CS477 Formal Software Development Methods

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How to build a Reduced Ordered BDD

- Let u, l, h be bdd nodes, i be a variable index
  - var(u) gives the variable at u
  - low(u) gives node pointed to by false out-edge of u
  - high(u) gives node pointed to by true out-edge of u
- Assume to tables: T and H
- T : u -> (i,l,h)
  - init T, update (T,(u,(i,l,h))),
- H : (i,l,h) -> u - Inverse of T
  - init H, member(H,(i,l,h)), lookup(H,(i,l,h)), insert (H,((i,l,h),u)

How to build a Reduced Ordered BDD

- For efficiency, use memoizing function
- function Mk[T,H](i,l,h) =

```
        if l = h then return l
        else if member (H,(i,l,h))
                then return (lookup (H,(i,l,h)))
       else update (T,(u,(i,l,h)));
                insert(H,((i,l,h),u);
                return u
    end
```

How to build a Reduced Ordered BDD

- Main function for building ROBDD
- p proposition; $x_1 > \ldots > x_n$ variables in p
- function Build[T,H](p,i) =

```
    if i = 0 then if p = False then False else True
    else let l = Build[T,H](p[False/x_i],(i-1)) in
         let h = Build[T,H](p[True/x_i],(oi-1)) in
         let u = Mk[T,H] (i,l,h) in
        return u
 end
```

(A ⋀ B) ⋁ (not C) Variables: C > B > A

(A ⋀ B) ⋁ (not C)

$(A \wedge B) \vee (\neg C)$ Variables: C > B > A

$(A \wedge B) \vee (\neg C)$

$((A \wedge B) \vee (\neg C))[True/C] = (A \wedge B)$

$((A \wedge B) \vee (\neg C))[False/C] =$ True

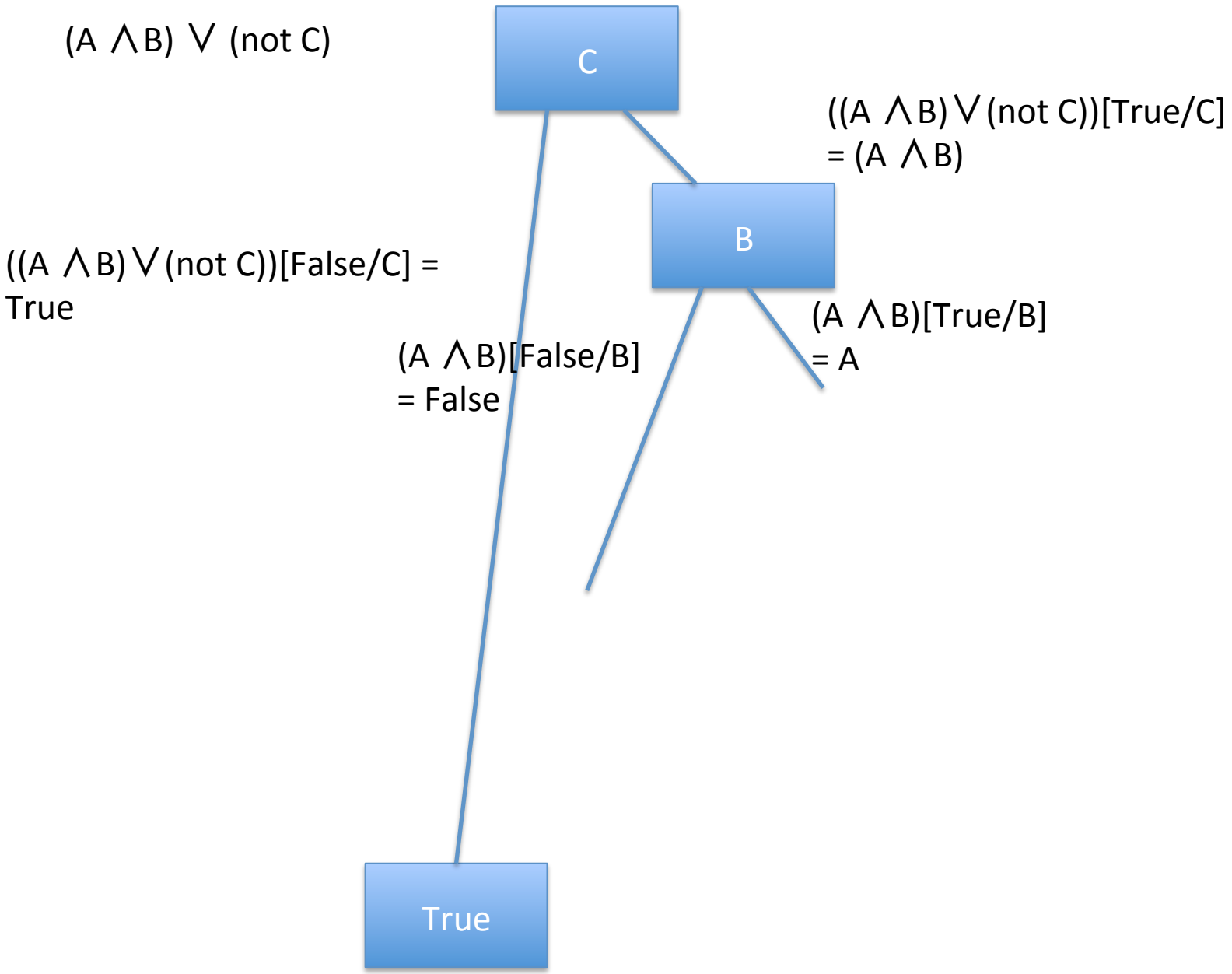
(A ∧ B) ∨ (not C) Variables: C > B > A

(A ∧ B) ∨ (not C)

((A ∧ B) ∨ (not C))[True/C] = (A ∧ B)

((A ∧ B) ∨ (not C))[False/C] = True

(A ∧ B) ∨ (not C) Variables: C > B > A



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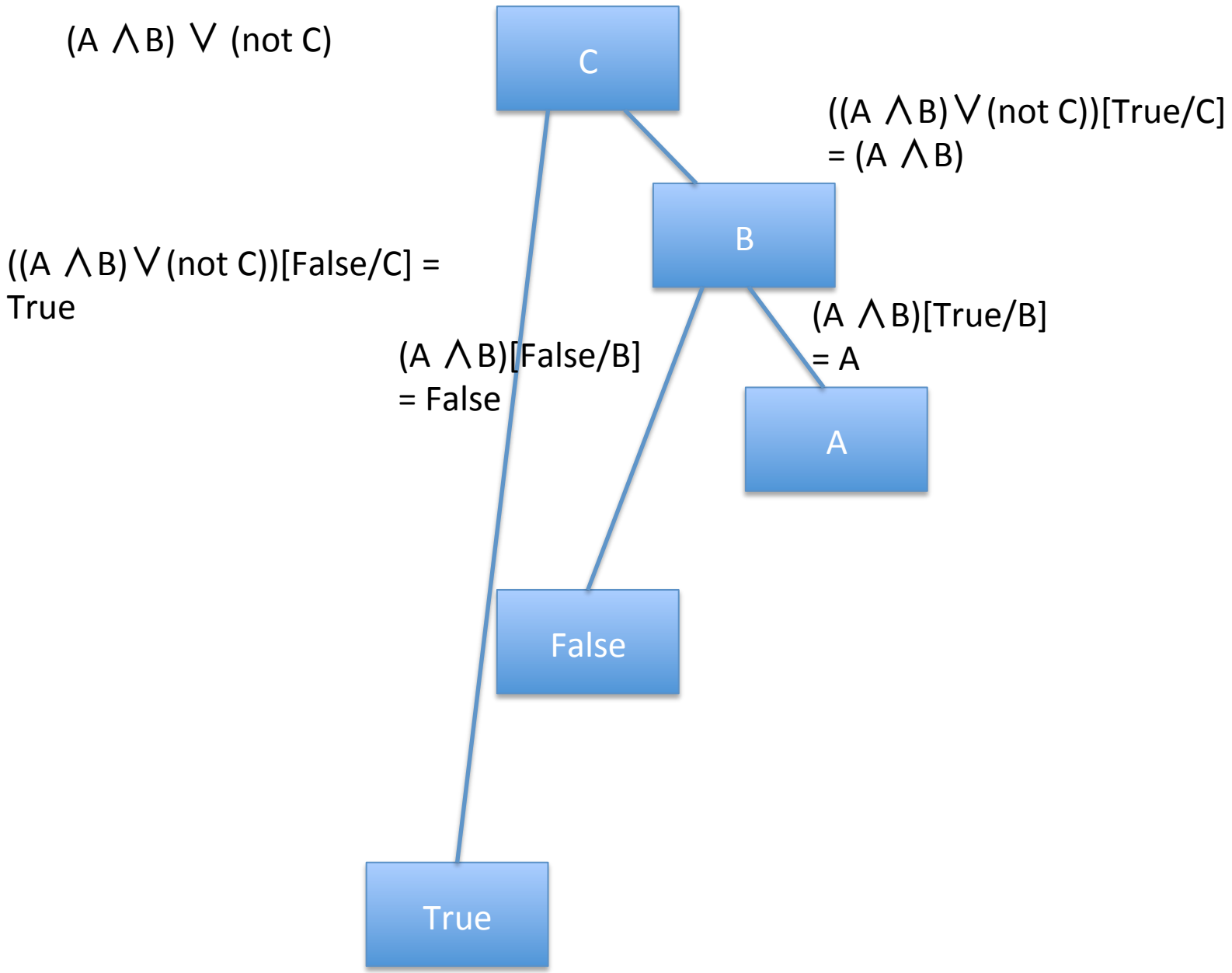
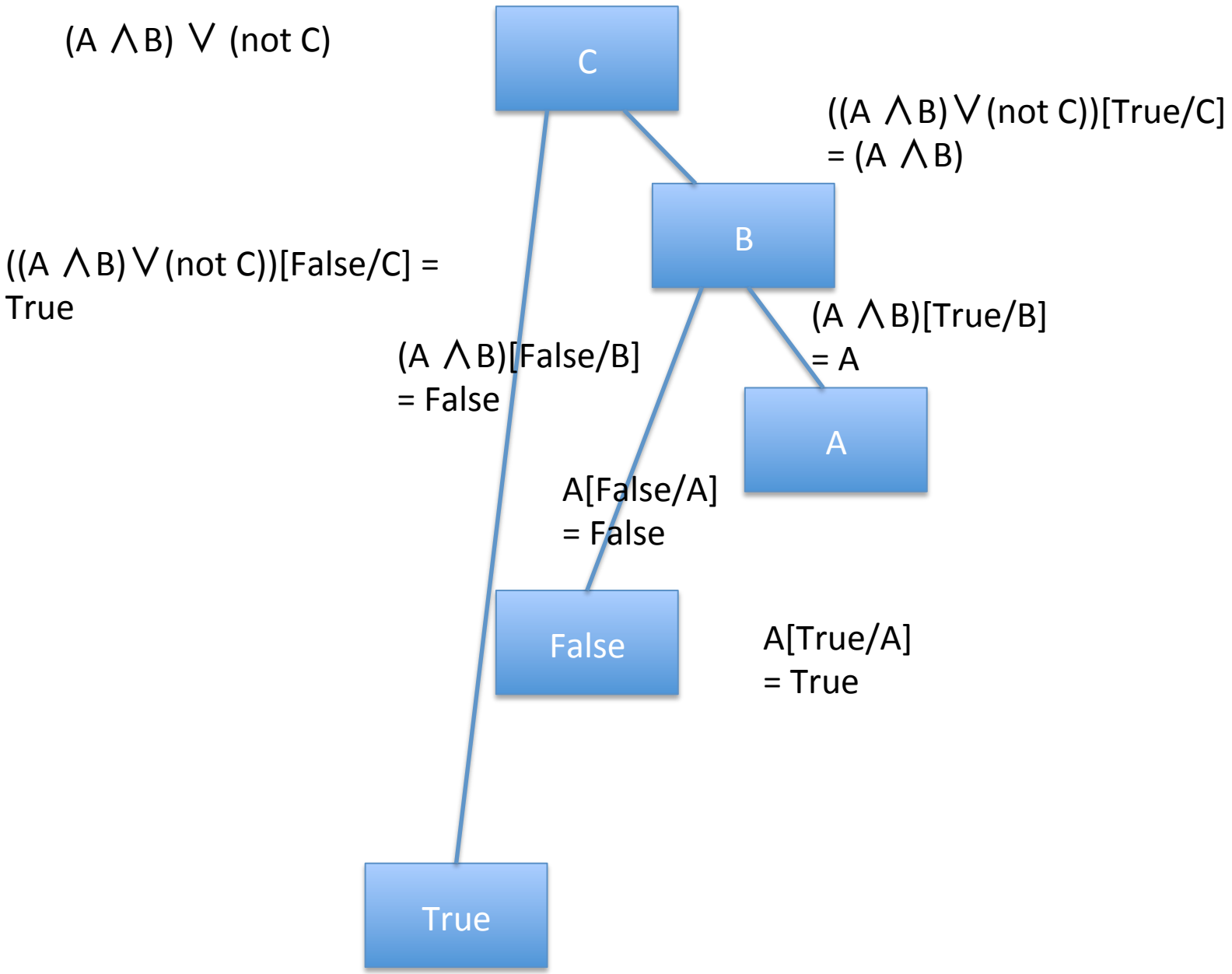


$(A \wedge B) \vee (\neg C)$ Variables: C > B > A



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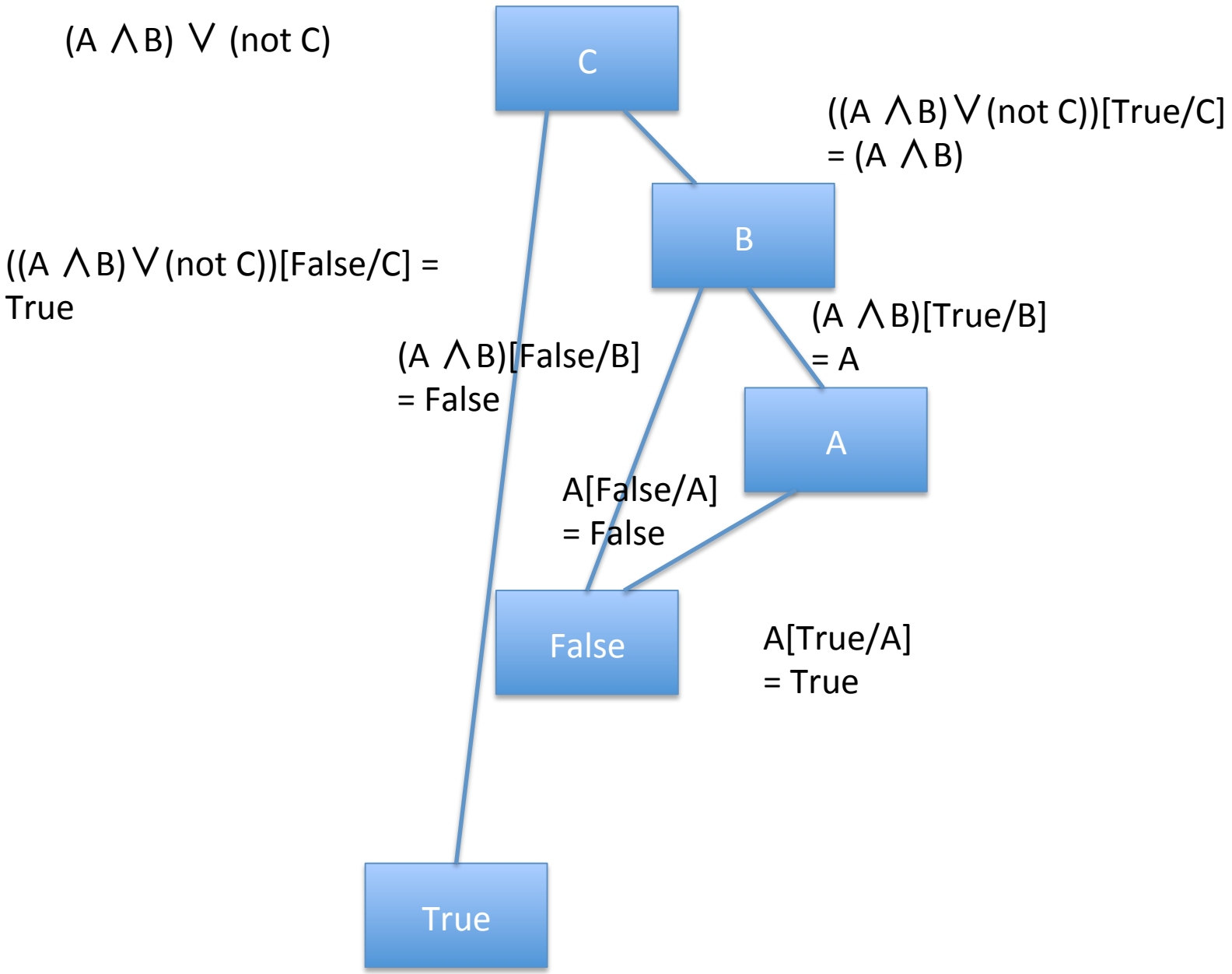

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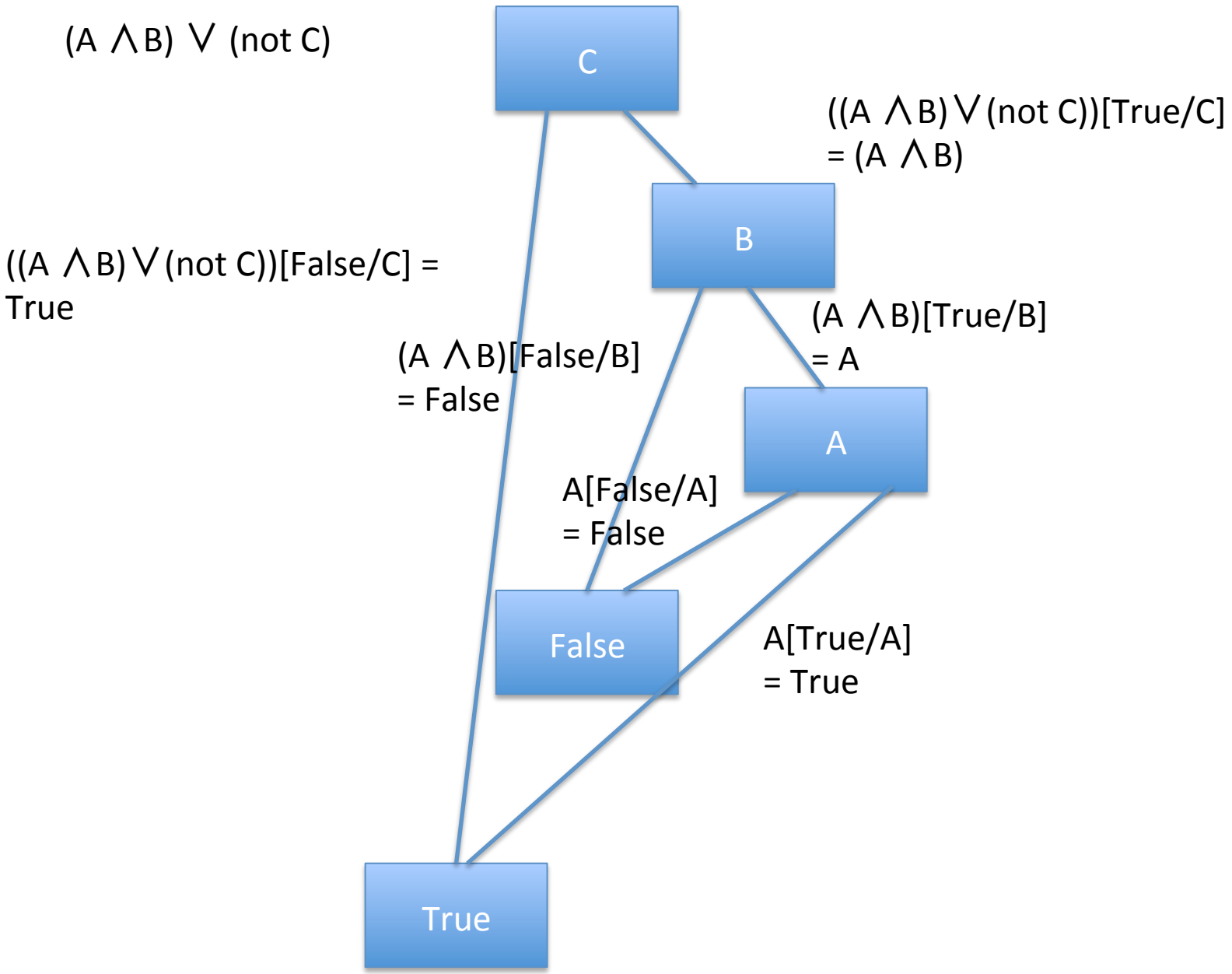

$(A \rightarrow B) \wedge ((\text{not } B) \rightarrow (\text{not } (A \wedge C)))$

$(A \rightarrow B) \wedge ((\text{not } B) \rightarrow (\text{not } (A \wedge C)))[\text{False}/C] =$
$(A \rightarrow B) \wedge ((\text{not } B) \rightarrow (\text{not } (A \wedge \text{False}))) =$
$(A \rightarrow B) \wedge ((\text{not } B) \rightarrow (\text{not False})) =$
$(A \rightarrow B) \wedge ((\text{not } B) \rightarrow \text{True}) =$
$(A \rightarrow B) \wedge \text{True} = (A \rightarrow B)$

(A ➔ B) ⋀ ((not B) ➔ (not (A ⋀ C)))

(A ➔ B) ⋀ ((not B) ➔ (not (A ⋀ C)))[False/C] =
(A ➔ B) ⋀ ((not B) ➔ (not (A ⋀ False))) =
(A ➔ B) ⋀ ((not B) ➔ (not False)) =
(A ➔ B) ⋀ ((not B) ➔ True) =
(A ➔ B) ⋀ True = (A ➔ B)

(A ➔ B) [False/B] =
(A ➔ False) = not A

(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))

(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))[False/C] =
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(A ➔ B) ∧ True = (A ➔ B)

(A ➔ B) [False/B] =
(A ➔ False) = not A

(not A)[False/A] = True
(not A)[True/A] = False

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$(A \rightarrow B) \wedge ((\text{not } B) \rightarrow (\text{not } (A \wedge C)))[\text{False}/C] =$
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$(A \rightarrow B) \wedge ((\text{not } B) \rightarrow \text{True}) =$
$(A \rightarrow B) \wedge \text{True} = (A \rightarrow B)$

$(A \rightarrow B)[\text{False}/B] =$
$(A \rightarrow \text{False}) = \text{not } A$

$(A \rightarrow B)[\text{True}/B] =$
$(A \rightarrow \text{True}) = \text{True}$

$(\text{not } A)[\text{False}/A] = \text{True}$
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$(A \rightarrow B) \wedge ((\text{not } B) \rightarrow (\text{not } (A \wedge C)))$

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$(A \rightarrow B) \wedge ((\text{not } B) \rightarrow (\text{not } (A \wedge C)))[\text{True}/C]$
$= (A \rightarrow B) \wedge ((\text{not } B) \rightarrow (\text{not } (A \wedge \text{True}))) =$
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$(\text{not } A)[\text{False}/A] = \text{True}$
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(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))

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(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))[True/C]
= (A ➔ B) ∧ ((not B) ➔ (not (A ∧True))) =
(A ➔ B) ∧ ((not B) ➔ (not A))

(A ➔ B) [False/B] =
(A ➔ False) = not A

(A ➔ B) [True/B] =
(A ➔ True) = True

(A ➔ B) ∧ ((not B) ➔ (not A)) [False / B]=
(A ➔ False) ∧ ((not False) ➔ (not A)) =
(not A) ∧ ((not True) ➔ (not A)) =
(not A) ∧ (False ➔ (not A)) = not A

(not A)[False/A] = True
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(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))

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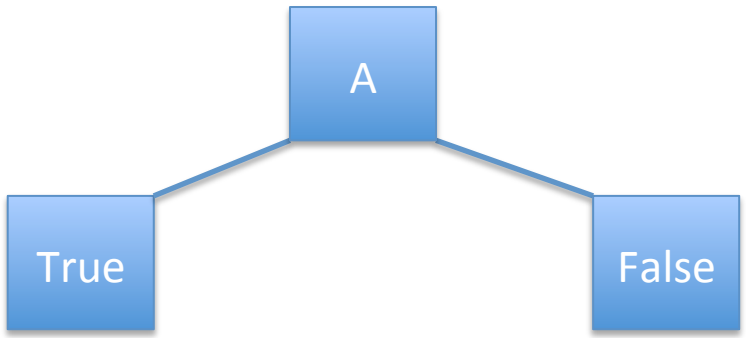

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(not A) ∧ (not A)

(A ➔ B) ∧ ((not B) ➔
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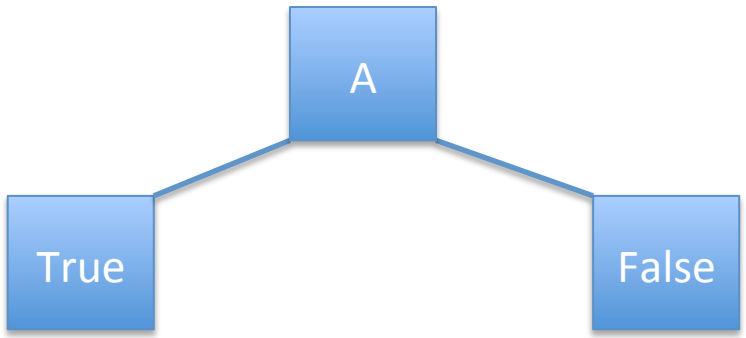

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(A ➔ B) ∧ ((not B) ➔
(not A)) [True / B]=
(A ➔ True) ∧
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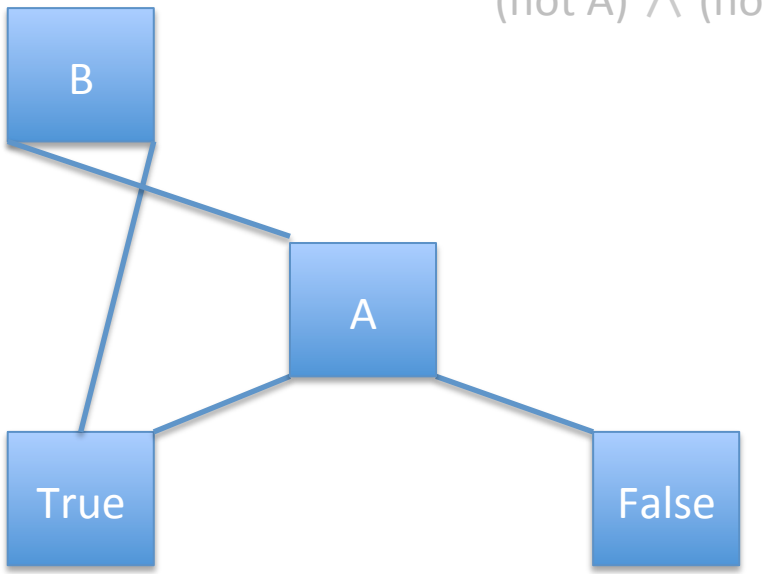

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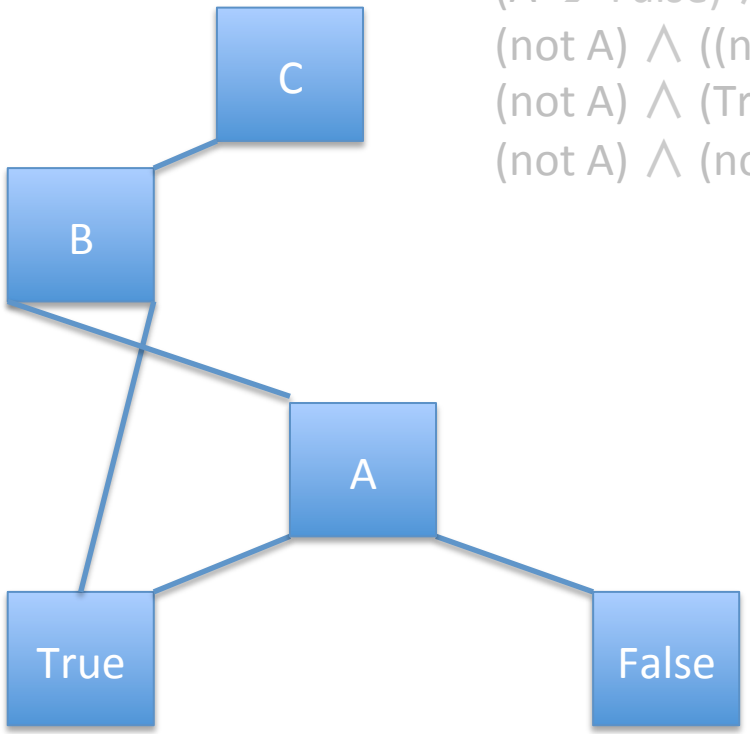

(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))[True/C]
= (A ➔ B) ∧ ((not B) ➔ (not (A ∧ True))) =
(A ➔ B) ∧ ((not B) ➔ (not A))

(A ➔ B) ∧ ((not B) ➔ (not A)) [False / B]=
(A ➔ False) ∧ ((not False) ➔ (not A)) =
(not A) ∧ ((not False) ➔ (not A)) =
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(not A) ∧ (not A)

(A ➔ B) ∧ ((not B) ➔
(not A)) [True / B]=
(A ➔ True) ∧
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True ∧ (False ➔ (not A)) =
True

(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))

(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))[False/C] =
(A ➔ B) ∧ ((not B) ➔ (not (A ∧ False))) =
(A ➔ B) ∧ ((not B) ➔ (not False)) =
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(A ➔ B) ∧ True = (A ➔ B)

(A ➔ B) [False/B] =
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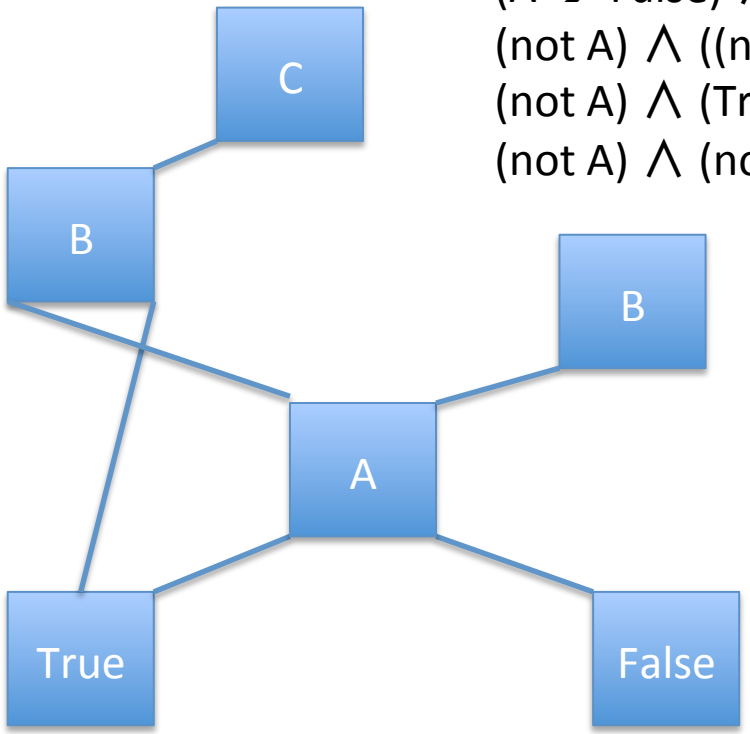

(A ➔ B) ∧ ((not B) ➔ (not A)) [False / B]=
(A ➔ False) ∧ ((not False) ➔ (not A)) =
(not A) ∧ ((not False) ➔ (not A)) =
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(A ➔ True) ∧ ((not True) ➔ (not A)) =
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(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))[True/C]
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(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))

(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))[False/C] =
(A ➔ B) ∧ ((not B) ➔ (not (A ∧ False))) =
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(A ➔ B) [True/B] =
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(A ➔ B) [False/B] =
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(not A)[False/A] = True
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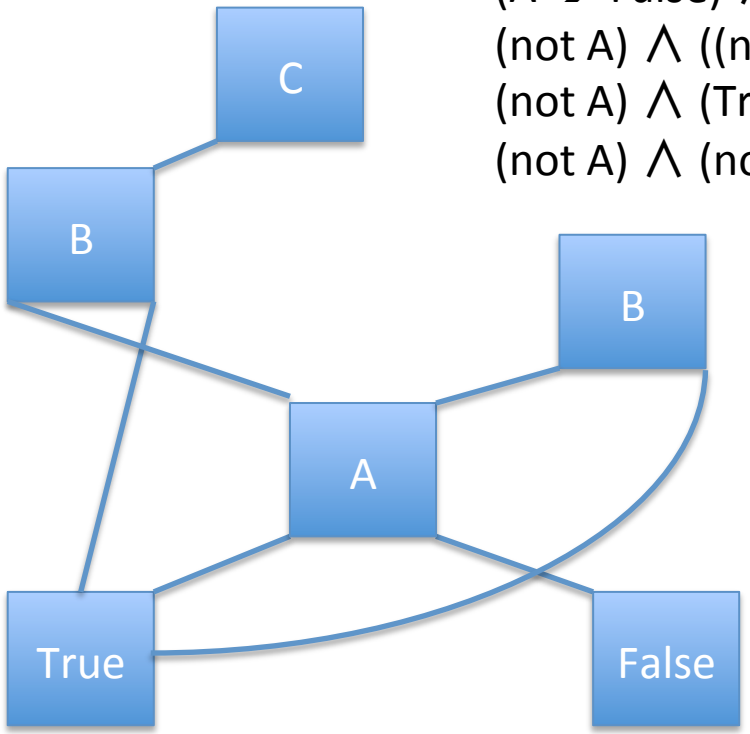

(A ➔ B) ∧ ((not B) ➔ (not A)) [False / B]=
(A ➔ False) ∧ ((not False) ➔ (not A)) =
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(A ➔ B) ∧ ((not B) ➔
(not A)) [True / B]=
(A ➔ True) ∧
((not True) ➔ (not A)) =
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(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))[True/C]
= (A ➔ B) ∧ ((not B) ➔ (not (A ∧True))) =
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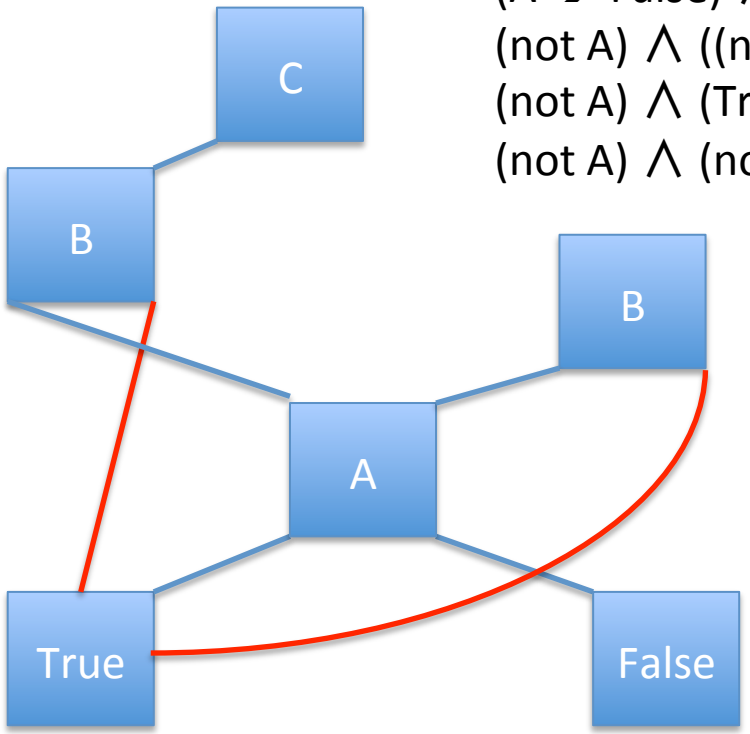
(not A)[False/A] = True
(not A)[True/A] = False



(A ➔ B) ∧ ((not B) ➔ (not A)) [False / B]=
(A ➔ False) ∧ ((not False) ➔ (not A)) =
(not A) ∧ ((not False) ➔ (not A)) =
(not A) ∧ (True ➔ (not A)) =
(not A) ∧ (not A)

(A ➔ B) ∧ ((not B) ➔
(not A)) [True / B]=
(A ➔ True) ∧
((not True) ➔ (not A)) =
True ∧ (False ➔(not A)) =
True

(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))

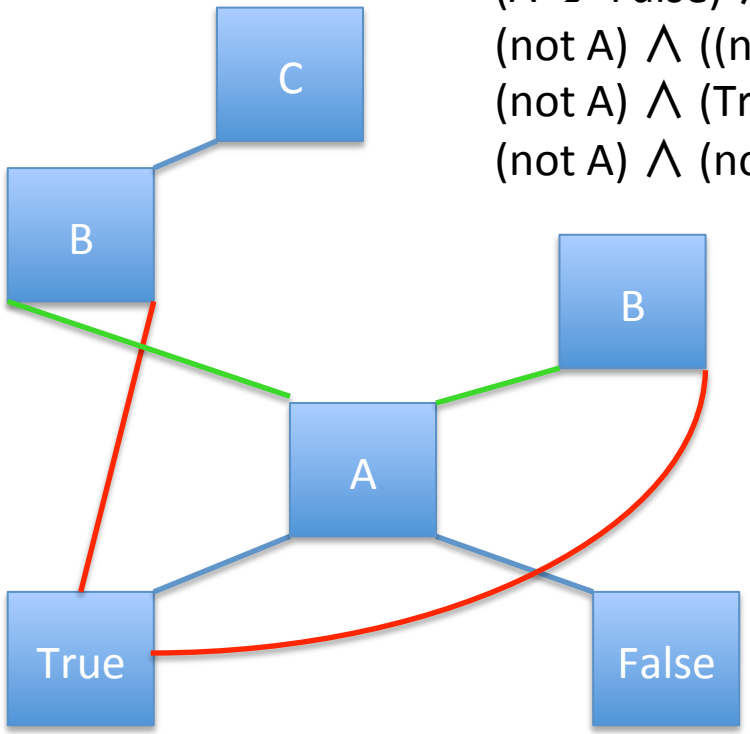
(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))[False/C] =
(A ➔ B) ∧ ((not B) ➔ (not (A ∧ False))) =
(A ➔ B) ∧ ((not B) ➔ (not False)) =
(A ➔ B) ∧ ((not B) ➔ True) =
(A ➔ B) ∧ True = (A ➔ B)

(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))[True/C]
= (A ➔ B) ∧ ((not B) ➔ (not (A ∧True))) =
(A ➔ B) ∧ ((not B) ➔ (not A))

(A ➔ B) [False/B] =
(A ➔ False) = not A

(A ➔ B) [True/B] =
(A ➔ True) = True

(not A)[False/A] = True
(not A)[True/A] = False



(A ➔ B) ∧ ((not B) ➔ (not A)) [False / B]=
(A ➔ False) ∧ ((not False) ➔ (not A)) =
(not A) ∧ ((not False) ➔ (not A)) =
(not A) ∧ (True ➔ (not A)) =
(not A) ∧ (not A)

(A ➔ B) ∧ ((not B) ➔
(not A)) [True / B]=
(A ➔ True) ∧
((not True) ➔ (not A)) =
True ∧ (False ➔(not A)) =
True

(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))

(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))[False/C] =
(A ➔ B) ∧ ((not B) ➔ (not (A ∧ False))) =
(A ➔ B) ∧ ((not B) ➔ (not False)) =
(A ➔ B) ∧ ((not B) ➔ True) =
(A ➔ B) ∧ True = (A ➔ B)

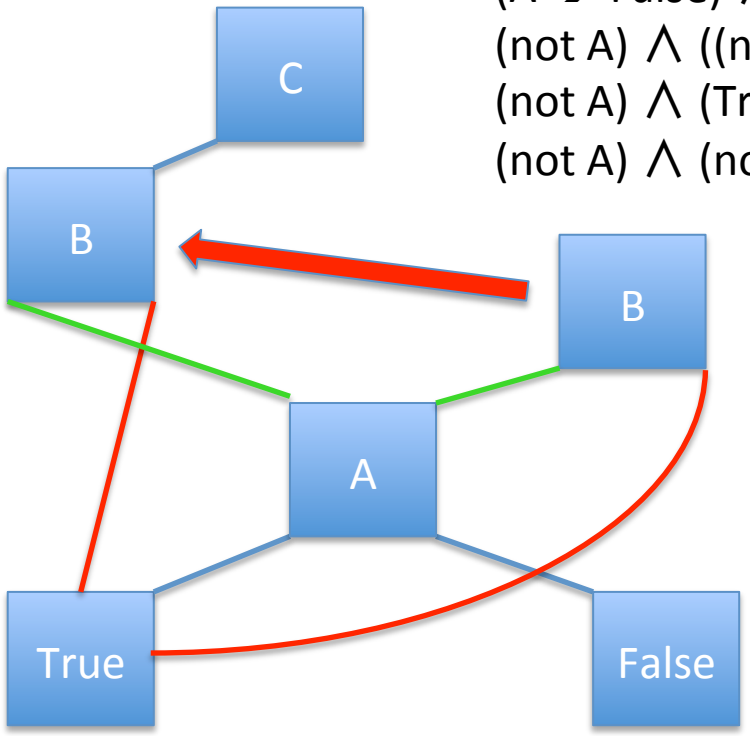
(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))[True/C]
= (A ➔ B) ∧ ((not B) ➔ (not (A ∧True))) =
(A ➔ B) ∧ ((not B) ➔ (not A))

(A ➔ B) [False/B] =
(A ➔ False) = not A

(A ➔ B) [True/B] =
(A ➔ True) = True

(not A)[False/A] = True
(not A)[True/A] = False

(A ➔ B) ∧ ((not B) ➔ (not A)) [False / B]=
(A ➔ False) ∧ ((not False) ➔ (not A)) =
(not A) ∧ ((not False) ➔ (not A)) =
(not A) ∧ (True ➔ (not A)) =
(not A) ∧ (not A)



(A ➔ B) ∧ ((not B) ➔
(not A)) [True / B]=
(A ➔ True) ∧
((not True) ➔ (not A)) =
True ∧ (False ➔(not A)) =
True

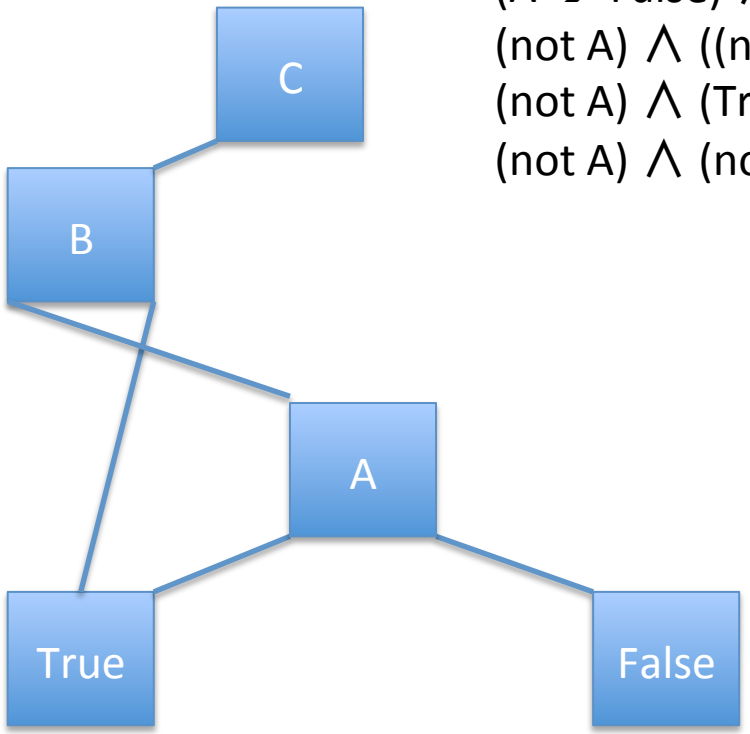
(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))

(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))[False/C] =
(A ➔ B) ∧ ((not B) ➔ (not (A ∧ False))) =
(A ➔ B) ∧ ((not B) ➔ (not False)) =
(A ➔ B) ∧ ((not B) ➔ True) =
(A ➔ B) ∧ True = (A ➔ B)

(A ➔ B) [False/B] =
(A ➔ False) = not A

(A ➔ B) [True/B] =
(A ➔ True) = True

(not A)[False/A] = True
(not A)[True/A] = False



(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))[True/C]
= (A ➔ B) ∧ ((not B) ➔ (not (A ∧ True))) =
(A ➔ B) ∧ ((not B) ➔ (not A))

(A ➔ B) ∧ ((not B) ➔ (not A)) [False / B]=
(A ➔ False) ∧ ((not False) ➔ (not A)) =
(not A) ∧ ((not False) ➔ (not A)) =
(not A) ∧ (True ➔ (not A)) =
(not A) ∧ (not A)

(A ➔ B) ∧ ((not B) ➔
(not A)) [True / B]=
(A ➔ True) ∧
((not True) ➔ (not A)) =
True ∧ (False ➔(not A)) =
True

(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))

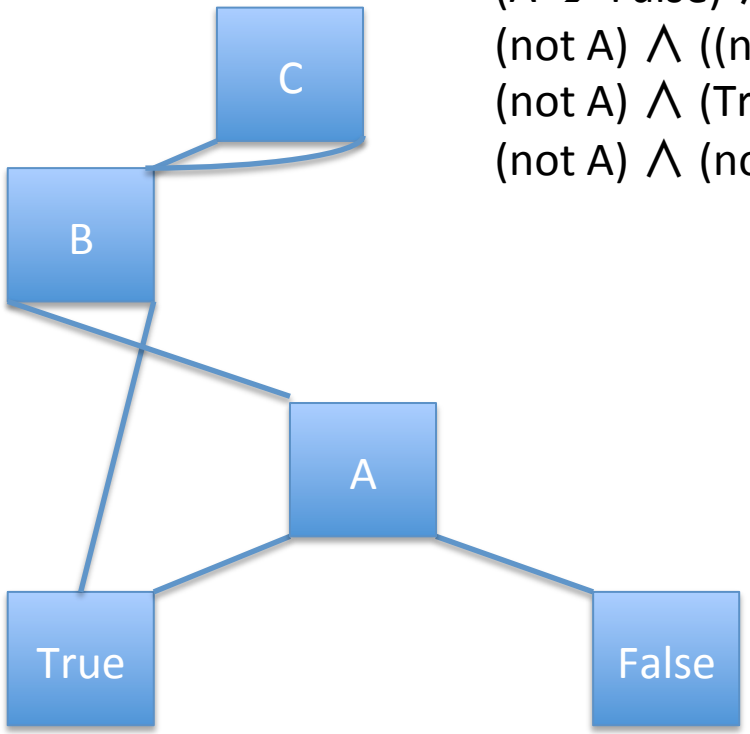
(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))[False/C] =
(A ➔ B) ∧ ((not B) ➔ (not (A ∧ False))) =
(A ➔ B) ∧ ((not B) ➔ (not False)) =
(A ➔ B) ∧ ((not B) ➔ True) =
(A ➔ B) ∧ True = (A ➔ B)

(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))[True/C]
= (A ➔ B) ∧ ((not B) ➔ (not (A ∧True))) =
(A ➔ B) ∧ ((not B) ➔ (not A))

(A ➔ B) [False/B] =
(A ➔ False) = not A

(A ➔ B) [True/B] =
(A ➔ True) = True

(not A)[False/A] = True
(not A)[True/A] = False



(A ➔ B) ∧ ((not B) ➔ (not A)) [False / B]=
(A ➔ False) ∧ ((not False) ➔ (not A)) =
(not A) ∧ ((not False) ➔ (not A)) =
(not A) ∧ (True ➔ (not A)) =
(not A) ∧ (not A)

(A ➔ B) ∧ ((not B) ➔
(not A)) [True / B]=
(A ➔ True) ∧
((not True) ➔ (not A)) =
True ∧ (False ➔(not A)) =
True

(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))

(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))[False/C] =
(A ➔ B) ∧ ((not B) ➔ (not (A ∧ False))) =
(A ➔ B) ∧ ((not B) ➔ (not False)) =
(A ➔ B) ∧ ((not B) ➔ True) =
(A ➔ B) ∧ True = (A ➔ B)

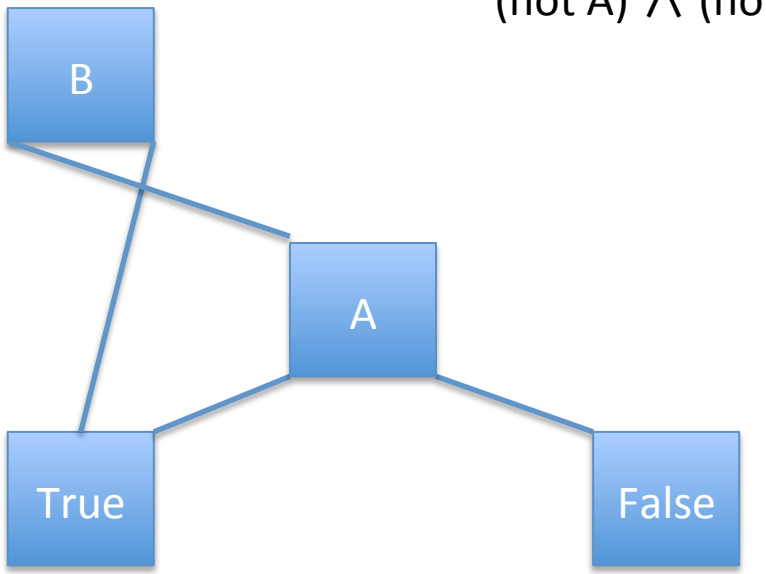
(A ➔ B) ∧ ((not B) ➔ (not (A ∧ C)))[True/C]
= (A ➔ B) ∧ ((not B) ➔ (not (A ∧True))) =
(A ➔ B) ∧ ((not B) ➔ (not A))

(A ➔ B) [False/B] =
(A ➔ False) = not A

(A ➔ B) [True/B] =
(A ➔ True) = True

(A ➔ B) ∧ ((not B) ➔ (not A)) [False / B]=
(A ➔ False) ∧ ((not False) ➔ (not A)) =
(not A) ∧ ((not False) ➔ (not A)) =
(not A) ∧ (True ➔ (not A)) =
(not A) ∧ (not A)

(not A)[False/A] = True
(not A)[True/A] = False



(A ➔ B) ∧ ((not B) ➔
(not A)) [True / B]=
(A ➔ True) ∧
((not True) ➔ (not A)) =
True ∧ (False ➔(not A)) =
True