Turing Machines

a. Consider the Turing Machine

$$(\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_{acc}, q_{rej}\}, \{a, b\}, \{a, b, X, \llcorner\!\lrcorner\}, \delta, q_0, q_{acc}, q_{rej})$$

where δ is defined as:

(q,s)	$\delta((q,s))$	(q,s)	$\delta((q,s))$	(q,s)	$\delta((q,s))$	(q,s)	$\delta((q,s))$
(q_0, a)	$(q_1, \llcorner\!\lrcorner, R)$	(q_2, a)	$(q_{rej}, \llcorner\!\lrcorner, R)$	(q_4, a)	(q_4, a, L)	(q_6, a)	$(q_{rej}, \llcorner\!\lrcorner, R)$
(q_0, b)	$(q_{rej}, \llcorner\!\lrcorner, R)$	(q_2, b)	(q_3, b, L)	(q_4, b)	$(q_{rej}, \llcorner\!\lrcorner, R)$	(q_6, b)	$(q_{rej}, \llcorner\!\lrcorner, R)$
(q_0, X)	$(q_{rej}, \llcorner\!\lrcorner, R)$	(q_2, X)	$(q_{rej}, \llcorner\!\lrcorner, R)$	(q_4, X)	(q_5, X, R)	(q_6, X)	(q_6, X, L)
$(q_0, \llcorner\!\lrcorner)$	$(q_{rej}, \llcorner\!\lrcorner, R)$	$(q_2, \llcorner\!\lrcorner)$	$(q_6, \llcorner\!\lrcorner, L)$	$(q_4, \llcorner\!\lrcorner)$	$(q_5, \llcorner\!\lrcorner, R)$	$(q_6, \llcorner\!\lrcorner)$	$(q_{acc}, \llcorner\!\lrcorner, R)$
(q_1, a)	(q_1, a, R)	(q_3, a)	(q_4, a, L)	(q_5, a)	(q_1, X, R)		
(q_1, b)	(q_2, X, R)	(q_3, b)	$(q_{rej}, \llcorner\!\lrcorner, R)$	(q_5, b)	$(q_{rej}, \llcorner\!\lrcorner, R)$		
(q_1, X)	(q_1, X, R)	(q_3, X)	(q_3, X, L)	(q_5, X)	$(q_{rej}, \llcorner\!\lrcorner, R)$		
$(q_1, \llcorner\!\lrcorner)$	$(q_{rej}, \llcorner\!\lrcorner, R)$	$(q_3, \llcorner\!\lrcorner)$	$(q_{rej}, \llcorner\!\lrcorner, R)$	$(q_5, \llcorner\!\lrcorner)$	$(q_{rej}, \llcorner\!\lrcorner, R)$		

and

$$\delta((q_{acc}, s)) = (q_{acc}, \llcorner\!\lrcorner, L) \quad \textit{and} \quad \delta((q_{rej}, s)) = (q_{rej}, \llcorner\!\lrcorner, L) \quad \text{for each } s \in \{a, b, X, \llcorner\!\lrcorner\}$$

i Draw the state diagram of this Turing machine. For clarity, in your diagram you may leave out all outgoing transitions from q_{acc}, and you can leave out q_{rej} entirely (including all transitions in and out).

(Hint: q_1, q_2, q_3, q_4, q_5 form a pentagon so draw that first.)

ii Trace through the computation of this machine on input $aabb$ by listing each configuration of the machine in turn.

iii Will this machine always halt? Why or why not?

iv What language does this machine recognize?

b. Give an implementation-level description of a TM which decides the language $\{w \mid w \text{ does not contain (exactly) twice as many 0s as 1s}\}$.

c. (Sipser Problem 3.22) Let A be the language containing only the single string s, where

$$s = \begin{cases} 0 & \text{if life never will be found on Mars.} \\ 1 & \text{if life will be found on Mars someday.} \end{cases}$$

Is A decidable? Why or why not? For the purposes of this problem, assume that the question of whether life will be found on Mars has an unambiguous yes or no answer.

d. Is the class of decidable languages closed under complement?

e. Give an implementation-level description of how to duplicate a string on the tape, i.e. if the tape starts with w then it should end with ww. *(You are not describing a complete machine here - it does not accept or reject at the end. The point is to have a useful subroutine to call upon in later problems.)*

f. Is the class of decidable languages closed under union? intersection? Give implementation-level descriptions for any TMs you need for this problem, except that you can use the command "do whatever machine M would do" or minor variants thereof (which would usually be too high level to really count as "implementation-level").

g. There exist languages which are recognizable but not decidable. (We will prove this later.) Do you think the class of recognizable languages is closed under complement? (We haven't developed all the tools to fully justify an answer yet; just give your intuitive reasoning.)