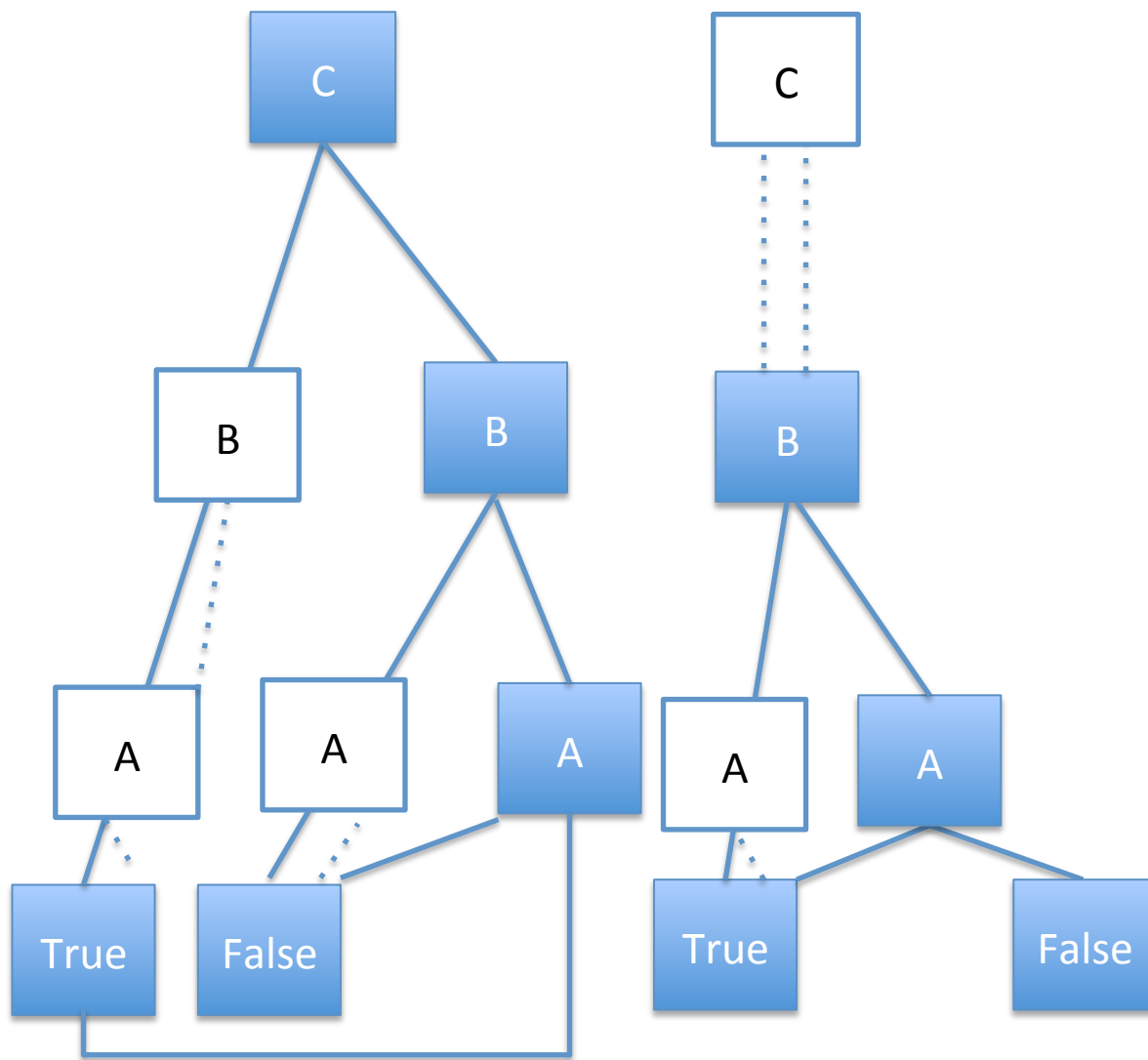




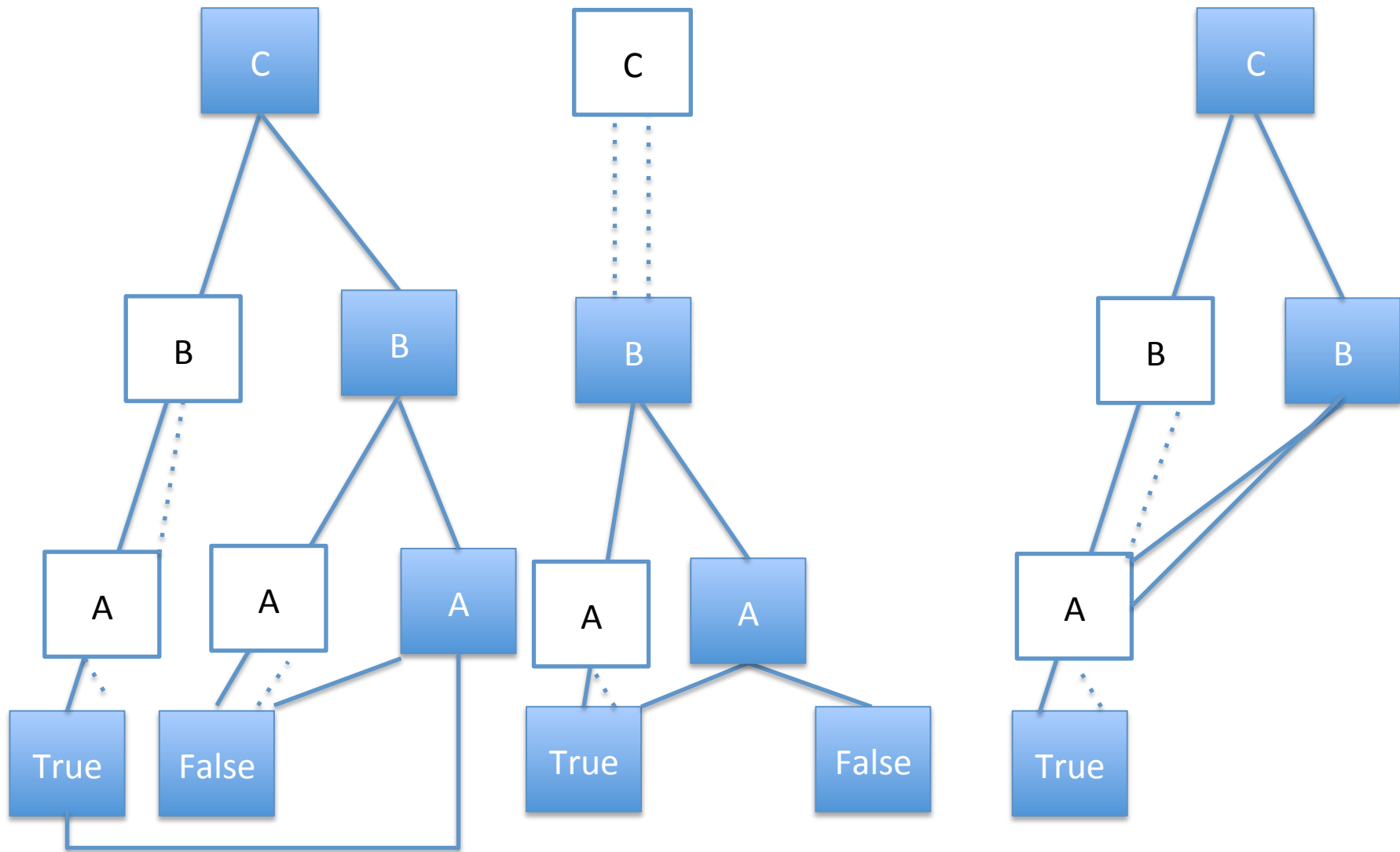




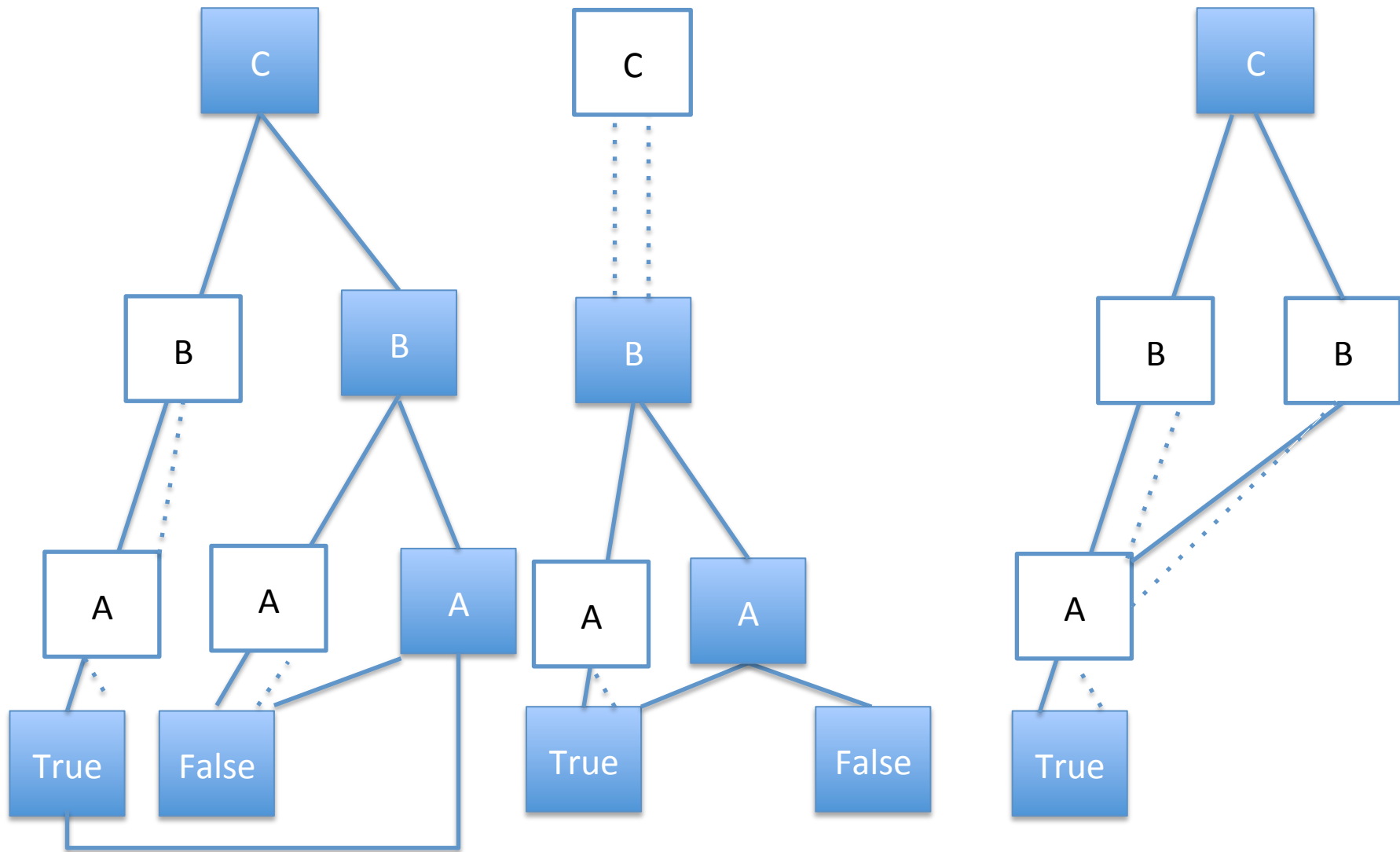




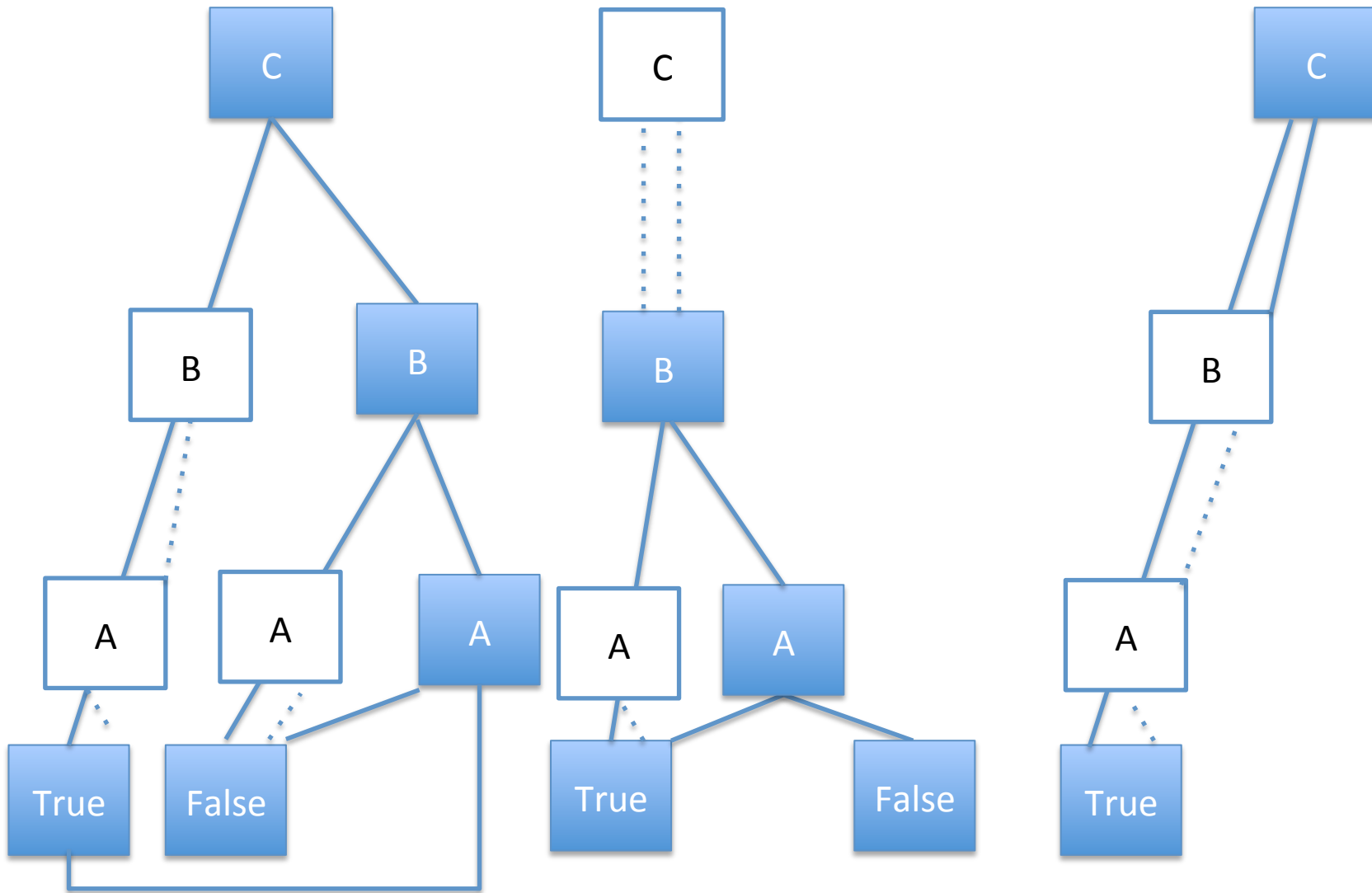
$((A \wedge B) \vee (\text{not } C)) \vee \text{not}(A \wedge B)$ Variables: $C > B > A$



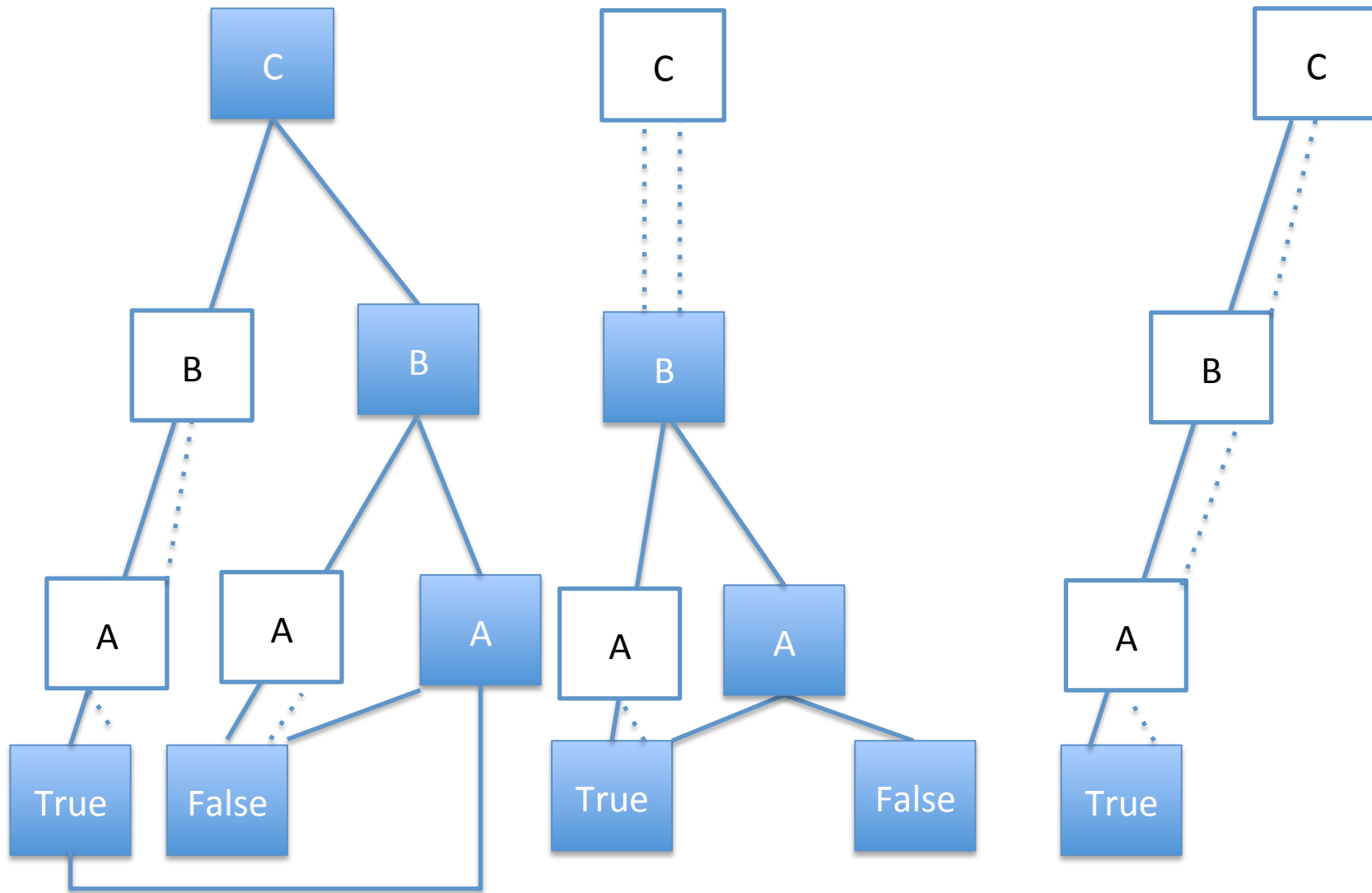
$((A \wedge B) \vee (\text{not } C)) \vee \text{not}(A \wedge B)$ Variables: $C > B > A$



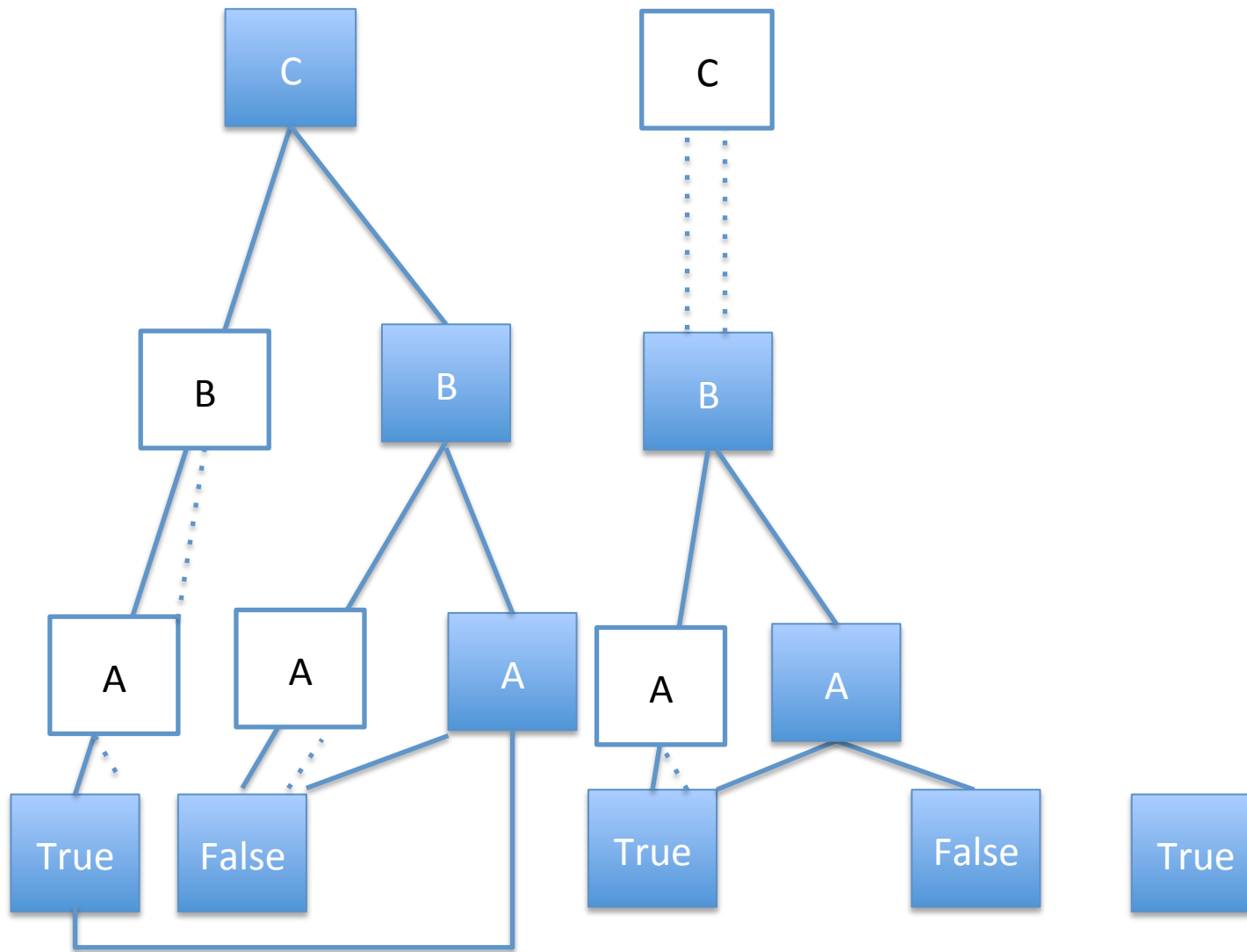
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$((A \wedge B) \vee (\text{not } C)) \vee \text{not}(A \wedge B)$ Variables: $C > B > A$



Uses of ROBDDs

- Reduced Order BDD for a proposition is unique for a fixed variable ordering
- Proposition is valid iff its ROBDD is just True
- Proposition is satisfiable its ROBDD is not just False
- Can check if a given valuation satisfies proposition in time linear to number of variables by walking the corresponding branch