# Program Verification: Lecture 19

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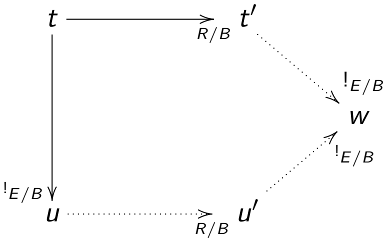

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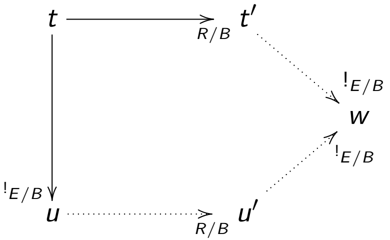


Maude's Coherence Checker tool checks this property.

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That is, the states reachable from state $[u]$ by a $\rightarrow_{\mathbb{C}_{\mathcal{R}}}$-transition are the normal forms of its 1-step $\rightarrow_{R/B}$-rewrites from $[u]$.

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Following Saul Kripke, analytic philosophers model this with a so-called possible worlds semantics.

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# Reachable Sets of States

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# An Expressive Language $\Pi$ for Modal Properties of $\mathbb{C}_{\mathcal{R}}$

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What should we choose as our property names $p \in \Pi$ for $\mathbb{C}_{\mathcal{R}}$? An expressive property language already available to Maude users allows defining properties with constrained constructor patterns of the form $u|\varphi$, with $u$ a constructor $\Omega$-term with variables $\vec{x}$ of sort $St$, and $\varphi$ a conjunction of $\Sigma$-equalities[1] on variables $\vec{x}$. The meaning function $\_{\mathbb{C}_{\mathcal{R}}}$ has the form: $\_{\mathbb{C}_{\mathcal{R}}} : (u|\varphi) \mapsto [\![u \mid \varphi]\!]$, where, by definition, $[\![u \mid \varphi]\!]$ is the computable subset:

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We can apply all this to verify invariants $\mathcal{R}, init \models \Box p^{Q}$ (resp. reachable sets, $\mathcal{R}, init \models \Diamond p^{Q}$) by verifying through explicit-state model checking that $\mathcal{R}, init \not\models \Diamond \neg p^{Q}$ (resp. $\mathcal{R}, init \models \Diamond \neg p^{Q}$) using Maude's `search` command.

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Let us illustrate this explicit-state model checking method with `QLOCK`, a mutual exclusion protocol proposed by K. Futatsugi, where the number of processes is unbounded.

# The QLOCK Mutual Exclusion Protocol

```
mod QLOCK is protecting NAT .
 sorts NatMSet NatList State .
 subsorts Nat < NatMSet NatList .
 op mt : -> NatMSet [ctor] .
 op _ _ : NatMSet NatMSet -> NatMSet [ctor assoc comm id: mt] .
 op nil : -> NatList [ctor] .
 op _;_ : NatList NatList -> NatList [ctor assoc id: nil] .
 op {_<_|_|_|_>} : NatMSet NatMSet NatMSet NatMSet NatList -> State [ctor] .
  op [_] : Nat -> NatMSet .  *** set of first n numbers
 op init : Nat -> State .   *** initial state, parametric on n

 vars n i j : Nat .  vars S U W C : NatMSet .  var Q : NatList .
 eq [0] = mt .
 eq [s(n)] = n [n] .
 eq init(n) = {[n] < mt | mt | mt | nil >} .

 rl [join] : {S i < U | W | C | Q >} => {S < U i | W | C | Q >} .
 rl [n2w] : {S < U i | W | C | Q >} => {S < U | W i | C | Q ; i >} .
 rl [w2c] : {S < U | W i | C | i ; Q >} => {S < U | W | C i | i ; Q >} .
 rl [c2n] : {S < U | W | C i | i ; Q >} => {S < U i | W | C | Q >} .
 rl [exit] : {S < U i | W | C | Q >} => {S i < U | W | C | Q >} .
endm
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Consider the following specification of a readers-writers system.

# Bounded Model Checking of Invariants (III)

```
mod R&W is
  protecting NAT .
  sort Config .
  op <_,_> : Nat Nat -> Config [ctor] .   --- readers/writers

  vars R W : Nat .

  rl < 0, 0 > => < 0, s(0) > .
  rl < R, s(W) > => < R, W > .
  rl < R, 0 > => < s(R), 0 > .
  rl < s(R), W > => < R, W > .
endm
```

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A state is represented by a tuple `< R, W >` indicating the number `R` of readers and the number `W` of writers accessing a critical resource.

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However, since the number of readers can grow unboundedly, Maude's `search` commands to find counterexamples instantiating either of these two patterns from `< 0, 0 >` search forever.

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No solution.
states: 1000002  rewrites: 2000001 in 36480ms cpu (50317ms real)

Maude> search [1, 1000000] < 0,0 > =>* < N:Nat, s(s(M:Nat)) > .
No solution.
states: 1000002  rewrites: 2000001 in 38910ms cpu (41650ms real)

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No solution.
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Thus verifying these three invariants up to depth $10^6$.