HW 5 – Evaluation Semantics

CS 477 – Spring 2014

Revision 1.0

Assigned April 2, 2014
Due April 9, 2014, 9:00 pm
Extension 48 hours (20% penalty)

1 Change Log

1.0 Initial Release.

2 Objectives and Background

The purpose of this HW is to test your understanding of

- Natural semantics evaluation, transition semantics evaluation, and program transition systems

Another purpose of HWs is to provide you with experience answering non-programming written questions of the kind you may experience on the final.

3 Turn-In Procedure

The pdf for this assignment (`hw5.pdf`) should be found in the `assignments/hw5/` subdirectory of your svn directory for this course. Your solution should be put in that same directory. Using your favorite tool(s), you should put your solution in a file named `hw5-submission.pdf`. If you have problems generating a pdf, please seek help from the course staff. Your answers to the following questions are to be submitted electronically from within `assignments/hw5/` subdirectory by committing the file as follows:

```
svn add hw5-submission.pdf
svn commit -m "Turning in hw5"
```

4 Problems

Each of the probelms will use the same program P give here:

```
i := 1;
while i != 2
 do
  i := i + 1
 od
```

1. (10 pts) Starting in the empty envirnoment, evaluate the program P using Natural Semantics, as described in class.

Solution:

$$\dfrac{\dfrac{\dfrac{}{(\mathtt{1},\{\ \})\Downarrow 1}\,\textsf{Num}}{(\mathtt{i\ :=\ 1},\{\ \})\Downarrow\{\mathtt{i}\mapsto 1\}}\,\textsf{Asgn} \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{(\mathtt{i},\{\mathtt{i}\mapsto 1\})\Downarrow 1}\,\textsf{Var} \quad \dfrac{}{(\mathtt{2},\{\mathtt{i}\mapsto 1\})\Downarrow 2}\,\textsf{Num} \quad (1=2)=\textsf{false}}{(\mathtt{i\ =\ 2},\{\mathtt{i}\mapsto 1\})\Downarrow\textsf{false}}\,\textsf{Rel}}{(\mathtt{i\ !=\ 2},\{\mathtt{i}\mapsto 1\})\Downarrow\textsf{true}}\,\textsf{Not-F} \quad \mathit{Asgn2} \quad \mathit{While2}}{(\mathtt{while\ i\ !=\ 2\ do\ i\ :=\ i+1\ od},\{\mathtt{i}\mapsto 1\})\Downarrow\{\mathtt{i}\mapsto 2\}}\,\textsf{While-T}}{(\mathtt{i\ :=\ 1;\ while\ i\ !=\ 2\ do\ i\ :=\ i+1\ od},\{\ \})\Downarrow\{\mathtt{i}\mapsto 2\}}\,\textsf{Seq}$$

where *Asgn2* =

$$\dfrac{\dfrac{\dfrac{}{(\mathtt{i},\{\mathtt{i}\mapsto 1\})\Downarrow 1}\,\textsf{Var} \quad \dfrac{}{(\mathtt{1},\{\mathtt{i}\mapsto 1\})\Downarrow 1}\,\textsf{Num} \quad (1+1)=2}{(\mathtt{i+1},\{\mathtt{i}\mapsto 1\})\Downarrow 2}\,\textsf{Arith}}{(\mathtt{i\ :=\ i+1},\{\mathtt{i}\mapsto 1\})\Downarrow\{\mathtt{i}\mapsto 2\}}\,\textsf{Asgn}$$

and *While2* =

$$\dfrac{\dfrac{\dfrac{\dfrac{}{(\mathtt{i},\{\mathtt{i}\mapsto 2\})\Downarrow 2}\,\textsf{Var} \quad \dfrac{}{(\mathtt{2},\{\mathtt{i}\mapsto 2\})\Downarrow 2}\,\textsf{Num} \quad (2=2)=\textsf{true}}{(\mathtt{i\ =\ 2},\{\mathtt{i}\mapsto 2\})\Downarrow\textsf{true}}\,\textsf{Rel}}{(\mathtt{i\ !=\ 2},\{\mathtt{i}\mapsto 2\})\Downarrow\textsf{false}}\,\textsf{Not-T}}{(\mathtt{while\ i\ !=\ 2\ do\ i\ :=\ i+1\ od},\{\mathtt{i}\mapsto 2\})\Downarrow\{\mathtt{i}\mapsto 2\}}\,\textsf{While-F}$$

2. (15 pts) Starting in the empty envirnoment, evaluate the program P using transition semantics, as described in class. You should use transition semantics for evaluating arithmetic and boolean expressions, as well.

Solution:

$$\dfrac{\dfrac{}{(\mathtt{i\ :=\ 1},\{\})\longrightarrow\{\mathtt{i}\mapsto 1\}}\,\textsf{Assign2}}{(\mathtt{i\ :=\ 1;\ while\ i\ !=\ 2\ do\ i\ :=\ i+1\ od},\{\})\longrightarrow(\mathtt{while\ i\ !=\ 2\ do\ i\ :=\ i+1\ od},\{\mathtt{i}\mapsto 1\})}\,\textsf{Seq}$$

$$\dfrac{}{\begin{array}{c}(\mathtt{while\ i\ !=\ 2\ do\ i\ :=\ i+1\ od},\{\mathtt{i}\mapsto 1\})\longrightarrow\\ (\mathtt{if\ \ i\ !=\ 2\ then\ i\ :=\ i+1;\ while\ i\ !=\ 2\ do\ i\ :=\ i+1\ od\ \ else\ skip},\{\mathtt{i}\mapsto 1\})\end{array}}\,\textsf{While}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{(\mathtt{i},\{\mathtt{i}\mapsto 1\})\longrightarrow(\mathtt{1},\{\mathtt{i}\mapsto 1\})}\,\textsf{Var}}{(\mathtt{i\ =\ 2},\{\mathtt{i}\mapsto 1\})\longrightarrow(\mathtt{1\ =\ 2},\{\mathtt{i}\mapsto 1\})}\,\textsf{Rel1}}{(\mathtt{i\ !=\ 2},\{\mathtt{i}\mapsto 1\})\longrightarrow(\mathtt{1\ !=\ 2},\{\mathtt{i}\mapsto 1\})}\,\textsf{Not1}}{\begin{array}{c}(\mathtt{if\ \ i\ !=\ 2\ then\ i\ :=\ i+1;\ while\ i\ !=\ 2\ do\ i\ :=\ i+1\ od\ \ else\ skip},\{\mathtt{i}\mapsto 1\})\longrightarrow\\ (\mathtt{if\ \ 1\ !=\ 2\ then\ i\ :=\ i+1;\ while\ i\ !=\ 2\ do\ i\ :=\ i+1\ od\ \ else\ skip},\{\mathtt{i}\mapsto 1\})\end{array}}\,\textsf{if1}$$

$$\dfrac{\dfrac{\dfrac{}{(\mathtt{1\ =\ 2},\{\mathtt{i}\mapsto 1\})\longrightarrow(\mathtt{false},\{\mathtt{i}\mapsto 1\})}\,\textsf{Rel3}}{(\mathtt{1\ !=\ 2},\{\mathtt{i}\mapsto 1\})\longrightarrow(\mathtt{!(false)},\{\mathtt{i}\mapsto 1\})}\,\textsf{Not1}}{\begin{array}{c}(\mathtt{if\ \ 1\ !=\ 2\ then\ i\ :=\ i+1;\ while\ i\ !=\ 2\ do\ i\ :=\ i+1\ od\ \ else\ skip},\{\mathtt{i}\mapsto 1\})\longrightarrow\\ (\mathtt{if\ \ !(false)\ then\ i\ :=\ i+1;\ while\ i\ !=\ 2\ do\ i\ :=\ i+1\ od\ \ else\ skip},\{\mathtt{i}\mapsto 1\})\end{array}}\,\textsf{if1}$$

It would have been acceptable to treat != as a single operation.

$$\dfrac{\dfrac{}{(\texttt{!(false)},\{\texttt{i}\mapsto 1\}) \longrightarrow (\texttt{true},\{\texttt{i}\mapsto 1\})}\ \textsf{Not3}}{\begin{array}{c}(\texttt{if !(false) then i := i+1; while i != 2 do i := i+1 od else skip},\{\texttt{i}\mapsto 1\}) \longrightarrow \\ (\texttt{if true then i := i+1; while i != 2 do i := i+1 od else skip},\{\texttt{i}\mapsto 1\})\end{array}}\ \textsf{if1}$$

$$\dfrac{}{\begin{array}{c}(\texttt{if true then i := i+1; while i != 2 do i := i+1 od else skip},\{\texttt{i}\mapsto 1\}) \longrightarrow \\ (\texttt{i := i+1; while i != 2 do i := i+1 od},\{\texttt{i}\mapsto 1\})\end{array}}\ \textsf{if-true}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{(\texttt{i},\{\texttt{i}\mapsto 1\}) \longrightarrow (1,\{\texttt{i}\mapsto 1\})}\ \textsf{Var}}{(\texttt{i+1},\{\texttt{i}\mapsto 1\}) \longrightarrow (\texttt{1+1},\{\texttt{i}\mapsto 1\})}\ \textsf{Arith1}}{(\texttt{i := i+1},\{\texttt{i}\mapsto 1\}) \longrightarrow (\texttt{i := 1+1},\{\texttt{i}\mapsto 1\})}\ \textsf{Assign1}}{\begin{array}{c}(\texttt{i := i+1; while i != 2 do i := i+1 od},\{\texttt{i}\mapsto 1\}) \longrightarrow \\ (\texttt{i := 1+1; while i != 2 do i := i+1 od},\{\texttt{i}\mapsto 1\})\end{array}}\ \textsf{Seq1}$$

$$\dfrac{\dfrac{\dfrac{}{(\texttt{1+1},\{\texttt{i}\mapsto 1\}) \longrightarrow (2,\{\texttt{i}\mapsto 1\})}\ \textsf{Arith3}}{(\texttt{i := 1+1},\{\texttt{i}\mapsto 1\}) \longrightarrow (\texttt{i := 2},\{\texttt{i}\mapsto 1\})}\ \textsf{Assign1}}{\begin{array}{c}(\texttt{i := 1+1; while i != 2 do i := i+1 od},\{\texttt{i}\mapsto 1\}) \longrightarrow \\ (\texttt{i := 2; while i != 2 do i := i+1 od},\{\texttt{i}\mapsto 1\})\end{array}}\ \textsf{Seq1}$$

$$\dfrac{\dfrac{}{(\texttt{i := 2},\{\texttt{i}\mapsto 1\}) \longrightarrow \{\texttt{i}\mapsto 2\}}\ \textsf{Assign2}}{\begin{array}{c}(\texttt{i := 2; while i != 2 do i := i+1 od},\{\texttt{i}\mapsto 1\}) \longrightarrow \\ (\texttt{while i != 2 do i := i+1 od},\{\texttt{i}\mapsto 2\})\end{array}}\ \textsf{Seq2}$$

$$\dfrac{}{\begin{array}{c}(\texttt{while i != 2 do i := i+1 od},\{\texttt{i}\mapsto 2\}) \longrightarrow \\ (\texttt{if i != 2 then i := i+1; while i != 2 do i := i+1 od else skip},\{\texttt{i}\mapsto 2\})\end{array}}\ \textsf{While}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{(\texttt{i},\{\texttt{i}\mapsto 2\}) \longrightarrow (1,\{\texttt{i}\mapsto 2\})}\ \textsf{Var}}{(\texttt{i = 2},\{\texttt{i}\mapsto 2\}) \longrightarrow (\texttt{2 = 2},\{\texttt{i}\mapsto 2\})}\ \textsf{Rel1}}{(\texttt{i != 2},\{\texttt{i}\mapsto 2\}) \longrightarrow (\texttt{2 != 2},\{\texttt{i}\mapsto 2\})}\ \textsf{Not1}}{\begin{array}{c}(\texttt{if i != 2 then i := i+1; while i != 2 do i := i+1 od else skip},\{\texttt{i}\mapsto 2\}) \longrightarrow \\ (\texttt{if 2 != 2 then i := i+1; while i != 2 do i := i+1 od else skip},\{\texttt{i}\mapsto 2\})\end{array}}\ \textsf{if1}$$

$$\dfrac{\dfrac{\dfrac{}{(\texttt{2 = 2},\{\texttt{i}\mapsto 2\}) \longrightarrow (\texttt{true},\{\texttt{i}\mapsto 2\})}\ \textsf{Rel3}}{(\texttt{2 != 2},\{\texttt{i}\mapsto 2\}) \longrightarrow (\texttt{!(true)},\{\texttt{i}\mapsto 2\})}\ \textsf{Not1}}{\begin{array}{c}(\texttt{if 2 != 2 then i := i+1; while i != 2 do i := i+1 od else skip},\{\texttt{i}\mapsto 2\}) \longrightarrow \\ (\texttt{if !(true) then i := i+1; while i != 2 do i := i+1 od else skip},\{\texttt{i}\mapsto 2\})\end{array}}\ \textsf{if1}$$

Again, it would have been fine to treat != as a single operation.

$$\cfrac{\cfrac{}{(\texttt{!(true)}, \{\texttt{i} \mapsto 2\}) \longrightarrow (\texttt{false}, \{\texttt{i} \mapsto 2\})}\ \textsf{Not2}}{\begin{array}{c}(\texttt{if !(true) then i := i+1; while i != 2 do i := i+1 od else skip}, \{\texttt{i} \mapsto 2\}) \longrightarrow \\ (\texttt{if false then i := i+1; while i != 2 do i := i+1 od else skip}, \{\texttt{i} \mapsto 2\})\end{array}}\ \textsf{if1}$$

$$\cfrac{}{\begin{array}{c}(\texttt{if false then i := i+1; while i != 2 do i := i+1 od else skip}, \{\texttt{i} \mapsto 2\}) \longrightarrow \\ (\texttt{skip}, \{\texttt{i} \mapsto 2\})\end{array}}\ \textsf{if-false}$$

$$\cfrac{}{(\texttt{skip}, \{\texttt{i} \mapsto 2\}) \longrightarrow \{\texttt{i} \mapsto 2\}}\ \textsf{Skip}$$

5 Extra Credit

3. (5 pts) Translate P into a program transition system. You will need to introduce at least one additional variable. If the value 2 were changed to another value in both P your program, the resulting programs should continue to behave the same as each other in terms of values assigned to `i`.

Solution: Let

- $V = \{\texttt{pc}, \texttt{i}\}$,
- $F = \{+,\ 1,\ 2\}$, $af = \{+ \mapsto 2; 1 \mapsto 0; 2 \mapsto 0\}$ (we will use $+$ infixed),
- $R = \{\ =\ \}$, $rf = \{= \mapsto 2\}$ (we will use it infixed),
- $\mathcal{G} = (V, F, af, R, ar)$,
- $\mathcal{D} = \mathbb{N}$,
- $\mathcal{F}$ contains the addtion function, and the Natural numbers 1 and 2
- $\phi(+) = +$ $\mathcal{R}$ is the singleton set containing equality on $\mathbb{N}$,
- $\rho(=) = =$, $\rho(1) =\ 1$, $\rho(2) =\ 2$
- $\mathcal{S} = (\mathcal{G}, \mathcal{D}, \mathcal{F}, \phi, \mathcal{R}, \rho)$.
- $init = (pc = 1)$

Such a program transition system may be given by $(\mathcal{S}, T, \textsf{true})$ where

$$T = \left\{ \begin{array}{rcl} \texttt{pc = 1} & \rightarrow & \texttt{(pc,i) := (2,1)} \\ \texttt{pc = 2 \& i != 2} & \rightarrow & \texttt{pc ::= 3} \\ \texttt{pc = 2 \& i = 2} & \rightarrow & \texttt{pc ::= 4} \\ \texttt{pc = 3} & \rightarrow & \texttt{(pc,i) := (2,i + 1)} \end{array} \right\}$$