CS477 Formal Software Development Methods

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α-equivalence

- $\psi \stackrel{\alpha}{\equiv} \psi$
- If $\psi_1 \stackrel{\alpha}{\equiv} \psi_2$ then $\psi_2 \stackrel{\alpha}{\equiv} \psi$.
- It $\psi_1 \stackrel{\alpha}{\equiv} \psi_2$ and $\psi_2 \stackrel{\alpha}{\equiv} \psi_3$ then $\psi_1 \stackrel{\alpha}{\equiv} \psi_3$
- If $x \notin fv(\psi)$ and $y \notin fv(\psi)$ then $\psi \stackrel{\alpha}{\equiv} \psi[x \leftrightarrow y]$.
- If $\psi_i \stackrel{\alpha}{\equiv} \psi'_i$ for $i = 1, 2$ then
  - $(\psi_1) \stackrel{\alpha}{\equiv} (\psi'_1)$ $\quad \neg\psi_1 \stackrel{\alpha}{\equiv} \neg\psi'_1$
  - $\psi_1 \otimes \psi_2 \stackrel{\alpha}{\equiv} \psi'_1 \otimes \psi'_2$ for $\otimes \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$
  - $\mathcal{Q}\, z.\, \psi_1 \stackrel{\alpha}{\equiv} \mathcal{Q}\, z.\, \psi'_1$ for $\mathcal{Q} \in \{\forall, \exists\}$

α-equivalence: Example

$$(x > 3 \wedge (\exists y.\ (\forall z.\ z \geq (y - x)) \vee (z \geq y))) \stackrel{\alpha}{\equiv} (x > 3 \wedge (\exists w.\ (\forall z.\ z \geq (w - x)) \vee (z \geq w)))$$

$$(x > 3 \wedge (\exists y.\ (\forall z.\ z \geq (y - x)) \vee (z \geq y))) \stackrel{\alpha}{\equiv} (x > 3 \wedge (\exists w.\ (\forall y.\ y \geq (w - x)) \vee (z \geq w)))$$

Proof Rules

Will give Sequent version of Natural Deduction rules
All rules from Propositional Logic included

$$\frac{\Gamma \vdash \psi'[t/x]}{\Gamma \vdash \exists x.\psi}\ \text{Ex I}$$
provided $\psi \stackrel{\alpha}{\equiv} \psi'$

$$\frac{\Gamma \vdash \exists x.\psi \quad \Gamma \cup \{(\psi[y/x])\} \vdash \varphi}{\Gamma \vdash \varphi}\ \text{Ex E}$$
provided
$y \notin fv(\varphi) \cup (fv(\psi) \setminus \{x\}) \cup \bigcup_{\psi' \in \Gamma} fv(\psi')$

$$\frac{\Gamma \vdash \psi[y/x]}{\Gamma \vdash \forall x.\psi}\ \text{All I}$$
provided
$y \notin (fv(\psi) \setminus \{x\}) \cup \bigcup_{\psi' \in \Gamma} fv(\psi')$

$$\frac{\Gamma \vdash \forall x.\psi \quad \Gamma \cup \{\psi'[t/x]\} \vdash \varphi}{\Gamma \vdash \varphi}\ \text{All E}$$
provided $\psi \stackrel{\alpha}{\equiv} \psi'$

Example

Show

$$\frac{}{\{\ \} \vdash (\exists x. \forall y. x \leq y) \Rightarrow (\forall x. \exists y. y \leq x)}$$

Example

Show

$$\frac{\dfrac{}{\{(\exists x. \forall y. x \leq y)\} \vdash \forall x. \exists y. y \leq x}}{\{\ \} \vdash (\exists x. \forall y. x \leq y) \Rightarrow (\forall x. \exists y. y \leq x)}\ \text{Imp I}$$

Example

Show

$$\{(\exists x.\,\forall y.\,x\le y)\}\vdash \exists y.\,y\le x$$

$$\{(\exists x.\,\forall y.\,x\le y)\}\vdash \forall x.\,\exists y.\,y\le x \quad \text{All I}$$

$$\{\ \}\vdash(\exists x.\,\forall y.\,x\le y)\Rightarrow(\forall x.\,\exists y.\,y\le x) \quad \text{Imp I}$$

Example

Show

$$\{\exists x.\,\forall y.\,x\le y\}\vdash \exists x.\,\forall y.\,x\le y \qquad \left\{\begin{matrix}\exists x.\,\forall y.\,x\le y;\\ \forall y.\,z\le y\end{matrix}\right\}\vdash \exists y.\,y\le x$$

$$\{(\exists x.\,\forall y.\,x\le y)\}\vdash \exists y.\,y\le x \quad \text{Ex E}$$

$$\{(\exists x.\,\forall y.\,x\le y)\}\vdash \forall x.\,\exists y.\,y\le x \quad \text{All I}$$

$$\{\ \}\vdash(\exists x.\,\forall y.\,x\le y)\Rightarrow(\forall x.\,\exists y.\,y\le x) \quad \text{Imp I}$$

Example

Show

$$\{\exists x.\,\forall y.\,x\le y\}\vdash \exists x.\,\forall y.\,x\le y \ \text{Hyp} \qquad \left\{\begin{matrix}\exists x.\,\forall y.\,x\le y;\\ \forall y.\,z\le y\end{matrix}\right\}\vdash \exists y.\,y\le x$$

$$\{(\exists x.\,\forall y.\,x\le y)\}\vdash \exists y.\,y\le x \quad \text{Ex E}$$

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Example

Show

$$\left\{\begin{matrix}\exists x.\,\forall y.\,x\le y;\\ \forall y.\,z\le y\end{matrix}\right\}\vdash \forall y.\,z\le y \qquad \left\{\begin{matrix}\exists x.\,\forall y.\,x\le y;\\ \forall y.\,z\le y;\ z\le x\end{matrix}\right\}\vdash \exists y.\,y\le x$$

$$\{\exists x.\,\forall y.\,x\le y\}\vdash \exists x.\,\forall y.\,x\le y \ \text{Hyp} \qquad \left\{\begin{matrix}\exists x.\,\forall y.\,x\le y;\\ \forall y.\,z\le y\end{matrix}\right\}\vdash \exists y.\,y\le x \quad \text{All E}$$

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Example

Show

$$\left\{\begin{matrix}\exists x.\,\forall y.\,x\le y;\\ \forall y.\,z\le y\end{matrix}\right\}\vdash \forall y.\,z\le y \ \text{Hyp} \qquad \left\{\begin{matrix}\exists x.\,\forall y.\,x\le y;\\ \forall y.\,z\le y;\ z\le x\end{matrix}\right\}\vdash \exists y.\,y\le x$$

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$$\{\ \}\vdash(\exists x.\,\forall y.\,x\le y)\Rightarrow(\forall x.\,\exists y.\,y\le x) \quad \text{Imp I}$$

Example

Show

$$\left\{\begin{matrix}\exists x.\,\forall y.\,x\le y;\\ \forall y.\,z\le y;\ z\le x\end{matrix}\right\}\vdash z\le x$$

$$\left\{\begin{matrix}\exists x.\,\forall y.\,x\le y;\\ \forall y.\,z\le y\end{matrix}\right\}\vdash \forall y.\,z\le y \ \text{Hyp} \qquad \left\{\begin{matrix}\exists x.\,\forall y.\,x\le y;\\ \forall y.\,z\le y;\ z\le x\end{matrix}\right\}\vdash \exists y.\,y\le x \quad \text{Ex I}$$

$$\{\exists x.\,\forall y.\,x\le y\}\vdash \exists x.\,\forall y.\,x\le y \ \text{Hyp} \qquad \left\{\begin{matrix}\exists x.\,\forall y.\,x\le y;\\ \forall y.\,z\le y\end{matrix}\right\}\vdash \exists y.\,y\le x \quad \text{All E}$$

$$\{(\exists x.\,\forall y.\,x\le y)\}\vdash \exists y.\,y\le x \quad \text{Ex E}$$

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Example

Show

$$\dfrac{\dfrac{\dfrac{}{\{\exists x.\forall y.x\le y\}\vdash \exists x.\forall y.x\le y}\text{Hyp}\quad \dfrac{\dfrac{}{\left\{\begin{matrix}\exists x.\forall y.x\le y;\\ \forall y.z\le y\end{matrix}\right\}\vdash \forall y.z\le y}\text{Hyp}\quad \dfrac{\dfrac{}{\left\{\begin{matrix}\exists x.\forall y.x\le y;\\ \forall y.z\le y;\ z\le x\end{matrix}\right\}\vdash z\le x}\text{Hyp}}{\left\{\begin{matrix}\exists x.\forall y.x\le y;\\ \forall y.z\le y;\ z\le x\end{matrix}\right\}\vdash \exists y.y\le x}\text{Ex I}}{\left\{\begin{matrix}\exists x.\forall y.x\le y;\\ \forall y.z\le y\end{matrix}\right\}\vdash \exists y.y\le x}\text{All E}}{\dfrac{\{(\exists x.\forall y.x\le y)\}\vdash \exists y.y\le x}{\{(\exists x.\forall y.x\le y)\}\vdash \forall x.\exists y.y\le x}\text{All I}}\text{Ex E}$$

$$\dfrac{\{(\exists x.\forall y.x\le y)\}\vdash \forall x.\exists y.y\le x}{\{\ \}\vdash (\exists x.\forall y.x\le y)\Rightarrow(\forall x.\exists y.y\le x)}\text{Imp I}$$

Example of Failure

Let's try to show 1

$$\dfrac{}{\{\ \}\vdash (\forall x.\exists y.y\le x)\Rightarrow(\exists x.\forall y.x\le y)}$$

Example of Failure

Let's try to show 2

$$\dfrac{\dfrac{}{\{\forall x.\exists y.y\le x\}\vdash \exists x.\forall y.x\le y}}{\{\ \}\vdash (\forall x.\exists y.y\le x)\Rightarrow(\exists x.\forall y.x\le y)}\text{Imp I}$$

Example of Failure

Let's try to show 3

$$\dfrac{\dfrac{\dfrac{}{\{\forall x.\exists y.y\le x\}\vdash \forall y.z\le y}}{\{\forall x.\exists y.y\le x\}\vdash \exists x.\forall y.x\le y}\text{Ex I}}{\{\ \}\vdash (\forall x.\exists y.y\le x)\Rightarrow(\exists x.\forall y.x\le y)}\text{Imp I}$$

Example of Failure

Let's try to show 4

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\{\forall x.\exists y.y\le x\}\vdash z\le x}}{\{\forall x.\exists y.y\le x\}\vdash \forall y.z\le y}\text{All I}}{\{\forall x.\exists y.y\le x\}\vdash \exists x.\forall y.x\le y}\text{Ex I}}{\{\ \}\vdash (\forall x.\exists y.y\le x)\Rightarrow(\exists x.\forall y.x\le y)}\text{Imp I}$$

Example of Failure

Let's try to show 5

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\{\forall x.\exists y.y\le x\}\vdash \forall x.\exists y.y\le x}\quad \dfrac{}{\left\{\begin{matrix}\forall x.\exists y.y\le x;\\ \exists y.y\le x\end{matrix}\right\}\vdash z\le x}}{\{\forall x.\exists y.y\le x\}\vdash z\le x}\text{All E}}{\{\forall x.\exists y.y\le x\}\vdash \forall y.z\le y}\text{All I}}{\{\forall x.\exists y.y\le x\}\vdash \exists x.\forall y.x\le y}\text{Ex I}}{\{\ \}\vdash (\forall x.\exists y.y\le x)\Rightarrow(\exists x.\forall y.x\le y)}\text{Imp I}$$

Example of Failure

Let’s try to show 6

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\{\forall x.\exists y.\,y\le x\}\vdash\forall x.\exists y.\,y\le x}\,\text{Hyp}\qquad \dfrac{}{\left\{\begin{matrix}\forall x.\exists y.\,y\le x;\\ \exists y.\,y\le x\end{matrix}\right\}\vdash z\le x}}{\{\forall x.\exists y.\,y\le x\}\vdash z\le x}\,\text{All E}}{\{\forall x.\exists y.\,y\le x\}\vdash\forall y.\,z\le y}\,\text{All I}}{\{\forall x.\exists y.\,y\le x\}\vdash\exists x.\forall y.\,x\le y}\,\text{Ex I}}{\{\ \}\vdash(\forall x.\exists y.\,y\le x)\Rightarrow(\exists x.\forall y.\,x\le y)}\,\text{Imp I}$$

Example of Failure

Let’s try to show 7

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\{\forall x.\exists y.\,y\le x\}\vdash\forall x.\exists y.\,y\le x}\,\text{Hyp}\qquad \dfrac{\dfrac{}{\mathit{Something}}\quad \left\{\begin{matrix}\mathit{Lab}\ \forall x.\exists y.\,y\le x;\\ \exists y.\,y\le x;\ z\le x\end{matrix}\right\}\vdash z\le x}{\left\{\begin{matrix}\forall x.\exists y.\,y\le x;\\ \exists y.\,y\le x\end{matrix}\right\}\vdash z\le x}\,\text{Ex E}}{\{\forall x.\exists y.\,y\le x\}\vdash z\le x}\,\text{All E}}{\{\forall x.\exists y.\,y\le x\}\vdash\forall y.\,z\le y}\,\text{All I}}{\{\forall x.\exists y.\,y\le x\}\vdash\exists x.\forall y.\,x\le y}\,\text{Ex I}}{\{\ \}\vdash(\forall x.\exists y.\,y\le x)\Rightarrow(\exists x.\forall y.\,x\le y)}\,\text{Imp I}$$

Example of Failure

Let’s try to show 8

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\{\forall x.\exists y.\,y\le x\}\vdash\forall x.\exists y.\,y\le x}\,\text{Hyp}\qquad \dfrac{\dfrac{}{\mathit{Something}}\quad \dfrac{}{\left\{\begin{matrix}\mathit{Lab}\ \forall x.\exists y.\,y\le x;\\ \exists y.\,y\le x;\ z\le x\end{matrix}\right\}\vdash z\le x}\,\text{Hyp}}{\left\{\begin{matrix}\forall x.\exists y.\,y\le x;\\ \exists y.\,y\le x\end{matrix}\right\}\vdash z\le x}\,\text{Ex E}}{\{\forall x.\exists y.\,y\le x\}\vdash z\le x}\,\text{All E}}{\{\forall x.\exists y.\,y\le x\}\vdash\forall y.\,z\le y}\,\text{All I}}{\{\forall x.\exists y.\,y\le x\}\vdash\exists x.\forall y.\,x\le y}\,\text{Ex I}}{\{\ \}\vdash(\forall x.\exists y.\,y\le x)\Rightarrow(\exists x.\forall y.\,x\le y)}\,\text{Imp I}$$

Example of Failure

Let’s try to show 9

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\{\forall x.\exists y.\,y\le x\}\vdash\forall x.\exists y.\,y\le x}\,\text{Hyp}\qquad \dfrac{\dfrac{}{\mathit{Something}}\quad \dfrac{}{\left\{\begin{matrix}\mathit{Lab}\ \forall x.\exists y.\,y\le x;\\ \exists y.\,y\le x;\ z\le x\end{matrix}\right\}\vdash z\le x}\,\text{Hyp}}{\left\{\begin{matrix}\forall x.\exists y.\,y\le x;\\ \exists y.\,y\le x\end{matrix}\right\}\vdash z\le x}\,\text{Ex E}}{\{\forall x.\exists y.\,y\le x\}\vdash z\le x}\,\text{All E}}{\{\forall x.\exists y.\,y\le x\}\vdash\forall y.\,z\le y}\,\text{All I}}{\{\forall x.\exists y.\,y\le x\}\vdash\exists x.\forall y.\,x\le y}\,\text{Ex I}}{\{\ \}\vdash(\forall x.\exists y.\,y\le x)\Rightarrow(\exists x.\forall y.\,x\le y)}\,\text{Imp I}$$

Example of Failure

Let’s try to show 10

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\{\forall x.\exists y.\,y\le x\}\vdash\forall x.\exists y.\,y\le x}\,\text{Hyp}\qquad \dfrac{\dfrac{}{\mathit{Something}}\quad \dfrac{}{\left\{\begin{matrix}\mathit{Lab}\ \forall x.\exists y.\,y\le x;\\ \exists y.\,y\le x;\ z\le x\end{matrix}\right\}\vdash z\le x}\,\text{Hyp}}{\left\{\begin{matrix}\forall x.\exists y.\,y\le x;\\ \exists y.\,y\le x\end{matrix}\right\}\vdash z\le x}\,\text{Ex E}}{\{\forall x.\exists y.\,y\le x\}\vdash z\le x}\,\text{All E}}{\{\forall x.\exists y.\,y\le x\}\vdash\forall y.\,z\le y}\,\text{All I}}{\{\forall x.\exists y.\,y\le x\}\vdash\exists x.\forall y.\,x\le y}\,\text{Ex I}}{\{\ \}\vdash(\forall x.\exists y.\,y\le x)\Rightarrow(\exists x.\forall y.\,x\le y)}\,\text{Imp I}$$

Floyd-Hoare Logic

- Also called Axiomatic Semantics
- Based on formal logic (first order predicate calculus)
- Logical system built from axioms and inference rules
- Mainly suited to simple imperative programming languages
- Ideas applicable quite broadly

Floyd-Hoare Logic

- Used to formally prove a property (post-condition) of the state (the values of the program variables) after the execution of program, assuming another property (pre-condition) of the state holds before execution

Floyd-Hoare Logic

- Goal: Derive statements of form

$$\{P\}\ C\ \{Q\}$$

  - P, Q logical statements about state, P precondition, Q postcondition, C program
- Example:

$$\{x = 1\}\ x := x + 1\ \{x = 2\}$$

Floyd-Hoare Logic

- **Approach:** For each type of language statement, give an axiom or inference rule stating how to derive assertions of form

$$\{P\}\ C\ \{Q\}$$

  where C is a statement of that type
- Compose axioms and inference rules to build proofs for complex programs

Partial vs Total Correctness

- An expression $\{P\}\ C\ \{Q\}$ is a partial correctness statement
- For total correctness must also prove that C terminates (i.e. doesnt run forever)
  - Written: $[P]\ C\ [Q]$
- Will only consider partial correctness here

Simple Imperative Language

- We will give rules for simple imperative language

$$\begin{array}{l}\langle command\rangle\ ::=\ \langle variable\rangle := \langle term\rangle\\ \mid \langle command\rangle;\ \ldots;\ \langle command\rangle\\ \mid \textit{if}\ \langle statement\rangle\ \textit{then}\ \langle command\rangle\ \textit{else}\ \langle command\rangle\\ \mid \textit{while}\ \langle statement\rangle\ \textit{do}\ \langle command\rangle\end{array}$$

- Could add more features, like for-loops

Substitution

- Notation: $P[e/v]$ (sometimes $P[v \to e]$)
- Meaning: Replace every v in P by e
- Example:

$$(x + 2)[y - 1/x] = ((y - 1) + 2)$$

The Assignment Rule

$$\overline{\{P[e/x]\}\ x\ :=\ e\ \{P\}}$$

Example:

$$\overline{\{\quad ?\quad\}\ x\ :=\ y\ \{x\ = 2\}}$$

The Assignment Rule

$$\overline{\{P[e/x]\}\ x\ :=\ e\ \{P\}}$$

Example:

$$\overline{\{\boxed{} = 2\}\ x\ :=\ y\ \{\boxed{x} = 2\}}$$

The Assignment Rule

$$\overline{\{P[e/x]\}\ x\ :=\ e\ \{P\}}$$

Example:

$$\overline{\{\boxed{x} = 2\}\ x\ :=\ y\ \{\boxed{x} = 2\}}$$

The Assignment Rule

$$\overline{\{P[e/x]\}\ x\ :=\ e\ \{P\}}$$

Examples:

$$\overline{\{y = 2\}\ x\ :=\ y\ \{x = 2\}}$$

$$\overline{\{y = 2\}\ x\ :=\ 2\ \{y = x\}}$$

$$\overline{\{x + 1 = n + 1\}\ x\ :=\ x + 1\ \{x = n + 1\}}$$

$$\overline{\{2 = 2\}\ x\ :=\ 2\ \{x = 2\}}$$

The Assignment Rule – Your Turn

- What is the weakest precondition of
  $x\ :=\ x + y\ \{x + y = wx\}$?

$$\{\qquad ?\qquad\}$$
$$x\ :=\ x + y$$
$$\{x + y = wx\}$$

The Assignment Rule – Your Turn

- What is the weakest precondition of
  $x\ :=\ x + y\ \{x + y = wx\}$?

$$\{\ (x + y) + y = w(x + y)\ \}$$
$$x\ :=\ x + y$$
$$\{x + y = wx\}$$

Precondition Strengthening

$$\frac{(P \Rightarrow P')\,\{P'\}\ C\ \{Q\}}{\{P\}\ C\ \{Q\}}$$

- Meaning: If we can show that P implies P' (*i.e.* $(P \Rightarrow P')$ and we can show that $\{P\}\ C\ \{Q\}$, then we know that $\{P\}\ C\ \{Q\}$
- P is stronger than P' means $P \Rightarrow P'$

Precondition Strengthening

- Examples:

$$\frac{x = 3 \Rightarrow x < 7 \quad \{x < 7\}\ x\ :=\ x + 3\ \{x < 10\}}{\{x = 3\}\ x\ :=\ x + 3\ \{x < 10\}}$$

$$\frac{True \Rightarrow (2 = 2) \quad \{2 = 2\}\ x\ :=\ 2\ \{x = 2\}}{\{True\}\ x\ :=\ 2\ \{x = 2\}}$$

$$\frac{x = n \Rightarrow x + 1 = n + 1 \quad \{x + 1 = n + 1\}\ x\ :=\ x + 1\ \{x = n + 1\}}{\{x = n\}\ x\ :=\ x + 1\ \{x = n + 1\}}$$

Which Inferences Are Correct?

$$\frac{\{x > 0 \wedge x < 5\}\ x\ :=\ x * x\ \{x < 25\}}{\{x = 3\}\ x\ :=\ x * x\ \{x < 25\}}$$

$$\frac{\{x = 3\}\ x\ :=\ x * x\ \{x < 25\}}{\{x > 0 \wedge x < 5\}\ x\ :=\ x * x\ \{x < 25\}}$$

$$\frac{\{x * x < 25\}\ x\ :=\ x * x\ \{x < 25\}}{\{x > 0 \wedge x < 5\}\ x\ :=\ x * x\ \{x < 25\}}$$

Which Inferences Are Correct?

$$\frac{\{x > 0 \wedge x < 5\}\ x\ :=\ x * x\ \{x < 25\}}{\{x = 3\}\ x\ :=\ x * x\ \{x < 25\}}\ \textit{YES}$$

$$\frac{\{x = 3\}\ x\ :=\ x * x\ \{x < 25\}}{\{x > 0 \wedge x < 5\}\ x\ :=\ x * x\ \{x < 25\}}$$

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Which Inferences Are Correct?

$$\frac{\{x > 0 \wedge x < 5\}\ x\ :=\ x * x\ \{x < 25\}}{\{x = 3\}\ x\ :=\ x * x\ \{x < 25\}}\ \textit{YES}$$

$$\frac{\{x = 3\}\ x\ :=\ x * x\ \{x < 25\}}{\{x > 0 \wedge x < 5\}\ x\ :=\ x * x\ \{x < 25\}}\ \textit{NO}$$

$$\frac{\{x * x < 25\}\ x\ :=\ x * x\ \{x < 25\}}{\{x > 0 \wedge x < 5\}\ x\ :=\ x * x\ \{x < 25\}}$$

Which Inferences Are Correct?

$$\frac{\{x > 0 \wedge x < 5\}\ x\ :=\ x * x\ \{x < 25\}}{\{x = 3\}\ x\ :=\ x * x\ \{x < 25\}}\ \textit{YES}$$

$$\frac{\{x = 3\}\ x\ :=\ x * x\ \{x < 25\}}{\{x > 0 \wedge x < 5\}\ x\ :=\ x * x\ \{x < 25\}}\ \textit{NO}$$

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