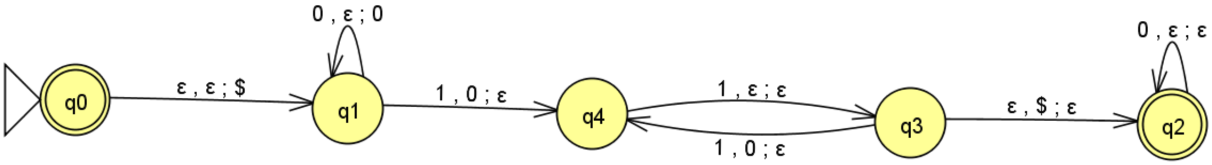
Pushdown Automata and Context-Free Languages

1. Pushdown Automata

a. Consider the following PDA:



Describe (in set-builder notation) the language it recognizes.

b. (Sipser Problem 2.47a) Let $\Sigma = \{0, 1\}$, and let B be the collection of strings that contain at least one 1 in their second half. In other words, $B = \{uv \mid u \in \Sigma^*, v \in \Sigma^*1\Sigma^*, \text{ and } |u| \geq |v|\}$. Give a PDA that recognizes B.

2. Context-Free Languages

a. Prove that every regular language is context-free by showing how to convert a DFA or NFA (your choice) to a PDA. (Use their formal definitions.)

b. Show that the class of context-free languages is closed under union and concatenation. That is, show that if L_1 and L_2 are CFLs, then so are $L_1 \cup L_2$ and $\{uv \mid u \in L_1, v \in L_2\}$.

3. Non-context-free Languages

[Theorem 2.34 in Sipser] If A is a context-free language, then there is a number p (the pumping length) where, if s is any string in A of length at least p, then s may be divided into five pieces, $s = uvxyz$ satisfying the conditions

1. for each $i \geq 0, uv^ixy^iz \in A$,
2. $|vy| > 0$, and
3. $|vxy| \leq p$.

a. i Use this CFL pumping lemma to show that $\{0^n1^n2^n \mid n \geq 0\}$ is not context-free.

ii At a informal handwavy high level, *why* is L not context-free? after all we did add arbitrarily large memory to our automata and we've shown it can be used to count things and recognize languages like $\{0^n1^n \mid n \geq 0\}$, so we can't we handle $\{0^n1^n2^n \mid n \geq 0\}$? What might we need to include in future more powerful automata to recognize more languages like this one?

b. Prove that the class of context-free languages is *not* closed under intersection. *(Hint: Find two CFLs whose intersection is the language from the previous problem. Demonstrate your two languages are indeed CFLs by providing high-level descriptions of PDAs which recognize them.)* Then show the class of CFLs is not closed under complementation.