Exam 3 (Complexity) Additional Practice

a (from Sipser Problem 8.10) The Japanese game go-moku is played by two players, "X" and "O", on a 19×19 grid. Players take turns placing markers, and the first player to achieve five of her markers consecutively in a row, column, or diagonal is the winner. Consider this game generalized to an $m \times m$ board. Let $GM = \{\langle B \rangle \mid B$ is a position in generalized go-moku, where player "X" has a winning strategy$\}$. By a position we mean a board with markers placed on it, such as may occur in the middle of a play of the game, together with an indication of which player moves next. Show that $GM \in SPACE(n^2)$. *(Hint: Note that the input size n is not the same as the board side length m. You can choose to work with ns from the beginning, or instead to think in terms of m first, and then convert to ns at the end.)*

b (from Sipser Problem 7.46) As on the homework, let $MIN\text{–}FORMULA$ be the collection of minimal Boolean formulas. Show that if $P = NP$, then $MIN\text{–}FORMULA \in P$.

c (Sipser Problem 7.15) Show that P is closed under the star operation. (Hint: Use dynamic programming. On input $y = y_1 \cdots y_n$ for $y_i \in \Sigma$, build a table indicating for each $i \leq j$ whether the substring $y_i \cdots y_j \in A^*$.)

d (from Sipser Problem 7.24b)

Let $CNF_3 = \{\langle\phi\rangle \mid \phi$ is a satisfiable cnf-formula where each variable appears in at most 3 places$\}$. Show that CNF_3 is NP-complete. *(Hint: What clauses can you add to a cnf-formula to ensure that two variables must be assigned the same value? Then generalize this strategy to more than two variables.)*

e (from Sipser Problem 7.32) In the *mine consistency problem*, you are given a graph G along with numbers labeling some of G's nodes. You must determine whether a placement of mines on the remaining nodes is possible, so that any node v that is labeled m has exactly m neighboring nodes containing mines. Show that (the language corresponding to) this decision problem is NP-complete. *(Hint: Reduce from* $3SAT$*, where placing a mine means marking something as True. You'll want variable gadgets, clause gadgets, and then a way to make sure that each literal in each clause gadget is* not inconsistent with *the assignment from the variable gadget. You can always create a node that is adjacent to a subset of other nodes of your choice, so it may help to just think in terms of subsets of nodes rather than adjacency.)*