

# CS 476 Homework #11 Due 10:45am on 11/7

**Note:** Answers to the exercises listed below and all Maude code as well as screenshots of tool interactions should be emailed to `clarage2@illinois.edu`.

1. Solve **Ex.21.1** in pg. 11 of Lecture 21. To avoid any ambiguities, the Lifting Lemma modulo  $B$  (which you do not need to prove but can *assume* when solving **Ex.21.1**) is explicitly stated below:

**Theorem** (Lifting Lemma modulo  $B$ ). Let  $(\Sigma, B, R)$  be a rewrite theory,  $t \in T_\Sigma(X)$ , and  $\theta$  an  $R/B$ -irreducible substitution (i.e., if  $x \in \text{dom}(\theta)$ , then  $\theta(x)$  cannot be rewritten with  $R$  modulo  $B$ ). Then, for each rewrite step modulo  $B$ ,  $t\theta \rightarrow_{R/B} u$  there is a narrowing step modulo  $B$ ,  $t \rightsquigarrow_{R/B}^\alpha v$  and an  $R/B$ -irreducible substitution  $\delta$  such that  $v\delta = u$ .

Note that the above theorem extends in a straightforward manner to narrowing sequences modulo  $B$ ,

$$t \rightsquigarrow_{R/B}^{\theta_1} t_1 \dots t_n \rightsquigarrow_{R/B}^{\theta_{n+1}} t_{n+1}$$

which do indeed cover *all*  $R/B$ -rewriting computations  $t\theta \rightarrow_{R/B}^* w$  as *instances*.

2. Recall the Readers and Writers mutual exclusion protocol in Lecture 19:

```
mod R&W is
  protecting NAT .
  sort Config .
  op <_,_> : Nat Nat -> Config [ctor] . --- readers/writers
vars R W : Nat .
  rl < 0, 0 > => < 0, s(0) > .
  rl < R, s(W) > => < R, W > .
  rl < R, 0 > => < s(R), 0 > .
  rl < s(R), W > => < R, W > .
endm
```

Prove by narrowing-based symbolic model checking the following two invariants from the initial state  $\langle 0, 0 \rangle$  which were only proved up to a  $10^6$  depth bound by explicit-state model checking in Lecture 19:

- **Mutual exclusion:** readers and writers never access the resource simultaneously: only readers or only writers can do so at any given time.
- **One writer:** at most one writer will be able to access the resource at any given time.

**Warning:** Please, do not fall into the pitfall of not giving the `[narrowing]` attribute to each of the rules in your module before giving `fvu-narrow` commands (see how this is done for the `BAKERY` protocol in pg. 14 of Lecture 21). If you forget to declare the `[narrowing]` attribute for each rule, what will happen is that *nothing will happen*, i.e., that no narrowing search will happen at all. Therefore you will get spurious `No solution` answers that do not mean anything and prove *nothing*.

**Extra Credit.** You can earn up to 50% extra credit for this problem if, using some method among those described in Appendix 3 of Lecture 21, you can also prove the following additional invariant:

- **Deadlock freedom:** there are no deadlocks.