Automata Exam Review

a. Prove $\{a^x \mid x \text{ is prime}\}$ is nonregular.

b. (Sipser 1.70) We define the avoids operation for languages A and B to be

A *avoids* $B = \{w \mid w \in A \text{ and } w \text{ doesn't contain any string in } B \text{ as a substring}\}$

Prove that the class of regular languages is closed under the avoids operation.

c. (Sipser 2.25) For any language A, let $SUFFIX(A) = \{v \mid uv \in A \text{ for some string } u\}$. Show that the class of context-free languages is closed under the $SUFFIX$ operation.

d. (based on Sipser 3.11) A **Turing machine with doubly infinite tape** is similar to an ordinary Turing machine, but its tape is infinite to the left as well as to the right. The tape is initially filled with blanks except for the portion that contains the input. Computation is defined as usual except that the head never encounters an end to the tape as it moves leftward. Show that this type of Turing machine can be simulated by a normal TM.

e. Give a CFG for this language:

$\{w \mid \text{ in every prefix of } w \text{ the number of } a\text{'s is at least the number of } b\text{'s}\}$

Can you find an *unambiguous* grammar?