Space Complexity

(from Sipser Definition 8.1) Let M be a (deterministic) TM that halts on all inputs. The space complexity of M is the function $f : \mathbb{N} \to \mathbb{N}$, where $f(n)$ is the maximum number of tape cells that M scans on any input of length n.
If N is an NTM wherein all branches halt on all inputs, we define its space complexity $f(n)$ to be the maximum number of tape cells that N scans on any branch of its computation for any input of length n.
(from Sipser Definition 8.2) Let $f : \mathbb{N} \to \mathbb{R}^+$ be a function.
$SPACE(f(n)) = \{L \mid L$ is a language decided by an $O(f(n))$ space (deterministic) TM$\}$.
$NSPACE(f(n)) = \{L \mid L$ is a language decided by an $O(f(n))$ space NTM$\}$.

a Recall that $SAT = \{\langle\phi\rangle \mid \phi$ is a satisfiable Boolean formula$\}$. Show that $SAT \in SPACE(n)$ (even though SAT is NP-complete!).

b Let $\widetilde{ALL_{NFA}} = \{\langle A\rangle \mid A$ is an NFA and $L(A) \neq \Sigma^*\}$. We will show that $\widetilde{ALL_{NFA}} \in NSPACE(n)$. ($\widetilde{ALL_{NFA}}$ is what I would previously have called *co*–ALL_{NFA}, but this notation was confusing since there are also complexity classes called things like coNP.)

 i Assume that some NFA $N = (Q, \Sigma, \delta, q_0, F)$ rejects at least one string, and let w be some shortest rejected string. Find an upper bound on the length of w in terms of some features of N. *(Hint: first convert the NFA to a DFA)* ii Why will our actual decider for $\widetilde{ALL_{NFA}}$ probably have to work with NFAs directly - why can't we just convert a given NFA into an equivalent DFA, and then run a (presumably-simpler) decider for $\widetilde{ALL_{DFA}}$ on that result?

iii Construct a linear-space NTM deciding $\widetilde{ALL_{NFA}}$. *Hint: the rough strategy is to nondeterministically guess a string of length ≤ your bound from part i, and accept if that string is rejected. But notice that you can't actually store the whole string.*

c We will now show that for $f(n) \geq n$, $NSPACE(f(n)) \subseteq SPACE(f^2(n))$.

i Let $TIMED\text{–}YIELDS_N = \{\langle c_1, c_2, t\rangle \mid N$ can transition from configuration c_1 to c_2 in $\leq t$ steps$\}$. For a given NTM T, we will deterministically decide $L(T)$ as follows: "On input w, construct N from T as follows: ______________________. Then run a decider for $TIMED\text{–}YIELDS_N$ on ______________________ and *accept/reject* as it does." Fill in the blanks. (N should be a relatively small modification of T, with $L(N) = L(T)$.)

ii Construct a space-efficient decider for $TIMED\text{–}YIELDS_N$.